Endogenous Financing and the Long Run Impact of Money Growth on Prices and Output

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Abstract

Most monetary models make use of the quantity theory of money along with a phillips curve. This implies a strong correlation between money growth and output in the short run (with little or no correlation between money and prices) and a strong long run correlation between money growth and inflation (with little or nor correlation between money growth and output). The empirical evidence between money and inflation is very robust, but the long run money/output relationship is ambiguous at best. This paper attempts to explain this by looking at the impact of money growth on firm financing decisions.

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1 Introduction

The Federal Reserve System was established by congress in 1913 to provide the country with a safer, more flexible and more stable monetary system. While traditionally, the Federal Reserve’s first priority has always been a low, stable inflation rate, maintaining a steady rate of economic growth has never been far behind. Specifically, the Employment act of 1946 and the Growth Act of 1978 instruct the Federal Reserve to

"...Maintain long run growth of the monetary and credit aggregates commensurate with the economy’s long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long term interest rates."

The natural question to ask is: Does the Federal Reserve actually have the power to carry these instructions out? Most explorations into monetary economics usually have at their core, some version of the quantity equations. The quantity equation is written as

\[ MV = PY \]

Where \( M \) is the aggregate money stock, \( V \) is velocity, \( P \) is the aggregate price level, and \( Y \) is aggregate output. Alternately, we could write this equation in terms of percentage changes. This yields a linear relationship between money growth, output growth and inflation.
\[ \% \Delta M + \% \Delta V = \% \Delta P + \% \Delta Y \]

It is commonly assumed that velocity and prices are fixed in the short run - this results in a strong positive relationship between money and output. The long run, however, is more likely characterized by fixed (or at least exogenous) output and, hence, a strong positive relationship between money growth and inflation. In other words, there exists a Phillip’s curve.

The empirical evidence on the money/price relationship is quite robust. Lucas (1980) analyzes the relationship in US data between M1 and the CPI at several frequencies and finds that at low frequencies (long run), the correlation between money and price approaches one. Other studies have analyzed cross country data. Vogel (1974) finds that increases in money growth result in proportionate increases in inflation after a lag of two years. Dwyer and Hafer (1988), Barro (1990), and Poole (1994) also confirm this strong, positive correlation. Rolnick and Weber(1994) analyze the money/price relationship under both commodity and fiat money systems and find that while the correlation is positive in both cases, the relationship is much stronger under fiat systems (near unity in fiat systems while only .61 in commodity systems).

Empirical evidence on the long run relationship between money and output, however, is not so certain. Kormendi and Meguire (1985) find that the average rate of growth in the money supply and the standard deviation of money shocks are both negatively associated with real output growth. Dwyer and Hafer (1988) find that money growth is negatively with the level of real output, but uncorrelated with the growth of real output. Poirier (1991) discovers that money is neutral in some countries, but not in others.
McCandless and Weber (1995) analyze data for 110 countries over a 30 year period and discover three long run money facts:

- There is a high correlation between the growth rate of the money supply and the rate of inflation - regardless of the specific definition of money and across the full sample of countries. (Correlations range between .89 and .97)

- There is no correlation between growth rates of money and real output. This holds for all definitions of money, but not for the OECD countries in their sample. (Correlations range between -.243 and .707)

- There is no relationship between inflation and real output growth. This holds for across their entire sample. (Correlations range between -.34 and .39)

A well-documented fact in the industrial organization literature is the negative relationship between firm size and industry growth. That is, industries with smaller average establishment size are associated with higher rates of technological advance. Secondly, work by Cooley and Quadrini (2000) and Gertker and Gilchrist (1994) have shown that small firms react quite differently to monetary policy shocks than do larger firms. Cooley and Quadrini (2000) suggest that this difference has to do with financing decisions made at the firm level. They show that small firms tend to have larger debt/equity ratios than do larger firms. In this paper I argue that this fact could be potentially important for looking at the long run output effects of money growth. If high inflation rates cause firm’s to alter their financing choice towards debt, this will have a disproportionate effect on smaller firms, which have less access to equity markets and, hence, rely more on debt. A model of firm finance similar to Cooley and Quadrini (2001)
or Gomes (1999) is set up to investigate the impact of money growth on the financing decisions of firms and the resulting impact on economic growth and inflation.

2 The Model

2.1 The Production Sector

2.1.1 Incumbent Firms

The production sector is inhabited by a continuum of firms with access to the following technology

\[ y = F(k, \psi) = \psi k^\alpha \]  \hspace{1cm} (1)

\[ \alpha \leq 1 \]

where \( k \) represents capital which depreciates at rate \( \delta \). The assumption of \( \alpha \leq 1 \) implies that the production technology exhibits diminishing marginal returns to capital. Each firm can be distinguished by a technology parameter, \( \psi \). To simplify the analysis, assume that \( \psi \) can take on two possible values.

\[ \Pr(\psi = \psi) = \lambda \]

\[ \Pr(\psi = \overline{\psi}) = 1 - \lambda \]

At each point in time, firms are characterized by the amount of capital, \( e \), that they own.
Henceforth, owned capital is referred to as the firm’s equity. The amount of equity will change as a firm reinvests profits over time. The value of the firm will therefore depend on the productivity parameter as well as the firm’s dividend/equity policy. In addition to the capital that firms own, they can rent capital. Rented capital is financed by borrowing from the financial intermediary at rate $r_t$. By allowing firms to rent capital, it is insured that financial differences will be the only factor affecting the firms’ production plans. That is, a firm’s production plan is not constrained by the amount of capital it owns at a point in time.

The problem facing an incumbent firm is to choose capital, the amount of borrowing, $b$, from financial intermediaries, and equity to maximize shareholder value. The value of the firm derives from the flow of dividends which are paid out at the end of the period. Note that with dividends paid at the end of the period (after the goods market has closed), consumers can’t convert dividends into consumption until the following period. This implies that one unit of real dividend paid at time $t$ allow the shareholder to buy $\frac{p_t}{p_{t+1}}$ units of consumption in period $t+1$. Therefore, the expected utility of a unit of real dividend paid at time $t$ will be equal to

$$\beta E_t \left( \frac{p_t u_{t+1}}{p_{t+1}} \right) = \theta$$  

(2)

With this, we can write the dynamic problem facing the firm as follows.
\[ V(e, \psi) = \max_{b, e'} \theta \pi + \beta [(1 - \eta) V(e', \psi) + \eta e'] \]

s.t.

\[ \pi = F(k, \psi) - r_l b - I \]

\[ k = b + e \]

\[ e' = (1 - \delta) e + I \]

The problem has the associated first order conditions.

\[ F_1(k, \psi) \geq r_l, \text{ if } b > 0 \]  \hspace{1cm} (3)

\[ F_1(k, \psi) < r_l, \text{ if } b = 0 \]

\[ \theta \leq \beta \theta' [(1 - \eta) [(1 - \delta) + F_1(k, \psi)] + \eta], \text{ if } e > 0 \]  \hspace{1cm} (4)

\[ \theta > \beta \theta' [(1 - \eta) [(1 - \delta) + F_1(k, \psi)] + \eta], \text{ if } e = 0 \]

### 2.1.2 New Firms

The creation of a new firm requires a lump sum cost that is sunk. The fixed cost here is important because without it, heterogeneity in the model would disappear. With costless entry, the optimal strategy would be to operate an infinite number of arbitrarily small firms. The size of all firms in the economy would collapse to the size of a new entry. New firms choose the optimal size (in terms of equity) to maximize the expected lifetime profit of a new firm. Upon entry, the new firm
receives its technology parameter. New firms must borrow from the financial sector to finance their initial equity position at rate $r_l$. Therefore, the cost of a new firm of size $e$ is $r_l (\kappa + e)$. The value of the new firm the next period will be $\beta E[V(e)]$. Therefore, the size of a new firm is given by

$$e_0 = \arg \max_{e > 0} \beta E[V(e)] - r_l (\kappa + e)$$

(5)

Note further that in a stationary steady state, entry must equal exit. Therefore, we have the condition that the size of a new firm must equal the size of the firms that exit. Specifically, this is given by the following condition.

$$e_0 = \eta [(1 - p) k(\psi) + p(\overline{\psi})]$$

(6)

### 2.2 The Household Sector

Consumers in the economy have preferences defined over random streams of consumption and leisure represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

(7)
where $c$ represents consumption, $\beta < 1$ is the discount rate and $E_0$ represents the conditional expectation based on information available at time 0.

Households have deposits at the financial intermediary earning interest $r_d$ and own shares of existing firms. Households must obtain cash to purchase consumer goods.

$$ p_t c_t \leq m_t $$

(8)

where $p_t$ is the nominal price level. At the end of the period, households receive income from interest earned on their savings, as well as earnings from an investment portfolio of incumbent firms. If $\pi(\mu)$ denotes the profits earned by the portfolio $\mu$, then the households end of period money can be written as follows.

$$ m_{t+1} = m_t - p_t c_t + (1 + r_d) p_t d_t + p_t \pi_t(\mu) $$

(9)

the household’s decision problem is to choose a contingency plan for

$\{c_t, d_t\}_{t=0}^{\infty}$ that maximizes expected lifetime utility subject to the series of constraints.

The household problem can be written in the following recursive formulation. Note that to save on notation, time subscripts have been left out. Primed variables indicate their $t + 1$ values.
\[ J(m, \mu, d) = \max_{c,d} \{ W(c) + \beta EJ(m', \mu', d') \} \]

\[ \text{s.t.} \]

\[ pc \leq m \]

\[ p(c + d') + m' = m + p \left( (1 + r_d) d + \int_c \pi(e) (de) \right) \]

With the associated efficiency condition:

\[ W_1(c) = \beta W_1(c') \left( \frac{p}{p'} \right) (1 + r_d) \]

### 2.3 Financial Intermediaries

There exists a continuum of perfectly competitive financial intermediaries which collect deposits from households at the end of each period and loan the money out to entering firms at the beginning of the next period. They also receive cash injections of liquid funds from the government. The sum of deposits and monetary injections make up the total supply of loanable funds. If we denote the interest rate for lending and deposits as \( r_l \) and \( r_d \) respectively. Then the zero profit condition on financial intermediaries gives the following relation.

\[ (1 + r_d) = \frac{e_0 + \lambda b(\psi) + (1 - \lambda) b(\psi) + \tau}{d} (1 + r_l) \]
Where $\tau$ represents the change in the money stock.

### 2.4 Government

The government in this model has only one purpose which is to regulate the supply of money. The rule for money supply is given by

$$m_{t+1} = (1 + \mu m_t) m_t$$

The government adds to the money supply by transferring $\tau_t$ to the financial intermediary.

$$\tau_t = m_{t+1} - m_t = \mu m_t m_t$$ \hspace{1cm} (16)

### 2.5 Equilibrium

An Equilibrium for the economy is represented by

1. Household decision rules $c$ and $d$
2. Firms decision rules given by $k, e, b$, and $e_0$
3. Pricing functions $p, r_d, r_l$

which satisfy the restrictions:

1. Households and firms optimize taking prices as given
2. All markets clear.

2.6 Analysis

The key to this economy is the relative cost of equity financing (firm ownership of capital) versus debt financing in terms of foregone utility. The condition governing equity financing is given by equation (4). The left hand side represents the utility cost of a unit of dividends foregone to purchase capital. The right hand side represents the (discounted) future benefit. With probability \((1 - \eta)\) the firm will stay in business. In this case the benefits to a unit of investment is the marginal product in production plus the value of the undepreciated portion of the capital. However, with probability \(\eta\) the firm goes out of business and sells the capital. However because dividends cannot be spent until the following period (after prices have risen, they must be discounted by the inflation rate. Alternatively, a firm could choose to finance using debt. The advantage to debt in this economy is that debts are repaid prior to the consumption market closing and, hence, do not incur an inflation penalty. Note that optimal production will require equating the productivity of capital across both incumbent firms. Therefore, there are two possible types of equilibria. Either both incumbents finance entirely with equity \(e(\overline{\psi}) = k(\overline{\psi}), e(\psi) = k(\psi)\) or incumbents rely purely on borrowing \(e(\overline{\psi}) = e(\psi) = 0\). The analysis could be complicated by setting different capital scrap values for high technology and low technology firms. In this case there would be three potential equilibria: the two mentioned above plus a third range where only the high tech firm chooses to finance with equity. Given steady state capital, the cutoff point between the two equilibria will be determined by money growth. Specifically, we have the following condition governing the level of money growth above which equity financing ceases.
The left hand side measures the steady state real interest rate (the real return to debt finance) while the right hand side represents the real return to equity financing.

\[
\frac{1}{\beta} \geq \frac{1}{(1 + \mu)} \left\{ (1 - \delta) (1 - \eta) + (1 - \eta) \bar{\psi} \alpha K^{\alpha - 1} + \eta \right\}
\]  

(17)

The key parameter governing the switching point between debt and equity is \( \eta \) - the probability of exit. As the chance of exit rises, firms choose lower steady state capital stocks and, hence, have a higher internal rate of return. Therefore, they can pay higher inflation adjusted dividends. Below are the cutoff points for various exit rates.

<table>
<thead>
<tr>
<th>Exit Probability (( \eta ))</th>
<th>Money Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.11%</td>
</tr>
<tr>
<td>5%</td>
<td>8%</td>
</tr>
<tr>
<td>10%</td>
<td>14%</td>
</tr>
<tr>
<td>25%</td>
<td>37%</td>
</tr>
<tr>
<td>50%</td>
<td>51%</td>
</tr>
</tbody>
</table>

Figures (1) – (4) illustrate the impact of money growth on the economy. The parameters chosen for the experiment are as follows.
The values for alpha (capital share of income), beta (discount rate) and delta (depreciation) are taken are standard. The remaining parameters governing productivity and exit will effect the absolute steady state values, but do not qualitatively change the results. Figure (1) looks at the impact of money growth on the capital stock relative to an economy with no money. Values are in terms of percentage difference between the monetary and non-monetary economies. For low values of money growth, incumbent firms choose equity financing over debt financing and, hence, pay the inflation tax. As money growth increases, the inflation tax rises and steady state capital falls. If the option of debt financing weren’t available, the economy would continue down the dashed line. However, allowing incombant firms to switch from equity to debt to avoid the inflation tax allows the economy to stay on the solid line. At the point the switch is made, money becomes superneutral - it has no impact on the real economy. Figure (2) shows the impact of money growth on the steady state capital stocks of high technology firms, low technology firms, and new firms. Figures (3) and (4) show steady state consumption, output, and productivity. Lastly, consider the relationship between money growth and inflation in this economy. Convert the cash in advance constraint into percentage changes and re-arrange to get
π = %Δp = %Δm − %Δc

Due to the restriction on entry, consumption growth is constant regardless of the financing regime. Therefore, the correlation between money growth and inflation is one.

3 Conclusions

Empirically, the relationship between money growth and inflation is consistently found to be close to unity. The relationship between money growth and real output is much less certain. Previous studies have found correlations at low as -.2 and as high as .7. This observation is difficult to reconcile with theoretical monetary models. Here, a monetary framework is created in which the production sector chooses its method of finance for its capital stock. When a firm purchases its capital (equity financing), an inflation tax is paid if money growth is present. As money growth rises, the size of the tax increases which lowers steady state capital and output. However, if s firm chooses to finance via short term debt - these loans are paid back in the same period they are created and, hence, avoid the inflation tax - then money becomes superneutral and the correlation between money growth and output growth is 0. Therefore, an economy that is near the cutoff point between debt and equity could experience a wide variety of correlation between money and output.
4 References


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