Finance 30210
Problem Set #3 Solutions

1) According to a study by Niccie McKay, the average cost per patient day for nursing homes in the US is

$$C = A - .16X + .00137X^2$$

We want to minimize the cost per patient by our choice of patient days (X).

$$\min_X \left\{A - .16X + .00137X^2\right\}$$

The first order necessary condition (take the derivative with respect to X – emember, A is a constant) is as follows:

$$C'(X) = -.16 + .00274X = 0$$

Solving for X, we get $X = 58.4$. Further, if we take the second derivative, we get

$$C''(X) = .00274 > 0$$

Which is always positive – this guarantees a minimum!

2) Consumer research for Kraft foods has discovered the following relationships between sales of TANG (Yes, the orange drink used by astronauts!) and advertising expenditures for two districts. Sales (S) and advertising (A) are in millions.

$$S_1 = 10 + 5A_1 - 1.5A_1^2$$

$$S_2 = 12 + 4A_2 - .5A_2^2$$

We can write Kraft's maximization problem as:

$$\underset{A_1, A_2}{Max}\; 22 + 5A_1 + 4A_2 - 1.5A_1^2 - .5A_2^2$$
$$subject\ to\ \ A_1 + A_2 \le \$5$$

The lagrangian for this problem is

$$l(A_1, A_2) = 22 + 5A_1 + 4A_2 - 1.5A_1^2 - .5A_2^2 + \lambda(5 - A_1 - A_2)$$

Taking the first order conditions, we get

$$l_{A_1}(A_1, A_2) = 5 - 3A_1 - \lambda = 0$$
$$l_{A_2}(A_1, A_2) = 4 - A_2 - \lambda = 0$$
$$A_1 + A_2 = 5$$

Solving the above system for A_1 and A_2 we get

$$A_1 = 1.5$$
$$A_2 = 3.5$$
$$\lambda = .5$$

The value of the multiplier of .5 tells us that a $1M increase in the adversising budget would raise sales by .5M (given that we are acting optimally).

3) Stafford rug company produces wool rugs and cotton rugs. Total cost (in dollars) is given by

$$C = 7X_1^2 + 9X_2^2 - 1.5X_1X_2$$

We can write Stafford's minimization problem as:

$$\underset{X1, X2}{Min}\{7X_1^2 + 9X_2^2 - 1.5X_1X_2\}$$
$$\textit{subject to } \; X_1 + X_2 \geq 10$$

The lagrangian for this problem is

$$l(X_1, X_2) = 7X_1^2 + 9X_2^2 - 1.5X_1X_2 - \lambda(X_1 + X_2 - 10)$$

Taking the first order conditions, we get

$$l_{X_1}(X_1, X_2) = 14X_1 - 1.5X_2 - \lambda = 0$$
$$l_{X_2}(X_1, X_2) = 18X_2 - 1.5X_1 - \lambda = 0$$
$$X_1 + X_2 = 10$$

Solving for X1, X2 and λ we get

$$X_1 = 5.57$$
$$A_2 = 4.42$$
$$\lambda = 71.35$$

A 50 percent increase in orders (i.e. an increase of 5 rugs would raise Stafford's costs by (approximately) 71.35*5 = $356.75.

4) Consider the following sets of prices. Determine which of the groups is inconsistent with utility maximization.

First, calculate the cost of each bundle at each set of prices:

	Choice A	**Choice B**	**Choice C**
	X = (3,2,1)	X = (2,2,1)	X = (1,2,1)
P = (1,2,3)	**$10**	$9	$8
P = (2,1,2)	$10	**$8**	$6
P = (3,5,1)	$20	$17	**$14**

At P = (1,2,3), A was chosen when B and C were cheaper: A>B, A>C
At P = (2,1,2), B was chosen when C was cheaper: B>C

First, calculate the cost of each bundle at each set of prices:

	Choice A	**Choice B**	**Choice C**
	X = (5,1,3)	X = (3,3,3)	X = (4,2,2)
P = (3,4,1)	**$22**	$24	$22
P = (2,3,2)	$19	**$21**	$18
P = (5,3,1)	$31	$28	**$28**

At P = (3,1,4), A was chosen when C was the same price: A>C
At P = (2,3,2), B was chosen when both A and C were cheaper: B>A, B>C
At P = (5,3,1), C was chosen when B was the same price: C > B

We have a contradiction here. At one set of prices, B>C, but at another, C>B. This is inconsistent with rational behavior!!

	Choice A	**Choice B**	**Choice C**
	X = (2,2,2)	X = (1,3,3)	X = (1,3,2)
P = (4,3,2)	**$18**	$19	$17
P = (5,3,3)	$22	**$23**	$20
P = (5,2,3)	$20	$20	**$17**

At P = (4,3,2), A was chosen when C was cheaper: A>C
At P = (5,3,3), B was chosen when A and C were cheaper: B>C, B>A

5) Suppose that the price of good X is $4 and the price of good Y is $6. You have $100 to spend and your preferences over X and Y are defined as

$$U(x,y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$$

Solve for your optimal choice of X and Y.

I will solve this the long way first, and then the short way we have a problem of the form:

$$\max_{x,y} f(x,y)$$
$$\textit{subject to} \quad g(\mathrm{x},\mathrm{y}) \geq 0$$

Where $f(x,y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$ and $g(x,y) = 100 - 4x - 6y$. Therefore, we can set up the lagrangian as

$$\ell(x,y) = x^{\frac{2}{3}} y^{\frac{1}{3}} + \lambda(100 - 4x - 6y)$$

Taking the first order conditions

$$\ell_x(x,y) = \frac{2}{3} x^{-\frac{1}{3}} y^{\frac{1}{3}} - 4\lambda = 0$$
$$\ell_y(x,y) = \frac{1}{3} x^{\frac{2}{3}} y^{-\frac{2}{3}} - 6\lambda = 0$$

Solving for lambda…

$$\lambda = \frac{2x^{-\frac{1}{3}} y^{\frac{1}{3}}}{12}$$
$$\lambda = \frac{x^{\frac{2}{3}} y^{-\frac{2}{3}}}{18}$$

Setting the above two equal to each other

$$\frac{2x^{-\frac{1}{3}}y^{\frac{1}{3}}}{12} = \frac{x^{\frac{2}{3}}y^{-\frac{2}{3}}}{18}$$

If we simplify this down a bit:

$$\frac{2x^{-\frac{1}{3}}y^{\frac{1}{3}}}{12} = \frac{x^{\frac{2}{3}}y^{-\frac{2}{3}}}{18} \quad \text{(Multiply both sides by 18)}$$

$$3x^{-\frac{1}{3}}y^{\frac{1}{3}} = x^{\frac{2}{3}}y^{-\frac{2}{3}} \quad \text{(Divide both sides by } x^{-\frac{1}{3}}\text{)}$$

$$3y^{\frac{1}{3}} = xy^{-\frac{2}{3}} \quad \text{(Divide both sides by } y^{-\frac{2}{3}}\text{)}$$

$$3y = x$$

We could get to this point quicker by using the rule:

$$MRS = \frac{U_x}{U_y} = \frac{P_x}{P_y}$$

Where

$$U_x = \frac{2}{3}x^{-\frac{1}{3}}y^{\frac{1}{3}}$$

$$U_y = \frac{1}{3}x^{\frac{2}{3}}y^{-\frac{2}{3}}$$

Now use the constraint to solve the rest of the problem.

$$\left.\begin{array}{l} 3y = x \\ 4x + 6y = 100 \end{array}\right\} \quad 4(3y) + 6y = 100 \Rightarrow y = 5.55$$

Y = 5.55, X = 16.65

Note that all income is spent. ($4)(16.65) + ($6)(5.55) = $100. Further, note that you are spending (2/3) of your income on X, (1/3) of your income on Y.