1) Suppose that you have the following representations of aggregate savings and investment: \((Y = \text{Aggregate Income}, r \text{ is the interest rate as a percentage})\)

\[
S = .2Y + 300r \\
I = 4,000 - 200r
\]

a) Solve for the equilibrium interest rate as a function of aggregate income.
b) Suppose that income is $10,000. What is the equilibrium value for the interest rate?
c) Calculate the income elasticity and the interest elasticity of savings at the equilibrium.
d) What would be the impact on interest rates, savings, and investment of a $500 government deficit?

2) Suppose we can represent individual preferences (for current and future consumption) as follows.

\[
U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \left( \frac{c_2^{1-\sigma}}{1-\sigma} \right)
\]

Individuals have a known current and future income denoted by \((y_1, y_2)\) and are free to borrow/lend at the risk free rate of interest \((r)\). Therefore, we have the following constraints on consumption:

\[
c_1 + S = y_1 \\
c_2 = (1 + r)S + y_2
\]

There is no production sector or government, so in equilibrium, net saving must equal zero.

a) Write down the above problem as a constrained maximization problem.
b) What is the efficiency condition governing consumption/savings.
c) Solve for current consumption, future consumption, and savings and a function of lifetime wealth. What role does \(\sigma\) play? How is this related to income/substitution effects?
d) Solve for the equilibrium interest rate. How do the values of \(\sigma\) and \(\beta\) influence the long run mean and variance of the interest rate?
e) Suppose that \(\sigma = 1.5\) and \(\beta = .98\). If GDP is expected to grow at an annual rate of 3\%, what should the interest rate be?
3) Suppose you were to estimate the following interest rate equation using percentage change in GDP, percentage change in total government debt and change in the Federal Funds rate (for simplicity, assume that inflation is always zero so that real and nominal variables are the same). All variables are expressed in percentages and are annual rates.

\[ \Delta r = -.6 + (.2)(\%\Delta Y) + (.35)(\%\Delta DEBT) + (.9)(\Delta FF) \]

Currently, the interest rate is 3% and we have the following data for current values and forecasts.

<table>
<thead>
<tr>
<th>Year</th>
<th>GDP (in Trillions)</th>
<th>Federal Deficit (in Billions)</th>
<th>Federal Funds Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>11.6</td>
<td>500</td>
<td>2%</td>
</tr>
<tr>
<td>2005</td>
<td>12.6</td>
<td>540</td>
<td>3%</td>
</tr>
<tr>
<td>2006</td>
<td>13.2</td>
<td>100</td>
<td>4%</td>
</tr>
</tbody>
</table>

a) Given the above data, calculate the expected path for the interest rate.
b) Using your forecasted interest rates from (a), calculate the current yield curve.
c) Assuming that the above model is correct and that no changes in fiscal policy (i.e. no changes in the federal debt) and no changes in monetary policy were expected, what would the following yield curve tell us about the market forecasts for the economy (i.e. calculate the anticipated rates of economic growth)?

<table>
<thead>
<tr>
<th>Term</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1yr</td>
<td>5%</td>
</tr>
<tr>
<td>2yr</td>
<td>4.50%</td>
</tr>
<tr>
<td>3yr</td>
<td>4.25%</td>
</tr>
</tbody>
</table>

4) Consider a firm with the following production technology:

\[ Y = .25k^{1/3}l^{2} \]

Where \( k \) is the inflation adjusted value of the firm’s capital stock and \( l \) is the firm’s total employment (in hours). Further, it is assumed that capital depreciates at a 10% annual rate. For simplicity, assume that the relative price of capital is equal to one.

a) Assume the firm currently employs 100 labor hours and has a current capital stock of $950. The firm is considering an investment of $50. Calculate the internal rate of return on this investment.
b) Growth in capital services have been increases by 4% per year in the US (1980-2004) while employment growth has averaged around 1.5% per year. What should this imply about the interest rate over the same time period?

5) Suppose that individuals have *adaptive expectations* towards inflation. That is, inflation expectations are determined as follows:

\[ \pi_t^e = 0.5\pi_{t-1} + 0.5\pi_{t-3} \]

Consider the following scenario. The real interest rate is fixed at 5%. Inflation is initially zero, but unexpectedly changes to 5%. Given inflation expectations, calculate the path for the real and nominal interest rate following the inflation increase.