Product Innovation, Technology Adoption and Monetary Policy

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Abstract

Many industries, particularly the computer industry have experienced large increases in total factor productivity since the early seventies. Simultaneously, average firm size has decreased in these high growth sectors. This paper investigates the relationship between the adoption of new technologies, firm level employment, and how these relationships might interact with the conduct of monetary policy.

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1 Introduction

Early models illustrating the relationship between money supply and economic activity utilized the paradigm of the perfectly competitive representative plant. Stockman (1981), for example, shows that money growth can affect real growth by acting as a ”tax” on investment by the representative firm. Cooley and Hansen (1989) show that when there is a delay between the time a firm pays its workers and the time that those wages can be converted into consumption, then the ”inflation tax” acts on wages as well as capital. While are useful in thinking about the effects of inflation on the real economy, they might not be a realistic picture of the manufacturing sector of the U.S. economy.

Many industries, particularly the computer industry have experienced large increases in total factor productivity since the early seventies. Figure one illustrates TFP growth in the computer industry (SIC 3573) from 1961-1997. Note the dramatic rise starting with the development of the microprocessor in 1971. Mitchell (2000) documents the fact that during this same time period, average firm size in the computer industry has dramatically fallen.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean Plant Size</th>
</tr>
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<tbody>
<tr>
<td>1972</td>
<td>290</td>
</tr>
<tr>
<td>1977</td>
<td>261</td>
</tr>
<tr>
<td>1982</td>
<td>228</td>
</tr>
<tr>
<td>1987</td>
<td>197</td>
</tr>
<tr>
<td>1992</td>
<td>127</td>
</tr>
</tbody>
</table>
From 1972 to 1992, average firm size in the computer industry dropped from 2990 to 127. Mitchell further documents examples beyond the computer industry and establishes a negative relationship between average industry productivity growth and firm size. The relationship between firm size and the rate of innovation is an old issue in industrial organization. Many researchers have approached the question from the direction of size causing productivity. That is, smaller firms are better able to organize, monitor, etc. labor and, hence, have higher rates of growth. However, causality in the other direction is equally likely. Specifically, the rate of technological innovation affects the optimal firm size. Either way, a model of modey that fails to include these types of technology development and industry dynamics might be missing some potentially important consequences of monetary policy. This paper attempts to take a first step in this direction.

The model developed here is a variant of Mitchell (2000). The stance taken is that the rate of technological development affects the rate in which firms adopt new technologies, and, therefore, the optimal firm size. The rate of technological progress will be taken as exogenous, so money will not affect long run growth. However, because the adoption rule and labor demand is endogenous, the resulting distribution of firm size will be affected by monetary policy. Specifically, a higher rate of monetary expansion will widen the firm size distribution. This could have potentially important consequences which will be discussed later on.

The structure of the paper is as follows. Section two lays out the model. This is done in steps. First, the adoption rule is taken to be exogenous so that the general equilibrium of the model is more transparent. Next, the adoption rule is endogenized. Finally, money is added to the model via a standard chs in advance constraint. In Section three the model is analyzed.
Finally, section four offers some final comments and directions for further study.

## 2 The Economic Environment

Consider an industry inhabited by a continuum of perfectly competitive firms. Let \( p \) be the price of the firm’s output and \( r \) the market interest rate. Labor is the only factor of production and is the numeraire good. Capital could be added to this framework, but the results would be qualitatively similar and the notation would be much more cumbersome. New technologies for utilizing labor are periodically introduced into the economy. As in Greenwood, et al (1997), new technologies cannot be combined with old technologies. Adopting a new production technology requires scrapping the old technogy. For a plant with technology level \( v \), the output produced by employing \( l \) units of labor is

\[
f(l; v) = \gamma^v l^\alpha
\]

\[
\alpha \prec 1 \tag{1}
\]

Where \( \gamma \) represents the rate of technological advance. The firm’s problem is to choose the amount of labor to hire each period as well as when to adopt a new technology in order to maximize lifetime profits.
2.1 Exogenous Upgrading

2.1.1 Firms

For simplicity, first consider the following. A new technology arrives every $T$ periods and automatically replaces the firm’s existing technology. With the dynamic decision removed, all that remains is the labor decision. The firm chooses $l$ to maximize profits. Remember, labor is numeraire.

$$\pi(p, v) = \max_l \{p\gamma^vl^\alpha - l\}$$  \hfil (2)

The upshot of this problem is the following efficiency condition.

$$\alpha pl^{\alpha - 1} = 1$$  \hfil (3)

Therefore, the optimal labor decision and profits are given by

$$l^*(p, v) = (\alpha p\gamma^v)^{\frac{1}{1-\alpha}}$$
$$\pi^*(p, v) = \left(\alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)(\alpha p\gamma^v)^{\frac{1}{1-\alpha}}$$
$$= \chi(\alpha p\gamma^v)^{\frac{1}{1-\alpha}}$$  \hfil (4)
The firm discounts future profits at the rate \( \frac{1}{1+r} \). Therefore, the value function for the firm can be written as follows.

\[
V_t(v) = \pi^*_t(p_t, v) + \left( \frac{1}{1+r_t} \right) \begin{cases} 
V_{t+1}(v), & \text{for } t \leq T - 1 \\
V_{t+1}(v + T), & \text{for } t = T 
\end{cases}
\]  \tag{5}

Suppose that there are \( N \) plants of vintage \( i \). Then total industry \( j \) output is given by

\[
Y_{jt} = N \sum_{i=t}^{t+T} \gamma^i (l^*(p, t))^\alpha
\]  \tag{6}

2.1.2 Consumers

A representative consumer has preferences over consumption of commodities produced by the various industries. Let \( c_i \) be the consumption of good \( i \). The consumer’s instantaneous utility can be given by

\[
U(c) = \int_0^1 \log c_i \, di
\]  \tag{7}

Future consumption is discounted at the rate \( \beta \). The consumer receives income from wages as well as profits from the various firms (in which the consumer owns). Labor is supplied inelastically at one unit. The consumer’s budget constraint therefore is given by

6
\[
\int_0^1 p_t c_t \, dt = 1 + \int_0^1 \pi_i \, di \tag{8}
\]

### 2.1.3 Equilibrium

Equilibrium requires that consumers and firms optimize and all markets clear. This is given by the following set of restrictions:

1. Consumers choose \( \{c_t\}_{t=0}^{\infty} \) to maximize utility subject to the budget constraint.

2. Firms choose labor to maximize profits

3. \( c_i = Y_i \) for all \( i \) (Goods Markets Clear)

4. \( \sum_j \sum_i N_{ij} = 1 \) (Labor Market Clears)

First, consider one industry by itself. Facing a set of fixed prices \( w, r, \) and \( p_t = \gamma^{-t}p_0 \) for the industry’s output. This satisfies a balanced growth path with a constant size distribution of plants (relative to the ”frontier”). Industry output growing at rate \( \gamma \) and industry price falling at rate \( \gamma \). Along this path, \( p_t \gamma^{d} \) is constant. Let \( b = t - v \) (the distance a particular is from the technological frontier). Then, firm profits can be written as a function of \( b \).

\[
\tilde{\pi} (p_0, b) = \max_l \{ p_0 \gamma^{-b}l^\alpha - l \} \tag{9}
\]

and the corresponding value function can be written as
The transformed problem is stationary. Each plant adopts a simple upgrade rule where it uses its existing technology until some critical point \( b^* \). At that point, it upgrades to the frontier. The distribution of plants across vintages \([0, b^*]\) is stationary and uniform. Suppose that there are \( M \) firms of each vintage 0 (state of the art) to \( b^* \) (ready for upgrade). Then industry output can be written as

\[
Q(b^*, M) = M \sum_{b=0}^{b^*} \gamma^{b-b^*} (l^*(p_0, t-b))^\alpha
\]  

Now for the consumer. Consider the static decision (composition of consumption bundle)

\[
U(I) = \max \left\{ \int_0^1 \log c_idi \right\}
\]

\[
I = \int_0^1 p_i c_idi
\]  

The associated first order conditions yield the following efficiency condition for consumption of good \( i \) relative to good 0.
Now, consider the dynamic problem facing the consumer. The euler equation for consumption (in terms of good 0) can be written as

\[ c_t p_t = c_0 p_0 \]  \hspace{1cm} (13)

The left hand side is the marginal value, in terms of good zero, one unit of the numeraire (labor) at time \( t \) while the right hand side is the marginal value at time \( t + 1 \). Therefore, consider the following candidate equilibrium: wages are constant, the output price for industry \( i \) falls at rate \( \gamma_i \), consumption of good \( i \) rises at rate \( \gamma_i \) (as does output of good \( i \)), and the interest rate is constant at the value \( \left( \frac{1}{\beta} - 1 \right) \). Note that these conditions satisfy the euler equation above for good zero as well as the static condition for all other goods.

### 2.2 Endogenous upgrading

While the above example is useful to understand how the general equilibrium operates in the framework, it is not very interesting in that the dynamic upgrading decision of the firm is trivial - a new technology comes along every \( T \) periods and replaces the firm’s existing technology. Now, consider the following. A new technology becomes available to the firm every period. Each period, the firm must decide whether or not to upgrade to the newly available technology. Note
that in the current framework, the cost of upgrading is zero. With free upgrades, each firm would upgrade every period and the problem would collapse to the above case with $T = 0$. To make the upgrade decision non-trivial, a cost of upgrading must be added. This could be done several ways. One such cost would be that knowledge is vintage specific. In other words, workers at a firm learn over time to be more productive with a given technology. That knowledge, however, is not transferable to any newer technology. Therefore, when a firm upgrades, a learning process must take place to allow workers to ”upgrade” their knowledge. During this learning period, the firm experiences a temporary drop in productivity. Specifically, consider the following modified production function.

$$f(l; v) = \lambda \gamma^\alpha l^\alpha$$

$$\alpha < 1$$

$$\lambda = \left[ \lambda - (\theta_v + \varepsilon_t - q_t)^2 \right]$$  \hspace{1cm} (15)

Whenever a plant upgrades its technology, it draws a technology parameter $\theta_v$ from a normal distribution which is unknown by the firm. Each period, the firm chooses a ”production technique” $q_t$ as well as $l_t$. Of course, if the firm could perfectly observe $\lambda$, the learning process is trivial. Therefore, $\varepsilon_t$ is a random shock that strikes the firm every period.

As in Jovanovic and Nyarko (1996), the plant can observe $\theta_v + \varepsilon_t$ costlessly each period. Suppose that the unconditional distribution from which the firm draws $\theta_v$ has a variance $\sigma_0^2$. Also, the random shock is drawn from a normal distribution with variance $\sigma_\varepsilon^2$. The firm begins period $t$ with a belief of the true value of $\theta_v$, denote this as $\mu$, chooses $q$, observes $\theta_v + \varepsilon_t$, \hspace{1cm} 10
then updates its beliefs. Bayes rule implies that beliefs will be normally distributed with mean \( \overline{\mu} \) and variance \( \sigma^2_\mu \). The optimal choice of production technique is \( q = \overline{\mu} \). Therefore, expected productivity can be written as

\[
[\overline{\lambda} - (\sigma^2_\mu + \sigma^2_\varepsilon)]
\]

(16)

Let \( \overline{\lambda} = 1 - \sigma^2_\varepsilon \). Therefore, \( \lambda = 1 - \sigma^2_\mu \).

Experience with a technology will improve a firm’s productivity over time. Specifically, as firms update their beliefs using Bayes rule, we have the evolution of beliefs as follows

\[
\sigma^2_{\mu+1} = \frac{\sigma^2_\mu \sigma^2_\varepsilon}{\sigma^2_\mu + \sigma^2_\varepsilon}
\]

(17)

Therefore, we can rewrite the plant’s static and dynamic problem as follows.

\[
\pi(p, \sigma^2_\mu, v) = \max_l \{p\gamma^v (1 - \sigma^2_\mu) l^\alpha - l \}
\]

(18)

Labor is chosen according to the appropriately modified efficiency condition

\[
l^* (p, v) = (\alpha p\gamma^v (1 - \sigma^2_\mu))^{\frac{1}{1-\alpha}}
\]

(19)
Finally, the dynamic problem is given as follows

\[
V_t (v, \sigma^2) = \bar{\pi} (p, v, \sigma^2) + \left( \frac{1}{1+r} \right) \max \{ V_{t+1} (t+1, \sigma^2), V_{t+1} (v, \sigma^2) \} \tag{20}
\]

Due to the increased complexity of this problem, a closed form solution cannot be obtained. Therefore, the following section reports the results as simulated by computer.

2.3 Calibration

The time period is chosen to be a year. Fortunately, the model has a very limited set of parameters to be calibrated. First, \(\beta\) was chosen so that the annual interest rate is 4\%. This implies the restriction

\[
1 - \frac{1}{\beta} = .04
\]

\[
\beta = .9615
\]

The technological parameter \(\alpha\) was chosen to match labor’s share of income which is .66. Learning studies summarized in Jovanovic and Nyarko (1996) indicate that learning doubles over the learning. This gives the restriction
Finally, \( \sigma^2 \) is chosen so that the ratio of second year productivity to first year productivity is line with studies such as summarized in Auerswald et al (1998). A value of 2 was selected.

2.4 Technology Growth and Plant Size

First, the model was simulated for a baseling growth rate of 1% per year. The Dynamics are summarized by figures 1-4. As in the simpler model, the firms upgrade decision is characterized by an optimal switching rule. The switching point is determined by the point where the marginal cost of switching (in terms of short term productivity losses) equals the marginal benefit (increased long run productivity). Once the upgrade and labor decisions are determined, output and profits follow immediately.

The next experiment was to compare plant size (as given by employment) as a function of technology growth. The model was run using several growth rates and the average plant size was calculated. The results below normalize the base plant size (1% growth).
3 A Monetary Framework: Price Setting under Monop-olistic Competition

3.0.1 Consumers

As in the above model, consumers have preferences over an index of consumption of commodities produced by the various industries, real money balances. Note that including real balances is a shortcut for introducing the real effects of money on aggregate demand for goods. Utility depends positively on consumption of the index and on real balances.

\[ U = \left( \frac{c}{g} \right)^g \left( \frac{m/P}{1 - g} \right)^{1-g} \]  

(21)
Let $c_i$ be the consumption of good $i$. Each good enters utility symmetrically. The functional form implies a constant elasticity of substitution equal to $\theta$. If $\theta$ is large, goods are close substitutes. To guarantee a solution, $\theta$ must be larger than one. Otherwise, the elasticity of demand facing each producer would have an elasticity less than one and the producer would want to choose an infinite price.

$$c = n^{\frac{1}{\theta}} \left( \sum_{j=1}^{n} (c_i)^{\frac{\theta - 1}{\theta}} \right)^{\frac{1}{1-\theta}}$$  \hspace{1cm} (22)

and

$$P = \frac{1}{n} \left( \sum_{j=1}^{n} (P_i)^{1-\theta} \right)^{\frac{\theta}{\theta - 1}}$$

Future consumption is discounted at the rate $\beta$. The consumer receives income from wages as well as profits from the various firms (in which the consumer owns). Labor is supplied inelastically at one unit. The consumer’s budget constraint therefore is given by

$$\sum_{j=1}^{n} P_{jt}c_{jt} + M_{t+1} = W_t + \sum_{j=1}^{n} \pi_i d_i + M_t = I_t$$  \hspace{1cm} (23)

Where $P_{jt}$ is the nominal price of good $j$ at time $t$ and $w$ is the nominal wage rate. Given wealth, $I_t$, maximization of utility subject to the budget constraint yields the following demands
for consumer goods and money.

\[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} \left( \frac{gI_t}{nP_t} \right) \]
\[ M_t = (1 - g) I_t \tag{24} \]

The demand for each good is linear in wealth and a function of the relative price of that good with an elasticity of \(-\theta\). The demand for real balances is also a linear function of wealth. Note that aggregate demand in the economy is given by summing the above expression over goods.

\[ Y = \sum_{j=1}^{n} \frac{P_{jt}c_{jt}}{P_t} = \sum_{j=1}^{n} \left( \frac{P_{jt}}{P_t} \right)^{-\theta} \left( \frac{gI_t}{nP_t} \right) = g \frac{I_t}{P_t} \tag{25} \]

In equilibrium, when aggregate demand equals output, the budget constraint implies that \(I/P + Y = M/P\). This implies

\[ Y = \left( \frac{g}{1 - g} \right) \left( \frac{M}{P} \right) \tag{26} \]

Thus, there is a simple quantity theory relationship among nominal prices, output, and the price level.

The demand facing individual producers is given by
\[
Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} \left( \frac{Y_t}{n} \right) = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} \left( \frac{g}{n(1 - g)} \right) \left( \frac{M_t}{P_t} \right)
\]

\[
\left( \frac{P_{jt}}{P_t} \right) = Y_{jt}^{\frac{1}{\sigma}} \left( \frac{g}{n(1 - g)} \right)^{\frac{\gamma}{\sigma}} \left( \frac{M_t}{P_t} \right)^{\frac{1}{\sigma}}
\]  

(27)

3.0.2 Firms

Firms face the same problem as above with the exception that they now face a less than perfectly elastic demand curve. Firms enter period \( t \) with a given technological level and a stock of knowledge about that technology. They must choose how much labor to employ as well as when to upgrade to a new technology.

Therefore, we can rewrite the plants static and dynamic problems as follows.

\[
\pi \left( \sigma_{\mu}^2, v, M/P \right) = \max_l \left\{ \left( \frac{P_j}{P} \right)^{\gamma} (1 - \sigma_{\mu}^2) \left[ l^\alpha - \left( \frac{w}{P} \right) l \right] \right\}
\]

(28)

Labor is chosen according to the appropriately modified efficiency condition

\[
l \left( \sigma_{\mu}^2, v, M/P \right) = \left( \alpha \left( 1 - \frac{1}{\theta} \right) (\gamma^v (1 - \sigma_{\mu}^2))^{-\frac{1}{\sigma}} \left( \frac{M_t}{P_t} \right)^{\frac{1}{\sigma}} \right)^{\frac{\theta}{1+\theta-\alpha\theta}}
\]

(29)

Finally, the dynamic problem is given as follows.
\[
V_t(v, \sigma^2_{\mu t}, M/P) = \tilde{\pi}(v, \sigma^2_{\mu t}, M/P) + \left( \frac{1}{1 + i} \right) \max \{ V_{t+1}(t + 1, \sigma^2_0), V_{t+1}(v, \sigma^2_{\mu t+1}) \} \tag{30}
\]

Where \( i \) is the nominal interest rate.

3.0.3 Government

The government in this model has only one purpose which is to regulate the supply of money.

The rule for money supply is given by

\[
M_{t+1} = (1 + \mu_{mt}) M_t
\]

The government distributes the extra money to the economy via a lump sum transfer to consumers.

\[
\tau_t = M_{t+1} - M_t = \mu_{mt} M_t \tag{31}
\]

4 References


