

$$\begin{aligned}
R(\theta_1, \theta_2) &= \frac{g_1(\theta_1)}{g_2(\theta_2)} = \frac{\mu_1 - \mu_2}{\sigma} \\
\theta_1 &= \mu_1 - \mu_2 \\
\theta_2 &= \sigma^2 \\
g_1(x) = x &\implies g_1'(x) = 1 \\
g_2(x) = \sqrt{x} &\implies g_2'(x) = \frac{1}{2\sqrt{x}}
\end{aligned}$$

$$\mathbf{\Sigma} = \begin{bmatrix} 2\sigma^2 & \frac{1}{2}(\mu_{13} - \mu_{23}) \\ \frac{1}{2}(\mu_{13} - \mu_{23}) & \frac{1}{4}(\mu_{14} + \mu_{24} - 2\sigma^4) \end{bmatrix}$$

and

$$\mathbf{D}' = \begin{bmatrix} g_1'(\theta_1) & -g_1(\theta_1)g_2'(\theta_2) \\ g_2(\theta_2) & g_2^2(\theta_2) \end{bmatrix} = \begin{bmatrix} 1 & -(\mu_1 - \mu_2) \\ \sigma & \frac{1}{2\sigma} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma} & -\frac{(\mu_1 - \mu_2)}{2\sigma^3} \end{bmatrix}.$$

$$\mathbf{x}'\mathbf{A}\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$$

therefore

$$\begin{aligned}
\mathbf{D}'\mathbf{\Sigma}\mathbf{D} &= \begin{bmatrix} \frac{1}{\sigma} & -\frac{(\mu_1 - \mu_2)}{2\sigma^3} \end{bmatrix} \begin{bmatrix} 2\sigma^2 & \frac{1}{2}(\mu_{13} - \mu_{23}) \\ \frac{1}{2}(\mu_{13} - \mu_{23}) & \frac{1}{4}(\mu_{14} + \mu_{24} - 2\sigma^4) \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma} \\ -\frac{(\mu_1 - \mu_2)}{2\sigma^3} \end{bmatrix} \\
&= (2\sigma^2) \left(\frac{1}{\sigma}\right)^2 + 2 \left[\frac{1}{2}(\mu_{13} - \mu_{23})\right] \left[\frac{1}{\sigma}\right] \left[-\frac{(\mu_1 - \mu_2)}{2\sigma^3}\right] + \left[\frac{1}{4}(\mu_{14} + \mu_{24}) - \frac{\sigma^4}{2}\right] \left[-\frac{(\mu_1 - \mu_2)}{2\sigma^3}\right]^2 \\
\xi^2 &= 2 - \underbrace{\frac{(\mu_1 - \mu_2)(\mu_{13} - \mu_{23})}{2\sigma^4}}_{t_1} + \underbrace{\frac{(\mu_1 - \mu_2)^2}{4\sigma^6}}_{t_2} \underbrace{\left(\frac{\mu_{14} + \mu_{24}}{4} - \frac{\sigma^4}{2}\right)}_{t_3}
\end{aligned}$$