

Finance 360 Problem Set #5 Solutions

1) Suppose that the demand curve for video rentals has been estimated to be

$$Q = 2500 - 250P$$

Further, your average costs of supplying videos is equal to

$$AC = 8 - .006Q + .000002Q^2$$

Calculate your optimal price, quantity and profits.

First, derive the inverse demand curve (solve demand for price)

$$Q = 2500 - 250P$$

$$250P = 2500 - Q$$

$$P = 10 - .004Q$$

Total revenues equal price times quantity.

$$P = 10 - .004Q$$

$$TR = PQ = 10Q - .004Q^2$$

Marginal revenue is the derivative of total revenue

$$TR = PQ = 10Q - .004Q^2$$

$$MR = 10 - .008Q$$

To get total cost, multiply average cost by quantity

$$AC = 8 - .006Q + .000002Q^2$$

$$TC = 8Q - .006Q^2 - .000002Q^3$$

Marginal cost is the derivative of total cost

$$TC = 8Q - .006Q^2 - .000002Q^3$$

$$MC = 8 - .012Q - .000006Q^2$$

Now, set marginal revenue equal to marginal cost and solve for Q

$$8 - .012Q - .000006Q^2 = 10 - .008Q$$

$$.000006Q^2 - .004Q - 2 = 0$$

$$Q = 1,000$$

Substitute quantity into the demand curve to get P=\$6.

- 2) Suppose that you are a monopoly faced with a demand curve given by

$$Q = 100 - 2P$$

You have a constant marginal cost equal to \$10. Calculate your optimal price and quantity. Show that your price adheres to the optimal markup rule based on demand elasticity.

$$Q = 100 - 2P$$

$$P = 50 - .5Q$$

$$TR = PQ = 50Q - .5Q^2$$

$$MR = 50 - Q = MC = 10$$

$$Q = 40$$

$$P = \$30$$

Now, calculate the elasticity at P = \$30, Q = 40

$$\varepsilon = \frac{dQ}{dP} \frac{P}{Q} = 2 \left(\frac{30}{40} \right) = 1.5$$

Now plug 1.5 into the markup rule

$$P = \frac{MC}{\left(1 + \frac{1}{\varepsilon}\right)} = \frac{\$10}{\left(1 - \frac{1}{1.5}\right)} = \frac{\$10}{\frac{1}{3}} = \$30$$

- 3) Suppose that the demand for loans depends on the annual interest rate charged (r_l), the annual fees charged (F), and the unemployment rate. Each loan is a \$100,000, 30 year fixed APR mortgage.

$$Q = 125 - 624r_l - .026F - 90.4UR$$

The bank has a monthly fixed cost of \$10,000 plus an annual variable cost (interest paid on deposits plus various administrative expenses equal to 5% of the loans created)

- Suppose that the bank charges nothing in fees. Solve for the profit maximizing interest rate. What are the banks monthly profits?
- Calculate the interest elasticity of loan demand at the profit maximizing point.
- Now, suppose that the bank decides to charge \$1200 per year in Fees. Calculate the bank's profit maximizing interest rate and monthly profits.
- How would (c) change if the Fee were a one time (i.e. closing costs)?

With no fees, we have the following demand for loans:

$$Q = 125 - 624r_l - 90.4UR$$

First, to simplify, set the unemployment rate to 5% (.05)

$$Q = 125 - 624r_l - 90.4(.05) = 120.48 - 624r_l$$

To calculate the elasticity, we first need the derivative with respect to the interest rate.

$$\frac{dQ}{dr_l} = -624$$

Next, divide by the quantity of loans and multiply by the interest rate to get the elasticity.

$$\frac{dQ}{dr_l} \frac{r_l}{Q} = -624 \left(\frac{r_l}{Q} \right)$$

To get total revenues as a function of L, first solve the demand curve for the interest rate.

$$r_l = \left(\frac{120.48}{624} \right) - \left(\frac{1}{624} \right) Q = .193 - .0016Q$$

Monthly revenues equal the interest rate charged (divided by twelve) times the quantity of loans issued times \$100,000.

$$TR = \$100,000(Q) \frac{r_l}{12} = 1608Q - 13Q^2$$

Marginal revenue is the derivative with respect to L

$$MR = 1608 - 2(13)Q = 1608 - 26Q$$

Now, take the cost function

$$TC = 10,000 + \left(\frac{.05}{12}\right)(100,000)Q = 10,000 + 416Q$$

Marginal cost is the derivative with respect to Q

$$MC = 416$$

Now, to get the optimal amount of loans, set MR=MC and solve for Q

$$1608 - 26Q = 416$$

$$Q = 46$$

Now, given L, the interest rate can be found using the demand curve.

$$r_i = .193 - .0016Q = .193 - .0016(46) = .1194 = 11.94\%$$

Therefore,

$$TR = \$100,000 \left(\frac{.1194}{12}\right) 46 = \$45,770$$

$$TC = 10,000 + 416(46) = \$29,136$$

$$\text{Profits} = \$16,634$$

At the profit maximizing point, elasticity of demand is

$$\frac{dQ}{dr_i} \frac{r_i}{Q} = -624 \left(\frac{.1194}{46}\right) = 1.61$$

If we add the fees, the procedure is the same, by the demand curve becomes:

$$Q = 125 - 624r_i - .026(1200) - 90.4(.05) = 89 - 624r_i$$

Solving for the interest rate, we get

$$r_i = \left(\frac{89}{624}\right) - \left(\frac{1}{624}\right)Q = .1431 - .0016Q$$

Total Revenues now include interest income and fee income:

$$TR = \$100,000(Q) \frac{r_i}{12} + \left(\frac{\$1200}{12} \right) Q = 1292Q - 13Q^2$$

Total Costs are unchanged at

$$TC = 10,000 + \left(\frac{.05}{12} \right) (100,000) Q = 10,000 + 416Q$$

Set marginal revenue equal to marginal cost as in part (a) and the optimum is 33 loans and an interest rate of 9%. Profits are equal to \$4322.

In part (d), things get interesting. The demand curve doesn't change, but now, Total revenues become:

$$TR = \$100,000(Q) \frac{r_i}{12} + (\$1200)Q = 2392Q - 13Q^2$$

Because all the fees are being paid up front. The optimal interest rate becomes 2.5%, 74 loans are created, and profits are \$63,000!

4) Suppose the demand for Bananas is given by

$$Q = 50 - 5P$$

The marginal cost of producing bananas is equal to \$2.

- Calculate the price and quantity that would occur if this was a perfectly competitive market.
- Calculate the price and quantity a monopoly would produce
- Calculate the difference in consumer surplus between a perfectly competitive outcome and a monopolistic outcome

A perfectly competitive firm sets price equal to marginal cost (\$2). At this price, we have sales equal to

$$Q = 50 - 5(2) = 40$$

And Consumer surplus = $(1/2)(40)(10 - 2) = \$160$

A monopolist produces $Q = 20$ and charges a price equal to \$6.

Consumer surplus equals $(1/2)(20)(6-2) = \$40$.

Note that some of that goes to profits which equal $(\$6-\$2)(20) = \$80$ while the remaining \$40 is lost.

- 5) What characteristics are important when determining whether a market will be competitive or monopolistic?
- Increasing returns to scale (decreasing costs)
 - Network Externalities
 - Small Markets
 - Government protection