Finance 360
Problem Set #7 Solutions

1) Consider the familiar "Rock, Paper, Scissors" game. Two players indicate either "Rock", "Paper", or "Scissors" simultaneously. The winner is determined by

- Rock crushes scissors
- Paper covers rock
- Scissors cut paper

Indicate a -1 if you lose and +1 if you win. Write down the strategic (matrix) form of the game. What is the Nash equilibrium of the game?

Here's the strategic form of the game (a description of the payouts from each combination of moves) – Player One's payouts are in bold.

		Player Two		
		Rock	**Paper**	**Scissors**
Player One	**Rock**	**0** 0	**-1** 1	**1** -1
	Paper	**1** -1	**0** 0	**-1** 1
	Scissors	**-1** 1	**1** -1	**0** 0

Note that neither player has a dominant strategy.

- If Player one chooses rock, Player two should play paper
- If Player one chooses paper, Player two responds with scissors
- If Player one chooses scissors, Player two chooses rock

Further, this game is symmetric, so Player two's optimal responses are the same. Both players randomly select rock, paper, or scissors

In an episode of Seinfeld, Kramer played a version of this game with his friend Mickey except that the rules were a little different:

- Rock crushes scissors
- Rock Flies Right through paper
- Scissors cut paper

How does this modification alter the Nash equilibrium of the game?
Here's the strategic form of the game (a description of the payouts from each combination of moves) – Player One's payouts are in bold.

		Player Two		
Player One		**Rock**	**Paper**	**Scissors**

	Rock	0	0	1	-1	1	-1
	Paper	-1	1	0	0	-1	1
	Scissors	-1	1	1	-1	0	0

Note that neither player has a dominant strategy.

- If Player one chooses rock, Player two should choose rock
- If Player one chooses paper, Player two responds with rock (or paper)
- If Player one chooses scissors, Player two responds with rock

Notice that playing rock is a dominant strategy for both players (i.e its best to choose rock, regardless of what your opponent is playing!

Therefore, the equilibrium for this game is unique:

Both players always select rock.

This was confirmed in Seinfeld.

2) Consider the following version of the prisoners dilemma game (Player one's payoffs are in bold):

		Player Two	
		Cooperate	Cheat
Player One	Cooperate	**$10** $10	**$0** $12
	Cheat	**$12** $0	**$5** $5

a) What is each player's dominant strategy? Explain the Nash equilibrium of the game.

Start with player one:

- If player two chooses cooperate, player one should choose cheat ($12 versus $10)
- If player two chooses cheat, player one should also cheat ($0 versus $5).

Therefore, the optimal strategy is to always cheat (for both players) this means that (cheat, cheat) is the only nash equilibrium.

b) Suppose that this game were played three times in a row. Is it possible for the cooperative equilibrium to occur? Explain.

If this game is played multiple times, then we start at the end (the third playing of the game). At the last stage, this is like a one shot game (there is no future). Therefore, on the last day, the dominant strategy is for both players to cheat. However, if both parties know that the other will cheat on day three, then there is no reason to cooperate on day 2. However, if both cheat on day two, then there is no incentive to cooperate on day one.

c) Now, suppose that this game was played an infinite number of times. For what values of the interest rate is the present value of cooperating higher than the value of cheating (so that a cooperative equilibrium could occur)

Once the possibility of an "endgame scenario" disappears, there is an incentive to cooperate.

Suppose that player 2 follows the following strategy:

"I will cooperate today. If player one cooperates, I will trust him and cooperate forever. However, if player 1 cheats, then I will never trust him again."

Now consider player 1's response:

- If I cooperate, I get $10 a year forever.
- If I cheat, I get $12 today and then $5 a year forever.

Player one will cheat if the present value of cheating is greater than the present value of cooperating.

$$\$12 + \frac{\$5}{i} > \$10 + \frac{\$10}{i}$$

Player one will cheat if the interest rate is bigger than 250%.

3) Consider the following bargaining problem: $20 dollars needs to be split between Jack and Jill. Jill gets to make an initial offer. Jack then gets to respond by either accepting Jill's initial offer or offering a counter offer. Finally, Jill can respond by either accepting Jakes offer or making a final offer. If Jake does not accept Jill's final offer both Jack and Jill get nothing. Jack discounts the future at 10%

(i.e. future earnings are with 10% less than current earnings while Jill discounts the future at 20%. Calculate the Nash equilibrium of this bargaining problem.

The key to each of these games is as follows: At any stage, the offer made needs to be acceptable to both parties. We need to work backwards:

Stage 3: Note that if Jack rejects Jill's offer at this stage, the money disappears. Therefore, Jack will accept anything positive.

Jill offers: $20 to herself, $0 to Jack

Stage 2: Now, Jack must make an offer that Jill will accept (if the game gets to stage three, Jack gets nothing). Jill is indifferent between $20 in one year and $16 today (she discounts the future at 20%).

Jack offers: $16 to Jill, $4 to Himself

Stage 3: Now, Jill must make an offer that Jack will accept (and is preferable to her – if this is not possible, then she will make an offer jack rejects and the game goes to stage 2). Jack is indifferent between $4 in one year and $3.60 today (he discounts the future at 10%). Note that $16.40 is preferred by Jill to $16 in one year.

Jill offers: $16.40 to herself, $3.60 to Jack

4) Consider a variation on the previous problem:

You and your sister have just inherited $3M that needs to be split between the two of you.

The rules are the same as above (offer, counteroffer, and final offer) except that each period, $1M is removed from the total (each round of negotiation costs $1M in lawyers fees). Further, assume that both you and your sister value future payments just as much as current payments (i.e. no discount factor). Calculate the Nash equilibrium for this game.

Stage 3: Note that if your sister rejects your offer at this stage, the money disappears. Therefore, your sister will accept anything positive.

You offer: $1M to you, $0 to your sister

Stage 2: Now, your sister must make an offer that you will accept (if the game gets to stage three, she gets nothing). If it gets to stage three, you get $1M.

Your sister offers: $1M to you, $1M to her

Stage 3: Now, you must make an offer that your sister will accept (and is preferable to you – if this is not possible, then you will make an offer she rejects and the game goes to stage 2). Your sister gets $1M if the game reaches stage two.

You offer: $2M to you, $1M to your sister

5) Consider yet, another variation of the previous problem: Same rules as in (4), However, this time, you learn something about your sister: You discover that your sister has always hated you. All she cares about with regards to splitting the $3M is that she gets more than you do (i.e. an allocation of $500,000 for you and $1M for her is preferred by her to an allocation of $1.5M apiece!). Calculate the new Nash equilibrium of the game.

Stage 3: Note that now, your sister's happiness is based on relative earnings (earnings relative to you). You must come up with an offer she will accept or you both get nothing.

You offer: $500K to you, $500K to your sister

Stage 2: Now, your sister must make an offer that you will accept (if the game gets to stage three, she gets $500K). If it gets to stage three, you get $500K.

Your sister offers: $500K to you, $1.5M to her (three to one ratio)

Stage 3: Now, you must make an offer that your sister will accept (and is preferable to you – if this is not possible, then you will make an offer she rejects and the game goes to stage 2). Your sister gets $1.5M if the game reaches stage two.

You offer: $750,000 to you, $2.25M to your sister (three to one ratio)