Finance 360
Problem Set #6 Solutions

1) Suppose that you are the manager of an opera house. You have a constant marginal cost of production equal to $50 (i.e. each additional person in the theatre raises your costs by $50 – we will ignore any fixed costs for now.) You have estimated your demand curve for tickets as follows:

$$Q = 150 - P$$

   a) Calculate your profit maximizing ticket price.

Here, we simply take the demand curve as given, and solve for price:

$$P = 150 - Q$$

Now, Total revenues are Price*Quantity

$$PQ = 150Q - Q^2$$

Marginal costs are the derivative with respect to Q.

$$MR = 150 - 2Q$$

Set marginal revenue equal to marginal cost and solve for Q

$$\begin{aligned} 50 &= 150 - 2Q \\ Q &= 50 \end{aligned}$$

Plugging Q=50 back into the demand curve, which gives us a price of $100.

Profits = $100(50) - $50(50) = $2500

Now, suppose that you re-estimated your demand curve, but this time, you included a dummy variable for gender:

$$Q = 175 - P - 50D$$

$$D = \begin{cases} 1, \text{if consumer is male} \\ 0, \text{if consumer is female} \end{cases}$$

   b) Given your new information, calculate your profit maximizing price assuming that you can't distinguish between male and female customers (i.e. your tickets are sold online)

First we need to aggregate the two demand curves

$$Q = \begin{cases} 175 - P, & Q \le 50 \\ 300 - 2P, & \text{Q} \ge 50 \end{cases}$$

Now, calculate thee two inverse demand pieces:

$$P = \begin{cases} 175 - Q, & Q \le 50 \\ 150 - .5Q, & \text{Q} \ge 50 \end{cases}$$

Now, calculate TR for each piece

$$TR = PQ = \begin{cases} 175Q - Q^2, & Q \le 50 \\ 150Q - .5Q^2, & \text{Q} \ge 50 \end{cases}$$

Now, calculate marginal revenue for each piece:

$$MR = \begin{cases} 175 - 2Q, & Q \le 50 \\ 150 - Q, & \text{Q} \ge 50 \end{cases}$$

Lastly, set marginal revenue equal to marginal cost. Note that with MC = $50, you will always sell at least 50 tickets, so the only piece of the demand curve that matters is that with Q>50.

$$\begin{aligned} 50 &= 150 - Q \\ Q &= 100 \\ P &= \$100 \end{aligned}$$

Profits = $100(100) - $50(100) = $5,000

c) Now, calculate the prices you would charge if you could distinguish between male and female consumers (i.e. ticket purchasers show up to the box office to buy tickets.) Why might you be concerned about secondary markets forming for your product?

In This problem, just treat the individual demand curves separately, and solve for the price and quantity separately.

For Females:

$$Q = 175 - P$$
$$P = 175 - Q$$
$$TR = 175Q - Q^2$$
$$MR = 175 - 2Q = 50$$
$$Q = 62.5$$
$$P = 112.5$$

For Males:

$$Q = 125 - P$$
$$P = 125 - Q$$
$$TR = 125Q - Q^2$$
$$MR = 125 - 2Q = 50$$
$$Q = 37.5$$
$$P = 87.5$$

Profits = $112.5(62.5) + $87.5(37.5) - $50(100) = $5312.50

2) Continuing with the same example, suppose again, that you are faced with the same demand curve(s)

$$Q = \begin{cases} 175 - P, & \text{(women)} \\ 125 - P, & \text{(men)} \end{cases}$$

a) Suppose that you charged the same price to each consumer (the price calculated in part (b) above. Calculate the consumer surplus for both consumer types.

   Given a $100 price, females buy 75 tickets and generate a consumer surplus of (1/2)(175-100)(75) = $2,812.50 while men buy 25 tickets and generate a CS of (1/2)(125-100)(25) = $312.50.

b) Suppose that you were to set a price equal to your marginal cost. Calculate the consumer surplus derived by both consumers.

   Given a $50 price, females buy 125 tickets and generate a consumer surplus of (1/2)(175-50)(125) = $7812.50 while men buy 75 tickets and generate a CS of (1/2)(125-50)(75) = $2812.50.

c) If you could distinguish between the types of consumers, how would you set up your prices to maximize profits? (i.e. you could start up an "opera lover's society" and charge a membership fee)

The ideal situation would be to set up a membership fee for the opera society. Women would pay a membership equal to $7812.50 which would allow them to buy opera tickets for $50 apiece while men would pay a membership price of $2812.50 and would be able to buy tickets at $50. To make sure everybody joins, set a ridiculously high price for non-member ticket prices (say, $1,000 per ticket.)

d) How would your answer to (b) change if you could not distinguish between customer types? (i.e. you could sell different ticket packages.

If we sell a package of 75 tickets for $2,812.50 + 75($50) = $6562,50, then men would buy the package, and get no consumer surplus.
Suppose that a woman bought the 75 ticket package:

We can figure up the total willingness of a women to pay for 75 tickets (using the female demand curve), we get $10,312.50. Subtract off the $6562.50 cost of the 75 ticket package and we find that a women gets a CS of $3750 buying the 75 ticket package. We need to make sure she gets at least a CS of $3750 with the 125 ticket package.

Total Willingness to pay for 125 tickets: $14,062.50:
Minus required surplus: $3750
125 Ticket Package: $10,312.50

3) Suppose that you are George Lucas. You are in the process of packaging the final trilogy (actually the three prequels) of Star Wars for sale to the public. Your marginal costs of production are $2 per movie. Further, you know that there are two types of consumers that you face: children under the age of 10 and everybody else.

Children under 10: Love Jar Jar Binks
Everybody else: Would like to see Jar Jar crushed by a very large truck

Consequently, willingness to pay for each of the three movies is based on how many minutes Jar Jar is on the screen.

Movie	Under 10yrs old	Over 10 yrs Old
Episode 1	$60	$5
Episode 2	$30	$40
Episode 3	$10	$50

a) If you sold these three movies separately, what would your prices be?

Take the first movie:

Movie	Sales	Total Revenues
P=$5	2	$10
P=$60	1	$60

We should charge a price of $60 to maximize revenues. Using similar logic, we should charge a price of $30 for the second and $50 for the third. Calculate profits:

Profits = ($60)(1) + ($30)(2) + ($50)(1) - $2(4) = $162

b) If you only sold these movies as a box set, what should you charge?

As a box set, we can add up each consumers willingness to pay over the three movies:

Movie	Under 10	Over 10
Box Set	$100	$95

Setting a box set price equal to $95 get us two sales (6 total movies sold)

Profits = 2($95) - $2(6) = $178.

4) Suppose that you have two manufacturers: one company specializes in the production of left shoes (They have a store called “The Left Shoe Emporium”). Another company specializes in right shoes (“Right Shoes ‘R’ Us”). Consumers have a demand for shoes given by:

$$Q = 150 - P$$

Where P is the price of a pair of shoes: $P = P_L + P_R$. For simplicity, assume that marginal costs for each firm constant and equal to zero.

a) Write down each firm’s inverse demand curve (They price they can charge as a function of sales and their competitor’s price)

Take the “Left Shoe Emporium” First: They face the demand curve

$$Q = (150 - P_R) - P_L$$

Solving for P, we get

$$P_l = (150 - P_R) - Q_L$$

Total revenues are equal to P*Q

$$TR = P_l Q = (150 - P_R)Q - Q_L^2$$

Marginal revenue is the derivative.

$$MR = (150 - P_R) - 2Q_L$$

Setting this equal to zero (we assumed MC=0) and solving for Q

$$Q_l = \frac{150 - P_R}{2}$$

Substituting back into the demand curve gives us price:

$$P_l = \frac{150 - P_R}{2}$$

Similarly, repeating the process for Right Shoes are us:

$$P_R = \frac{150 - P_L}{2}$$

b) Solve each firm's profit maximization problem as a function of their competitor's price.

In equilibrium, both of the above expressions for price must be true: Take the first expression and plug it into the second:

$$P_R = P_R = \$50$$

Note that the cost of a pair of shoes is $100.

Now, suppose that these two companies merged. What would happen to the cost of a pair of shoes?

The monopolist faces a demand equal to

$$Q = 150 - P$$

Just as in previous exercises, solving for the optimum price gives up P = $75.

5) Spatial competition is an example of *non-price* competition. Specifically, in addition to the price firm charges, it also chooses a location in which to set up shop.

   a) Describe how this firm location problem is solved.

      Basically, it is assumed that consumers pay a money price of a product as well as a time cost for a product (traveling costs, etc). Therefore, if a consumer's reservation price is equal to the sum of the two, then consumers will pay higher prices at stores that are close top them. Firms, therefore, must decide (based of consumer's locations, where to builds their store to maximize profits).

   b) Suppose that you are a cosmetic manufacturer. How could you use a spatial competition model when choosing your cosmetic line?

      The same argument as above, however instead of physical location, we are talking about preference location (i.e. each consumer has an "optimal" version of a product). The closer a firm can locate to that "optimal" product, the more the firm can charge. Therefore the firm must decide how many version of a product to make and specifically what types to maximize profits.