

**Tutorial Worksheet**

1. Find all solutions to the linear system by following the given steps

$$\begin{cases} x + 2y + 3z = 2 \\ 2x + 3y + z = 4 \\ y + z = 8 \end{cases} .$$

Step 1. Write down the augmented matrix (coefficient and constants) of the system.

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ \hline - & - & - & - \\ \hline - & - & - & - \end{array} \right]$$

Step 2. Replace the second row of the matrix with the second row subtracted by 2 times the first row. In our notation, we would write  $R_2 \mapsto R_2 - 2R_1$  to denote this operation. This gives us the matrix

$$\xrightarrow{R_2 \mapsto R_2 - 2R_1} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & - & - & - \\ \hline - & - & - & - \end{array} \right]$$

Step 3. Keep performing the row operations until you can solve the equation.

2. As a generalization to the first question, can you find the solution to the system

$$\begin{cases} x + 2y + 3z = a \\ 2x + 3y + z = b \\ y + z = c \end{cases}$$

for arbitrary real numbers  $a$ ,  $b$ , and  $c$ ?

3. Which matrices below are in echelon form? Which are in reduced echelon form?

$$A = \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Echelon form : \_\_\_\_\_.

Reduced echelon form : \_\_\_\_\_.

4. Which columns of the matrix below are pivot and which are free?

$$\begin{bmatrix} 1 & 2 & -1 & -3 & 2 \\ 2 & 5 & -1 & -6 & 6 \\ 3 & 7 & -2 & -8 & 8 \end{bmatrix}$$

Remark: A *free column* is a column which corresponds to a free variable.

5. Find all solutions to the linear system

$$\begin{cases} x - 3y + z - w = 2 \\ 2x - 6y + 3z - w = 3 \\ 3x - 9y + 5z - w = 4 \end{cases} .$$