Math 20580
Final Exam
December 12, 2017

Name:______________________________
Instructor:______________________________
Section:______________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 2 hours to do the test. You may leave earlier if you are finished.

There are 20 multiple choice questions worth 7 points each. You will receive 10 points for being present and following the instructions. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

1.	a	b	c	d	e	11.	a	b	c	d	e
2.	a	b	c	d	e	12.	a	b	c	d	e
3.	a	b	c	d	e	13.	a	b	c	d	e
4.	a	b	c	d	e	14.	a	b	c	d	e
5.	a	b	c	d	e	15.	a	b	c	d	e
6.	a	b	c	d	e	16.	a	b	c	d	e
7.	a	b	c	d	e	17.	a	b	c	d	e
8.	a	b	c	d	e	18.	a	b	c	d	e
9.	a	b	c	d	e	19.	a	b	c	d	e
10.	a	b	c	d	e	20.	a	b	c	d	e

1. Find $T\begin{bmatrix}1\\1\end{bmatrix}$ if $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$T\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\0\\6\end{bmatrix} \qquad \text{and} \qquad T\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\-2\\4\end{bmatrix}.$$

(a) $\begin{bmatrix}2\\2\\2\end{bmatrix}$ (b) $\begin{bmatrix}2\\-2\\10\end{bmatrix}$ (c) $\begin{bmatrix}1\\-1\\5\end{bmatrix}$ (d) $\begin{bmatrix}-1\\-1\\-1\end{bmatrix}$

(e) cannot be determined from the given information.

2. Find the solution of the initial value problem

$$\begin{cases} 4y'' - 4y' + y = 0 \\ y(2) = 4e, \quad y'(2) = 3e \end{cases}$$

(a) $4e^{2t-3}$ (b) $(t+2)e^{t/2}$ (c) $4e^{t/2} + t/2 - 1$ (d) $5e^{t/2} - e^{-t/2}$ (e) $e \cdot t^2$

3. If $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 4 & 0 & -1 \\ 3 & 0 & 3 \\ 2 & -2 & 5 \end{bmatrix}$ then the corresponding eigenvalue is

(a) 3 (b) 1 (c) -1 (d) -3 (e) 0

4. Find the integrating factor that would make the following equation exact:

$$y^2 + \sin x + xy\frac{dy}{dx} = 0$$

(a) $\mu = e^{xy^2/2}$ (b) $\mu = e^{y^2}$ (c) $\mu = \frac{x^2y^2}{2}$ (d) $\mu = y\sin(x)$ (e) $\mu = x$

5. The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ are

(a) $-2 \pm i \cdot 2\sqrt{3}$ (b) 2 (with multiplicity 2) (c) $2 \pm i \cdot \sqrt{3}$ (d) $2 \pm i$

(e) A has no eigenvalues

6. Consider the equation

$$y'' - 2ty' + e^t y = 0.$$

Find the Wronskian of the fundamental set of solutions of this equation determined by the conditions $y_1(0) = 2$, $y_1'(0) = 1$ and $y_2(0) = -1$, $y_2'(0) = 3$.

(a) e^{t+7} (b) $7e^{t^2}$ (c) $-t^2 + 7$ (d) 7 (e) $(t + 7)e^t$

7. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & -3 \\ 1 & -1 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find the invertible matrix P such that $A = PDP^{-1}$.

(a) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & -3 \\ 2 & 2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 1 & 1 \\ -3 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(e) P cannot be determined from the given data.

8. Let $y(t)$ be the unique solution of the initial value problem

$$\ln t \cdot \frac{dy}{dt} - \frac{2y}{\cos t} = \frac{t^2}{2^t - 8} \qquad y(2) = \pi$$

What is the largest interval on which a solution y is guaranteed to exist?

(a) $t > 0$ (b) $\frac{\pi}{2} < t < 3$ (c) $t < 1$ (d) $1 < t < 3\pi/2$ (e) $3 < t < \frac{3\pi}{2}$

9. Which of the following statements is **not** true for an invertible $n \times n$ matrix?

(a) $\dim(\text{Row}\, A) = n$

(b) $\text{rank}\, A = n$

(c) 0 is an eigenvalue of A

(d) $A^t A^{-1}$ is invertible

(e) $\dim(\text{Nul}\, A) = 0$

10. Which formula describes the general solution of the differential equation

$$2t^2y'' + 3ty' - y = 0, \;\; t > 0$$

given the fact that $y_1(t) = t^{-1}$ is a solution of this equation.

(a) $c_1t^2 + c_2t^{-1}$ (b) $c_1t^{-1} + c_2$ (c) $c_1t^{-1} + c_2t^{2/3}$ (d) $c_1e^t + c_2t^{-1}$ (e) $c_1t^{-1} + c_2t^{1/2}$

11. Find the general solution of

$$y'' + 2y' + \frac{13}{4}y = 0$$

(a) $y(t) = c_1 e^{-t} + c_2(\cos(3t/2) + \sin(3t/2))$ (b) $y(t) = c_1 t e^{-t} + c_2 e^{-t}$
(c) $y(t) = c_1 e^{-t}\cos(3t/2) + c_2 e^{-t}\sin(3t/2)$ (d) $y(t) = c_1 e^{-t} + c_2 e^{3t/2}$
(e) $y(t) = c_1 \cos(-t) + c_2 \sin(3t/2)$

12. Which formula describes implicitly the solution of the initial value problem

$$3e^x \cdot \frac{dy}{dx} - \frac{x}{y^2} = 0, \quad y(0) = 1.$$

(a) $3ye^x = x^2 + 3$ (b) $3e^x = \frac{x}{y} + 3$ (c) $e^x(x + y) = y^2$
(d) $y^3 + (x+1)e^{-x} = 2$ (e) $y^3 + 2y = 3e^x + x$

13. Consider the matrices

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 & 11 \\ 3 & 6 & -3 & 1 & 15 \\ -1 & -2 & 1 & 2 & 2 \\ 4 & 8 & -4 & 4 & 28 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced echelon form of A. A basis for the orthogonal complement of the row space of A is given by

(a) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -4 \\ 4 \\ 28 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 4 \\ 6 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 11 \\ 15 \\ 2 \\ 28 \end{bmatrix} \right\}$

14. Based on the method of Undetermined Coefficients, find the form of a particular solution of the differential equation

$$y'' + 4y' + 5y = (t^2 + 1)e^{-2t}.$$

(a) $Y(t) = A_0(t^2 + 1)e^{-2t}\cos(t) + B_0(t^2 + 1)e^{-2t}\sin(t)$

(b) $Y(t) = (A_0t^2 + A_1t + A_2)e^{-2t}\cos(t) + (B_0t^2 + B_1t + B_2)e^{-2t}\sin(t)$

(c) $Y(t) = t(A_0t^2 + A_1t + A_2)e^{-2t}$

(d) $Y(t) = (A_0t^2 + A_1t + A_2)e^{2t} + (B_0t^2 + B_1t + B_2)e^{-2t}$

(e) $Y(t) = (A_0t^2 + A_1t + A_2)e^{-2t}$

15. The second column of the inverse of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 2 \\ -1 & 2 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

16. Consider the initial value problem

$$\frac{dy}{dt} = 2y^2 - 4y, \qquad y(5) = 1.$$

Which of the following describes the nature of the solution?

(a) $\lim_{t\to-\infty} y(t) = 2;$ $\lim_{t\to\infty} y(t) = 0;$ inflection point at $y = 1$

(b) $\lim_{t\to-\infty} y(t) = 2;$ $\lim_{t\to\infty} y(t) = \infty;$ concave up

(c) $\lim_{t\to-\infty} y(t) = 0;$ $\lim_{t\to\infty} y(t) = 4;$ inflection point at $y = 2$

(d) $\lim_{t\to-\infty} y(t) = -\infty;$ $\lim_{t\to\infty} y(t) = 0;$ concave down

(e) $\lim_{t\to-\infty} y(t) = 0;$ $\lim_{t\to\infty} y(t) = -\infty;$ inflection point at $y = 1/2$

17. Recall that $\mathbb{P}_n$ denotes the vector space of polynomials of degree at most n, and consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_3$ defined by

$$T(y) = ty'' - y' + (t+1)y.$$

The matrix of T relative to the basis $\{1, t, t^2\}$ of $\mathbb{P}_2$ and the basis $\{1, t, t^2, t^3\}$ of $\mathbb{P}_3$ is

(a) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} t \\ -1 \\ t+1 \end{bmatrix}$ (c) $\begin{bmatrix} 1-t \\ 1+t \\ t+t^2 \\ t^2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(e) it cannot be determined from the given information.

18. Solve the initial value problem

$$\begin{cases} ty' + (t+1)y = te^{-t}, \ t > 0 \\ y(1) = 2e^{-1} \end{cases}$$

(a) $2e^{-t}$ (b) $te^{-t} + 1$ (c) $(t^2+1)e^{-t}$ (d) $\dfrac{1+t}{e^t}$ (e) $\dfrac{t^2+3}{2te^t}$

19. Consider the lined L spanned by the vector $\vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. The distance from the vector $\vec{x} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$ to the line L is

(a) $\sqrt{45}$ (b) $\sqrt{5}$ (c) $2\sqrt{3}$ (d) 5 (e) $\sqrt{50}$

20. Using the method of Variation of Parameters, find a particular solution of the differential equation

$$x^2y'' - 3xy' + 4y = x^2\ln(x), \;\; x > 0,$$

knowing that $\{y_1, y_2\} = \{x^2, x^2\ln(x)\}$ is a fundamental set of solutions for the homogeneous equation $x^2y'' - 3xy' + 4y = 0$.

(a) $x\ln(x) + \dfrac{x^3}{3}$ (b) $\dfrac{x^3\ln(x)}{2}$ (c) $\dfrac{x^2\ln^3(x)}{6}$ (d) $2x\ln^2(x)$ (e) $\dfrac{(x+\ln(x))^2}{2}$