Finance 360
Problem Set #8 Solutions

1) Consider the game of chicken. Two players drive their cars down the center of the road directly at each other. Each player chooses SWERVE or STAY. Staying wins you the admiration of your peers (a big payoff) only if the other player swerves. Swerving loses face if the other player stays. However, clearly, the worst output is for both players to stay! Specifically, consider the following payouts.

   (Player one's payoffs are in bold):

		Player Two	
		Stay	Swerve
Player One	Stay	**-6** -6	**2** -2
	Swerve	**-2** 2	**1** 1

   a) Does either player have a dominant strategy? Explain.

      In this case, neither player has a dominant strategy. Suppose player two chooses to stay. Then player one's best response is to swerve (-6 vs. -2). However, if player two swerves, then player one should stay (2 vs. 1).

   b) Suppose that Player B has adopted the strategy of Staying 1/5 of the time and swerving 4/5 of the time. Show that Player A is indifferent between swerving and staying.

      We need to show that if player B follows the strategy (stay = ¼, swerve = 5/4) then player A is indifferent between swerving and staying. If we calculate the expected reward to player A from staying/swerving, we get

      $E(stay) = (1/5)(-6) + (4/5)(2) = 2/5$

      $E(swerve) = (1/5)(-2) + (4/5)(1) = 2/5$

      They are in fact equal.

   c) If both player A and Player B use this probability mix, what is the chance that they crash?

      Both players are staying 1/5 of the time. Therefore, the probability that the crash (stay, stay) is (1/5)(1/5) = 1/25 = 4%.

2) The two most common paradigms for strategic interaction betweens firms are cournot competition and Bertrand competition. Briefly describe the assumptions

underlying the two models. What industries would you classify as Bertrand? What industries would you classify as cournot? In which of the two models is competitions the "fiercest"??

Cournot: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses production levels conditional on what they expect their rival's production levels to be. The nash equilibrium is the result of all firms playing their best responses.

Bertrand: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses *price* conditional on what they expect their rival's *price* to be. The nash equilibrium is the result of all firms playing their best responses.

In the specific case of identical products you could say that Bertrand competition is the "fiercest". Two firms undercut each other until price falls to marginal cost and profits disappear. However, in the general case, cournot competition is the most aggressive. The firm with the cost advantage raises its market share while the weaker firm shrinks. In the Bertrand case, if a firm's costs increase, rivals respond by raising price and maintaining market share rather than stealing from their weaker rival.

3) Suppose that the (inverse) market demand for fax paper is given by

$$P = 400 - 2Q$$

Where Q is total industry output. There are two firms that produce fax paper. Each firm has a constant marginal cost of production equal to $40 and they are competing in quantities. That is, they each choose production levels simultaneously.

a) Calculate the best response function for each firm (i.e. each firm's profit maximizing choice of quantity given the other firm's production levels)

   First, rewrite the aggregate production as the sum of each firm's output.

$$P = 400 - 2(q_1 + q_2)$$

   Now, lets look at the demand facing firm 1 (remember, firm one treats firm two's output as a constant

$$P = (400 - 2q_2) - 2q_1$$

Total revenues equal price times quantity.

$$Pq_1 = (400 - 2q_2)q_1 - 2q_1^2$$

Marginal revenue is the derivative with respect to quantity.

$$MR = (400 - 2q_2) - 4q_1$$

Set marginal revenue equal to marginal cost and solve for quantity. To get market price, remember there are two firms.

$$(400 - 2q_2) - 4q_1 = 40$$
$$q_1 = \frac{400 - 2q_2}{4} = 90 - .5q_2$$

This is firm one's best response function. Note that firm two is perfectly symmetric to firm one.

$$q_1 = 90 - .5q_2$$
$$q_2 = 90 - .5q_1$$

Substitute one into the other to get the equilibrium. Then, substitute into the demand curve to get price (remember, there are two firms)

$$q_1 = q_2 = 60$$
$$P = 400 = 2(120) = 160$$
$$\pi_1 = \pi_2 = \$160(60) - \$40(60) = \$7200$$

b) Calculate the profit maximizing price/quantity for a monopolist facing the same demand curve (and with the same production costs). How does your answer compare to (b)?

For the monopolist, it faces the aggregate demand curve.

$$P = 400 - 2Q$$

Total revenues equal price times quantity.

$$PQ = 400Q - 2Q^2$$

Marginal revenue is the derivative with respect to quantity.

$$MR = 400 - 4Q$$

Set marginal revenue equal to marginal cost

$$400 - 4Q = 40$$
$$Q = 90$$
$$P = \$220$$
$$\pi = \$220(90) - \$40(90) = \$16{,}200$$

4) Suppose that the (inverse) demand curve for Viagra is given by

$$P = 200 - 2Q$$

Where Q is total industry output. The market is occupied by two firms, each with constant marginal costs equal to $8.

a) Calculate the equilibrium price and quantity assuming the two firms compete in quantities.

Repeating the process from (1), we get the following:

$$q_1 = q_2 = 32$$
$$P = 200 - 2(64) = \$72$$
$$\pi_1 = \pi_2 = \$72(32) - \$8(32) = \$2048$$

b) How would your answer to (a) change if one of the firm's costs rose to $10?

Here, we need to actually go through the steps: Assume that firm one's costs increase to $10.

$$P = (200 - 2q_2) - 2q_1$$

Total revenues equal price times quantity.

$$Pq_1 = (200 - 2q_2)q_1 - 2q_1^2$$

Marginal revenue is the derivative with respect to quantity.

$$MR = \left(200 - 2q_2\right) - 4q_1$$

Set marginal revenue equal to marginal cost and solve for quantity. To get market price, remember there are two firms.

$$\left(200 - 2q_2\right) - 4q_1 = 10$$
$$q_1 = \frac{190 - 2q_2}{4} = 47.5 - .5q_2$$

This is firm one's best response function. Note that firm two has the same problem to solve, but with MC =$8

$$q_1 = 47.5 - .5q_2$$
$$q_2 = 48 - .5q_1$$

Solving for quantities:

$$q_1 = 31.3$$
$$q_2 = 32.3$$
$$P = 200 - 2(63.6) = \$78.8$$
$$\pi_1 = (\$78.8 - \$10)(31.3) = \$2153.44$$
$$\pi_2 = (\$78.8 - \$8)(32.3) = \$2286.84$$

Note that firm 2 exploits firm one's cost increase by grabbing market share.

c) Repeat parts (a) and (b) assuming the competition is in prices rather than quantities.

This is a quick one. With identical products, both frms charge a price equal to marginal cost (in this case, $8) and profits are zero. If firm one's costs rise to $10, firm 2 charges a price equal to $9.99, takes the entire market and earns profits equal to ($9.99 - $8)(95) = $189.

5) With identical products and no capacity constraints, both firms charge a price equal to marginal cost and earn zero profits. To avoid this, firms have two strategies:

- Restrict capacity: by restricting capacity, they create excess demand for their products. This will drive the price up (see the example in the notes about the movie theatre).

- Variety: By offering different varieties, they can create market niches given consumer preferences. See the spatial competition section of the notes for a complete answer.