Finance 360
Problem Set #9 Solutions

1) Suppose that the market demand is described by

$$P = 100 - (Q + q)$$

Where Q is the output of the incumbent firm, q is the output of the potential entrant and P is the market price. The incumbent's cost function is given by

$$TC(Q) = 40Q$$

While the cost function of the entrant is given by

$TC(Q) = 40q + 100$ (100 is a sunk cost paid upon entering the market)

a) If the entrant observes the incumbent producing $\overline{Q}$ units of output and expects this level to be maintained, what is the equation for the entrant's residual demand curve?

$$P = (100 - \overline{Q}) - q$$

b) If the entrant maximizes profits using the residual demand in (a), what output will the entrant produce?

Total revenues equals price times quantity

$$TR = Pq$$
$$Pq = (100 - \overline{Q})q - q^2$$

Marginal revenue is the derivative with respect to q.

$$MR = (100 - \overline{Q}) - 2q$$

Set marginal revenue equal to marginal cost (= 40) and solve for q

$$(100 - \overline{Q}) - 2q = 40$$

$$q = \frac{60 - \overline{Q}}{2} = 30 - \frac{\overline{Q}}{2}$$

c) How much would the incumbent have to produce to keep the entrant out of the market? At what price will the incumbent sell this output?

Profits can be written as follows:

$$\pi = (P - c)q - 100 = 0$$
$$\pi = (100 - \overline{Q} - q - 40)q - 100 = 0$$
$$\pi = \left(30 - \frac{\overline{Q}}{2}\right)q - 100 = 0$$
$$\pi = \left(30 - \frac{\overline{Q}}{2}\right)^2 - 100 = 0$$

Solving for Q we get Q = 40, q = 10, P = 50

2) Suppose the inverse demand in an industry is given by

$$P = 120 - (q_1 + q_2)$$

Where q_1 is the output of the incumbent firm and q_2 is the output of the entrant. Let both the labor cost and capital cost be 30. That is,

$$w = r = 30$$

In addition, let each firm have a fixed cost of 200.

a) Suppose that in stage one, the incumbent invests in capacity $\overline{K}_1$. Show that in stage two, the incumbent's best response is

$$q_1 = 45 - \frac{1}{2}q_2 \quad (\text{for } q_1 < \overline{K}_1)$$
$$q_1 = 30 - \frac{1}{2}q_2 \quad (\text{for } q_1 > \overline{K}_1)$$

For $q_1 < \overline{K}_1$, firm one's marginal cost is equal to \$30. Therefore,

$$P = (120 - q_2) - q_1$$
$$TR = PQ = (120 - q_2)q_1 - q_1^2$$
$$MR = (120 - q_2) - 2q_1 = MC = 30$$
$$q_1 = \frac{90 - q_2}{2} = 45 - \frac{q_2}{2}$$

For $q_1 > \overline{K}_1$, firm one's marginal cost is equal to 60 (capital plus labor). Solving in the same manner as above yields

$$q_1 = 30 - \frac{1}{2}q_2$$

b) Show that the entrant's best response in stage two is $q_2 = 30 - \frac{1}{2}q_1$

Firm two's marginal costs are always 60. Therefore as in part(a)

$$q_2 = 30 - \frac{1}{2}q_1$$

c) Show that the monopoly or stackelberg leader output is equal to 30. If the incumbent commits to a production capacity of $\overline{K}_1 = 30$, show that in stage two, the entrant will come in and produce a level of output equal to 15. Show that in this case, the entrant earns a profit equal to \$25 while the incumbent earns profits equal to \$250.

If $q_1 \leq \overline{K}_1$ the monopoly profits will be

$$\pi = (P - c)q_1 - 30\overline{K}_1 - 200$$
$$\pi = (120 - q_1 - 30)q_1 - 30\overline{K}_1 - 200$$
$$\pi = (90 - q_1)q_1 - 30\overline{K}_1 - 200$$

Note that a monopolist would never have idle capacity, therefore, $q_1 = \overline{K}_1$

$$\pi = (90 - q_1)q_1 - 30q_1 - 200$$

Maximizing profits with respect to q_1

$$\pi = (90 - q_1)q_1 - 30q_1 - 200$$
$$\pi = (60 - q_1)q_1 - 200$$
$$\pi = 60q_1 - q_1^2 - 200$$
$$\frac{d\pi}{dq_1} = 60 - 2q_1 = 0$$
$$q_1 = 30$$

Firm two's response is

$$q_2 = 30 - \frac{1}{2}(30) = 15$$

Total output is 45 and price is \$75.

Profits are calculated as

$$\pi = (75-30)30 - 30(30) - 200 = 250$$
$$\pi = (75-15)15 - 30(15) - 200 = 25$$

d) Show that if the incumbent instead commits to a production capacity of $\overline{K}_1 = 40$ then in stage two, the entrant's best response is to produce output equal to 10. However, in this case, the entrant earns negative profits.

With $\overline{K}_1 = 40$, it turns out that $q_1 = 40$ (by the same process as above)

$$q_2 = 30 - \frac{1}{2}(40) = 10$$

Total production is 50, P = 70, and entrant's profits are

$$\pi = (70-30)10 - 30(10) - 200 = -100$$

Firm two earns negative profit.

e) Show that if the incumbent chooses $\overline{K}_1 = 32$ in stage one, the entrant can't earn a positive profit in stage two.

Similar to part (d)

3) Consider the following pricing game.

		Firm Two		
		P = \$105	P = \$130	P = \$160
Firm One	P = \$105	**\$7.31** \$7.31	**\$8.25** \$7.25	**\$9.38** \$5.52
	P = \$130	**\$7.25** \$8.25	**\$8.50** \$8.50	**\$10** \$7.15
	P = \$160	**\$5.52** \$9.38	**\$7.15** \$10	**\$9.10** \$9.10

Confirm the following is a Nash subgame perfect equilibrium when the firms interact 5 times.

For t = 1: Set a price of \$160

For 1 < t < 5: Set a price of \$160 as long as the previous price combination was (\$160, \$160). Otherwise, choose a price equal to \$105.

For t = 5: Set a price of $130 as long as the previous combination was ($160, $160). Otherwise, choose a price equal to $105.

First, let's define the optimal strategies. For firm 1:

Firm #2 Charges $105: charge $105
Firm #2 Charges $130: charge $130
Firm #2 Charges $160: charge $130

Both firms want to avoid the potential ($105, $105) equilibrium. In the fourth period of (in fact for periods 1 through 4), Both firms charge ($160, $160). We know this is a nash equilibrium because:

#1) neither firm has the incentive to deviate (both would like to avoid the $105 price in period 5). That is, the treat to charge $105 is credible.

In period 5, a competitive nash equilibrium is ($130, $130).

4) Which two of the following are most clearly common value auctions items: Viper sports cars, electricity, patent licenses, TBills, antiques, or fine art?

   Electricity and T-bills (there is little ambiguity about their worth)

5) If some auction participants for crude oil fields have estimates that the oil in the ground is worth $1.2M, $1.3M or $1.5M with certainty and other participants have estimates that the oil in the ground is worth $1.1M, $1.3M or $1.5M with certainty and a third group has estimates of $1.1M, $1.2M or $1.3M, and all three forecasts include the true value, what is the value? How would you, as an auctioneer design a set of auction rules to reduce strategic underbidding and realize the true value?

   The true value is $1.3M (the second group eliminates $1.2M as a possibility, the third group eliminates $1.5M and the first group eliminates $1.1M).

   To elicit the correct value, we would want an open auction with multiple bidding rounds so that the participants will learn from the bids of others and learn the true value.