1) Suppose that the market demand is described by

\[ P = 100 - (Q + q) \]

Where \( Q \) is the output of the incumbent firm, \( q \) is the output of the potential entrant and \( P \) is the market price. The incumbent’s cost function is given by

\[ TC(Q) = 40Q \]

While the cost function of the entrant is given by

\[ TC(Q) = 40q + 100 \] \((100 \text{ is a sunk cost paid upon entering the market})\)

a) If the entrant observes the incumbent producing \( Q \) units of output and expects this level to be maintained, what is the equation for the entrant’s residual demand curve?

b) If the entrant maximizes profits using the residual demand in (a), what output will the entrant produce?

c) How much would the incumbent have to produce to keep the entrant out of the market? At what price will the incumbent sell this output?

2) Suppose the inverse demand in an industry is given by

\[ P = 120 - (q_1 + q_2) \]

Where \( q_1 \) is the output of the incumbent firm and \( q_2 \) is the output of the entrant. Let both the labor cost and capital cost be 30. That is,

\[ w = r = 30 \]

In addition, let each firm have a fixed cost of 200.

a) Suppose that in stage one, the incumbent invests in capacity \( K_1 \). Show that in stage two, the incumbent’s best response is

\[ q_1 = 45 - \frac{1}{2}q_2 \] \((\text{for } q_1 < K_1)\)

\[ q_1 = 30 - \frac{1}{2}q_2 \] \((\text{for } q_1 > K_1)\)
b) Show that the entrant’s best response in stage two is \( q_2 = 30 - \frac{1}{2} q_1 \).

c) Show that the monopoly or stackelberg leader output is equal to 30. If the incumbent commits to a production capacity of \( \bar{K}_1 = 30 \), show that in stage two, the entrant will come in and produce a level of output equal to 15. Show that in this case, the entrant earns a profit equal to $25 while the incumbent earns profits equal to $250.

d) Show that if the incumbent instead commits to a production capacity of \( \bar{K}_1 = 40 \) then in stage two, the entrant’s best response is to produce output equal to 10. However, in this case, the entrant earns negative profits.

e) Show that if the incumbent chooses \( \bar{K}_1 = 32 \) in stage one, the entrant can’t earn a positive profit in stage two.

3) Consider the following pricing game.

<table>
<thead>
<tr>
<th>Firm One</th>
<th>Firm Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = $105</td>
<td>P = $105</td>
</tr>
<tr>
<td>P = $130</td>
<td>$7.25</td>
</tr>
<tr>
<td>P = $160</td>
<td>$5.52</td>
</tr>
</tbody>
</table>

Confirm the following is a Nash subgame perfect equilibrium when the firms interact 5 times.

For \( t = 1 \): Set a price of $160

For \( 1 < t < 5 \): Set a price of $160 as long as the previous price combination was ($160, $160). Otherwise, choose a price equal to $105.

For \( t = 5 \): Set a price of $130 as long as the previous combination was ($160, $160). Otherwise, choose a price equal to $105.

4) Which two of the following are most clearly common value auctions items: Viper sports cars, electricity, patent licenses, T Bills, antiques, or fine art?

5) If some auction participants for crude oil fields have estimates that the oil in the ground is worth $1.2M, $1.3M or $1.5M with certainty and other participants have estimates that the oil in the ground is worth $1.1M, $1.3M or $1.5M with certainty and a third group has estimates of $1.1M, $1.2M or $1.3M, and all three forecasts include the true value, what is the value? How would you, as an auctioneer design a set of auction rules to reduce strategic underbidding and realize the true value?