1) Suppose that the demand curve for video rentals has been estimated to be

\[ Q = 2500 - 250P \]

Further, your average costs of supplying videos is equal to

\[ AC = 8 - 0.006Q + 0.000002Q^2 \]

Calculate your optimal price, quantity and profits.

First, derive the inverse demand curve (solve demand for price)

\[
\begin{align*}
Q &= 2500 - 250P \\
250P &= 2500 - Q \\
P &= 10 - 0.004Q
\end{align*}
\]

Total revenues equal price times quantity.

\[
\begin{align*}
P &= 10 - 0.004Q \\
TR &= PQ = 10Q - 0.004Q^2
\end{align*}
\]

Marginal revenue is the derivative of total revenue

\[
\begin{align*}
TR &= PQ = 10Q - 0.004Q^2 \\
MR &= 10 - 0.008Q
\end{align*}
\]

To get total cost, multiply average cost by quantity

\[
\begin{align*}
AC &= 8 - 0.006Q + 0.000002Q^2 \\
TC &= 8Q - 0.006Q^2 - 0.000002Q^3
\end{align*}
\]

Marginal cost is the derivative of total cost

\[
\begin{align*}
TC &= 8Q - 0.006Q^2 - 0.000002Q^3 \\
MC &= 8 - 0.012Q - 0.000006Q^2
\end{align*}
\]

Now, set marginal revenue equal to marginal cost and solve for Q
\[ 8 - 0.012Q - 0.000006Q^2 = 10 - 0.008Q \]
\[ 0.000006Q^2 - 0.004Q - 2 = 0 \]
\[ Q = 1,000 \]

Substitute quantity into the demand curve to get \( P = 6 \).

2) Suppose that you are a monopoly faced with a demand curve given by

\[ Q = 100 - 2P \]

You have a constant marginal cost equal to \$10. Calculate your optimal price and quantity. Show that your price adheres to the optimal markup rule based on demand elasticity.

\[ Q = 100 - 2P \]
\[ P = 50 - 0.5Q \]
\[ TR = PQ = 50Q - 0.5Q^2 \]
\[ MR = 50 - Q = MC = 10 \]
\[ Q = 40 \]
\[ P = 30 \]

Now, calculate the elasticity at \( P = 30, Q = 40 \)

\[ \varepsilon = \frac{dQ}{dP} \frac{P}{Q} = 2 \left( \frac{30}{40} \right) = 1.5 \]

Now plug 1.5 into the markup rule

\[ P = \frac{MC}{1 + \frac{1}{\varepsilon}} = \frac{10}{1 + \frac{1}{1.5}} = \frac{10}{1 + \frac{1}{1.5}} = \frac{10}{1 + \frac{1}{3}} = 30 \]

3) Suppose that the demand for loans depends on the annual interest rate charged \( (r_i) \), the annual fees charged \( (F) \), and the unemployment rate. Each loan is a \$100,000, 30 year fixed APR mortgage.

\[ Q = 125 - 624r_i - 0.026F - 90.4UR \]
The bank has a monthly fixed cost of $10,000 plus an annual variable cost (interest paid on deposits plus various administrative expenses equal to 5% of the loans created)

a) Suppose that the bank charges nothing in fees. Solve for the profit maximizing interest rate. What are the banks monthly profits?
b) Calculate the interest elasticity of loan demand at the profit maximizing point.
c) Now, suppose that the bank decides to charge $1200 per year in Fees. Calculate the bank’s profit maximizing interest rate and monthly profits.
d) How would (c) change if the Fee were a one time (i.e. closing costs)?

With no fees, we have the following demand for loans:

\[ Q = 125 - 624r_i - 90.4UR \]

First, to simplify, set the unemployment rate to 5% (.05)

\[ Q = 125 - 624r_i - 90.4(0.05) = 120.48 - 624r_i \]

To calculate the elasticity, we first need the derivative with respect to the interest rate.

\[ \frac{dQ}{dr_i} = -624 \]

Next, divide by the quantity of loans and multiply by the interest rate to get the elasticity.

\[ \frac{dQ}{dr_i} \frac{r_i}{Q} = -624 \left( \frac{r_i}{Q} \right) \]

To get total revenues as a function of \( L \), first solve the demand curve for the interest rate.

\[ r_i = \left( \frac{120.48}{624} \right) - \left( \frac{1}{624} \right)Q = .193 - .0016Q \]

Monthly revenues equal the interest rate charged (divided by twelve) times the quantity of loans issued times $100,000.

\[ TR = \$100,000(Q) \frac{r_i}{12} = 1608Q - 13Q^2 \]

Marginal revenue is the derivative with respect to \( L \)
\[ MR = 1608 - 2(13)Q = 1608 - 26Q \]

Now, take the cost function

\[ TC = 10,000 + \left( \frac{.05}{12} \right)(100,000)Q = 10,000 + 416Q \]

Marginal cost is the derivative with respect to \( Q \)

\[ MC = 416 \]

Now, to get the optimal amount of loans, set MR=MC and solve for \( Q \)

\[ 1608 - 26Q = 416 \]
\[ Q = 46 \]

Now, given \( L \), the interest rate can be found using the demand curve.

\[ r_i = .193 - .0016Q = .193 - .0016(46) = .1194 = 11.94\% \]

Therefore,

\[ TR = 100,000 \left( \frac{.1194}{12} \right) 46 = 45,770 \]

\[ TC = 10,000 + 416(46) = 29,136 \]

Profits = $16,634

At the profit maximizing point, elasticity of demand is

\[ \frac{dQ}{dr_i} \frac{r_i}{Q} = -624 \left( \frac{.1194}{46} \right) = 1.61 \]

If we add the fees, the procedure is the same, by the demand curve becomes:

\[ Q = 125 - 624r_i - .026(1200) - 90.4(.05) = 89 - 624r_i \]

Solving for the interest rate, we get

\[ r_i = \left( \frac{89}{624} \right) - \left( \frac{1}{624} \right)Q = .1431 - .0016Q \]
Total Revenues now include interest income and fee income:

\[ TR = \$100,000(Q) \frac{r_i}{12} + \left( \frac{\$1200}{12} \right) Q = 1292Q - 13Q^2 \]

Total Costs are unchanged at

\[ TC = 10,000 + \left( \frac{.05}{12} \right) (100,000) Q = 10,000 + 416Q \]

Set marginal revenue equal to marginal cost as in part (a) and the optimum is 33 loans and an interest rate of 9%. Profits are equal to $4322.

In part (d), things get interesting. The demand curve doesn’t change, but now, Total revenues become:

\[ TR = \$100,000(Q) \frac{r_i}{12} + (\$1200) Q = 2392Q - 13Q^2 \]

Because all the fees are being paid up front. The optimal interest rate becomes 2.5%, 74 loans are created, and profits are $63,000!

4) Suppose the demand for Bananas is given by

\[ Q = 50 - 5P \]

The marginal cost of producing bananas is equal to $2.

a) Calculate the price and quantity that would occur if this was a perfectly competitive market.
b) Calculate the price and quantity a monopoly would produce
c) Calculate the difference in consumer surplus between a perfectly competitive outcome and a monopolistic outcome

A perfectly competitive firm sets price equal to marginal cost ($2). At this price, we have sales equal to

\[ Q = 50 - 5(2) = 40 \]

And Consumer surplus = \( \frac{1}{2} \)(40)(10 − 2) = $160

A monopolist produces Q = 20 and charges a price equal to $6.

Consumer surplus equals \( \frac{1}{2} \)(20)(6-2) = $40.
Note that some of that goes to profits which equal \((6-2)(20) = $80\) while the remaining $40 is lost.

5) What characteristics are important when determining whether a market will be competitive or monopolistic?

- Increasing returns to scale (decreasing costs)
- Network Externalities
- Small Markets
- Government protection