Finance 360
Problem Set #4 Solutions

1) Suppose that you are a firm that produces xylophones. You have a production technology to produce xylophones that can be written as:

$$y = k^{\frac{1}{2}} l^{\frac{1}{2}}$$

Where k represents the units of capital employed at your production facility, l is the number of labor hours employed and y is your total production of xylophones. Assume that labor costs $10 per hour and that capital costs $250 per unit.

In the short run, your capital is fixed (at K = 100). Therefore, your only decision to make is how much labor to employ.

We know that you must produce 1,000 units of output. Therefore, we know that

$$1000 = (100)^{\frac{1}{2}} l^{\frac{1}{2}}$$

We can solve this for labor.

$$1000 = 10 l^{\frac{1}{2}}$$

$$100 = l^{\frac{1}{2}}$$

$$l^{\frac{1}{2}} = (100)^2 = 10,000$$

Therefore, we know that in the short run, you will hire 10,000 hours of labor to go along with you 100 units of capital. To calculate total costs, we must add up expenditures on capital and labor.

$$TC = \$10L + \$250K = \$10(10,000) + \$250(100) = \$125,000$$

Average costs (unit costs) are equal to total costs divided by output (which we know is 1,000).

$$AC = \frac{\$125,000}{1000} = \$125$$

Marginal costs represent the additional costs incurred by producing a little bit more output. This equals

$$MC = \frac{\$10}{F_l(k,l)} = \frac{\$10}{\frac{1}{2}k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \frac{\$10}{\frac{1}{2}(100)^{\frac{1}{2}}(10{,}000)^{-\frac{1}{2}}} = \$200$$

Let's think about this number for a minute. Recall that $F_l(k,l)$ represents the marginal product of labor (MPL). That is, the additional output that each hour of labor can produce. Here, we have

$$MPL = \frac{1}{2}k^{\frac{1}{2}}l^{-\frac{1}{2}} = \frac{1}{2}(100)^{\frac{1}{2}}(10{,}000)^{-\frac{1}{2}} = .05$$

Therefore, each extra hour of labor can produce .05 units of output. Therefore, to produce 1 unit of output would require 20 hours of labor which would cost \$200. (Capital costs aren't included because capital is fixed).

In the long run, you are free to choose both capital and labor. Therefore, we need to set up the maximization problem

$$\min_{k,l}\{\$10L + \$250K\}$$

Subject to the constraint that output is at least 1,000 units. ($k^{\frac{1}{2}}l^{\frac{1}{2}} \geq 1{,}000$)

Setting up the problem:

$$\ell(k,l) = \$10L + \$250K - \lambda\left(k^{\frac{1}{2}}l^{\frac{1}{2}} - 1000\right)$$

Take the derivatives with respect to K and L and set them equal to zero

$$\ell_L(k,l) = \$10 - \lambda\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}} = 0$$

$$\ell_K(k,l) = \$250 - \lambda\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}} = 0$$

Now, solve each for lambda.

$$\frac{\$10}{\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \lambda = \frac{\$250}{\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}}}$$

Rearranging, we get

$$\frac{\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}}}{\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \lambda = \frac{\$250}{\$10}$$

This simplifies to

$$L = 25K$$

The cost minimizing combination of capital and labor is 25 hours of labor for every unit of capital. Now, use the output constraint

$$1000 = k^{\frac{1}{2}}l^{\frac{1}{2}} = k^{\frac{1}{2}}(25k)^{\frac{1}{2}}$$

Solve for k (and l)

$$k = \frac{1000}{5} = 200 \qquad l = 25k = 5{,}000$$

IN the long run, your optimal scale is 200 units of capital and 5,000 hours of labor. Now, if we recalculate your costs

$$TC = \$10L + \$250K = \$10(5{,}000) + \$250(200) = \$100{,}000$$

$$AC = \frac{\$100{,}000}{1000} = \$100$$

$$MC = \frac{\$10}{F_l(k,l)} = \frac{\$10}{\frac{1}{2}k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \frac{\$10}{\frac{1}{2}(200)^{\frac{1}{2}}(5{,}000)^{-\frac{1}{2}}} = \$100$$

A couple points to note. First, notice that you have dramatically lowered your costs by adjusting your capital stock. Second, note that AC = MC. This is due to the fact that your production function has constant returns to scale.

$$2y = (2k)^{\frac{1}{2}}(2l)^{\frac{1}{2}}$$

2) Suppose that you have two industries, each of witch has its own production function

Industry A: $y_A = k_A^{.25} l_A^{.35}$

Industry B: $y_B = k_B^{.55} l_B^{.65}$

a) Describe what each industries marginal costs should look like in the short run (i.e. when capital is fixed) – specifically, which industry's marginal costs are increasing at a faster rate?

Calculate each industry's marginal product of labor ($F_l(k,l)$)

$$MPL_A = (.35)k_A^{.25} l_A^{-.65\cdot}$$

$$MPL_B = (.65)k_B^{.55} l_B^{-.35}$$

Note that industry A's MPL falls at a faster rate than industry B (labor's productivity – output per hour). Therefore, industry A's costs will rise at a faster rate as more output is produced.

b) If both of these industries are perfectly competitive, which industry should have a lower elasticity of supply (price elasticity)

With industry A's costs rising at a faster rate (as output increases), they will have to charge a higher price – their supply will be less elastic. (they need larger price changes that industry B to increase production.)

c) What should these two industry's cost functions look like in the long run (i.e. when capital adjustments can be made)?

Industry A's cost functions (marginal and average) will be increasing in the long run because their technology exhibits decreasing returns to scale (raising production raises average costs)

Industry B's costs will be decreasing because their technology exhibits increasing returns to scale (raising production lowers average cost)

d) Which of the two industries would you expect to be monopolized by one or a few firms? Why?

In industry B, large firms have a natural cost advantage over smaller firms. Therefore, we would most likely see one or a few large firms in this industry.

3) Suppose that you are operating a firm with constant marginal costs of production equal to $5 and no fixed costs. You are facing a demand with a constant price elasticity of -3.

a) Calculate your optimal (i.e profit maximizing) price.

The markup rule is

$$P = \frac{MC}{\left(1 + \frac{1}{\varepsilon}\right)}$$

Where ε is the elasticity of demand. In this case, the firm should charge

$$P = \frac{\$5}{\left(1 - \frac{1}{3}\right)} = \$750$$

b) What would your firm's Lerner index be?

$$LI = \frac{P - MC}{P} = \frac{\$7.50 - \$5}{\$7.50} = .33$$

c) Due to easy entry to the market, you would expect your industry to become more competitive over time. What should happen to your profit maximizing price in the long run?

Over time, as more and more firms enter the market, the markup should fall to zero (P = $5)

4) Suppose that you have an industry with 5 firms. Below are the market shares of each firm:

Firm	Market Share
1	35
2	25
3	15
4	15
5	10

a) Calculate the concentration ratios for this industry.

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1	35
2	60 (= 35+25)
3	75 (= 60 + 25)
4	90 (= 75 + 10)
5	100 (= 90 +10)

b) Calculate the Herfindahl-Hirschman index for this industry.

Firm	Market Share	Square of MS
1	35	1225
2	25	625
3	15	225
4	15	225
5	10	100

HHI = 2,400