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95 Million *t* tests: The Empirical Findings when the Assumption of Independence has Been Violated in the Two-Sample *t* Test[1]

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A copy of the computer program used written in SAS, version 6.12, can be obtained from the author. The data used may also be requested.

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Abstract

A Monte Carlo computer simulation was used to evaluate the effect on the Type I error rate when the assumption of independence was not met in the two-sample *t* test. It was shown that when there is a positive correlation within groups the nominal alpha level is considerably smaller than the probability of the Type I error rate. This study used five values of ρ and 19 different ν's, computing 1,000,000 *t*'s for each of the 95 combinations used in the empirically generated critical value table provided. The critical values in the table are derived from distributions with a known ρ and ν. It is believed when the independence assumption is violated in scientific research, use of empirically generated critical values that match the characteristics within groups will be more appropriate then using the *t* table, which is of course based on the assumption of independence.

KEY WORDS: Assumptions of the *t* test, dependent samples, empirically generated *t* distribution, experimental unit, independence assumption, Monte Carlo simulations, nonindependence, robustness of the *t* test, *t* test, unit of analysis, violating assumptions, intraclass correlation.

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Virtually all areas of scientific research make use of inferential statistical methods to analyze and make decisions regarding empirically gathered data. These statistical methods are invaluable for the information that they convey to other researchers as well as for the conclusions that are drawn from them. What must be carefully considered when conducting research and making use of inferential statistics are the underlying assumptions that the tests are built upon.

The Student's *t* test is one of the most widely used inferential techniques for analyzing data from empirical research (Kurita, 1996; Sawilowsky & Blair, 1992; Zolman, 1993). However, for a given *t* test to be valid, the data (and the experiment) should be inspected to insure that the assumptions underlying the *t* distribution are not violated. When the assumptions underlying this mathematical model are not violated, the *t* test has the difference between two unbiased estimates in the numerator, $\bar{X}_1$ minus $\bar{X}_2$, and the denominator is the square root of an unbiased estimate, the variance of $\bar{X}_1$ minus $\bar{X}_2$. Further, Sato (1937) showed that when the assumptions of the *t* test are not violated, the *t* test is a uniformly most powerful statistical test (as cited in Hsu, 1938). Therefore, to properly use the *t* test to infer probabilistically that there is a difference between two means, the assumptions that must be met are as follows: the observations of the dependent variable must follow a normal distribution, the population variances for the two groups must be equal, that is, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, and the most crucial of the assumptions is that the observations must be independent (Hays, 1994; Stevens, 1996; Lissitz &

Chardos, 1975; Zimmerman, Williams & Zumbo, 1992).

Although it is important to carefully consider and follow the assumptions underlying the *t* distribution, the *t* test has been found to be robust to moderate violations of the normality and homogeneity of variance assumptions. The effects of violating these two assumptions has been extensively studied, and the results are fairly consistent in that moderately violating these two assumptions has relatively small consequences on the outcome of the *t* test, especially if the sample size in the two groups are nearly equal and not extremely small (Boneau, 1960; Hays, 1994).

Another consistent finding regarding one of the assumptions of the *t* test is that it is *not* robust when the assumption of independence has been violated, that is, when there is a degree of dependence amongst the observations within a group (Kurita, 1996; Lissitz & Chardos, 1975; Zimmerman, 1997; Zimmerman & et al. 1992). Studies utilizing Monte Carlo simulations have suggested that when the observations in a group are correlated with one another, the nominal alpha level is no longer the Type I error rate. The amount of discrepancy between the nominal alpha level and the Type I error rate is a function of the sample size, degree of nonindependence and also whether the correlation is positive or negative. Lissitz and Chardos (1975) showed that when a positive correlation is introduced within a group the Type I error rate increases, however if a negative correlation is an attribute within a group the Type II error rate increases. If there is a positive correlation within the two groups, the distribution develops into one that is platykurtic in appearance. That is, the distribution is more “flat” than a normal probability curve (mesokurtic), and it has a larger variance. On the other hand, if negative correlations exist amongst the observations within groups, the distribution is leptokurtic. This leptokurtic distribution has a smaller variance, the mode is “taller,” and the tails are shorter, when compared to the *t* distribution. This presumably leads to an increase in Type II errors. The greater the degree of correlation amongst the observations in each group the more platykurtic or leptokurtic the distribution becomes (depending of course on whether the correlation is positive or negative).

The reason that the assumption of independence is so crucial to understand and evaluate is because in scientific research nonindependence exists amongst observations on a consistent basis. Kruskal (1988) states that in “most real cases there is noticeable dependence between phenomena” and that “independence seems rare in nature” (p. 934). With this in mind, it is imperative to know and understand what happens to statistical tests, such as the *t* test, when the independence assumption is violated under specified and controlled conditions. A better knowledge base of the results obtained under controlled situations will increase the understanding of actual *t* tests performed on real data.

In a study of nonindependent samples using the one-sample *t* test, Zimmerman, Williams, and Zumbo (1992) suggested a correction term for the denominator of the *t* test. This correction term approximately returned the Type I error rate back to the nominal alpha level by making the denominator larger than it otherwise would have been using the standard one sample *t* test formula. Although this formula appears to help with the problem of alpha level distortion, this method does not appear to have gained much acceptance in psychology as of yet.

Independence can be defined as a lack of association between two or more occurrences. These “occurrences” can be events, variables, people, outcomes or any other type of observation(s). When these occurrences are independent, knowing information about one gives rise to no information about any of the other occurrences. Thus, when knowing information about one occurrence provides some information about other occurrences, by definition, the outcomes are not independent of one another (Yaremko, Harari, Harrison & Lynn, 1986).

Examples of nonindependence in empirical research are easy to conceptualize. One example from Lissitz & Chardos (1975) that seems fairly common is as follows. Consider an experiment in which the behavior of participants who have been involved in some sort of therapy group are to be evaluated for the effect of a certain type of treatment. The experimenter randomly assigns four therapy groups to treatment "A" and four to treatment "B." After the conclusion of the last therapy session the experimenter uses a *t* test to evaluate the difference between means from treatment "A" and "B" on some measure. If the experimenter uses the individual scores from each person within each treatment to calculate the *t* test, a violation of the crucial assumption of independence has occurred. Since the same environment, therapist, and the participants themselves have influenced one another, these people, within each of the four groups from treatment "A" and "B," are no longer independent of one another on many measures, one of course being the end result of the treatment. Therefore, the participants' scores on the post-therapy test will not be independent of one another. In this example the correct experimental unit is not each person, but each of the four therapy groups under treatment "A" and "B." The reason that the correct unit of analysis is each group instead of the individual scores is because the groups are presumably independent of one another, even though the observations within each group are correlated to a certain degree. The temptation to use each participant's score instead of each of the eight group's scores as the unit of analysis may seem logical at first, however, it is evident that the people within each group are no longer independent of one another. Utilizing a *t* test in this situation would violate the assumption of independence and as stated previously would cause the nominal alpha level to be different than the Type I error rate.

An example from Stevens (1996) concerning educational research can occur when two teaching methods are to be evaluated at the end of the year by some test. Like the therapy example, the correct unit of analysis is each classroom, not the individual scores from each of the students within each teaching method. Because of the classroom environment and the interaction that occurred amongst the students, on many measures the students would no longer be independent of one another. If the individual scores from the students were used to calculate the *t* test, the independence assumption would be severely violated, since the scores (and the students themselves) are no longer independent of one another. The proper way to analyze the data in this example would be to use each of the classroom means as the units of analysis to determine the *t* ratio. Thus, the degrees of freedom are the number of classrooms in method "A" plus the number of classrooms in method "B" minus two. Even careful researchers analyze experiments similar to this one incorrectly. In a review of the "best" journals since 1980, Hykle, Stevens, & Markle (1993) found that 80% of analyses of this type of study were done incorrectly (as cited in Stevens, 1996).

In view of the fact that it is easy to speculate how frequently in psychology or other social sciences data occur that are not independent, it should be noted that the natural sciences are by no means immune to violating the assumption of independence. An example from Zolman (1993) is as follows. Suppose a biologist took tissue samples from a few animals in order to experiment on them in various ways. The tissue samples taken are 40 kidney nephrons (nephrons are the basic unit of the kidney) from four rats (10 nephrons from each rat). After the kidney nephrons have been randomly assigned to two conditions (20 nephrons each) for experimental manipulation, a *t* test is used to determine if there are significant differences between the two treatment means. The unit of analysis in this example should not be the number or nephrons, but the number of rats from which they came. The reason is because kidney nephrons that come from the same rat are not independent. Since the nephrons came from the same environment, were subjected to the same lifestyle effects, and were formed from the same organism's biological functions, the nephrons from a given rat are, of course, not independent of one another.

Many more examples could be listed in which observations are somehow related to one another as a result of an interaction between them or from some natural process(es). However, for a final example in which the dependency amongst occurrences may not be so obvious is in common psychological experiments. Consider the students in an introductory psychology class that must participate in an experiment for course credit. If a participant does Experiment "X," likes it, then tells his/her friends to sign up for it because it is easy or fun, a certain degree of dependency can arise. The friends that were told to sign up for Experiment "X" may come to the experiment with a certain mental set that is a function of how the previous participant performed or what the previous participant told them. This introduces a certain degree of nonindependence into the study that usually is not known by the researcher(s). Depending on what type of experiment is being performed, dependency can arise from the effects of the area or college campus where the experiments are being conducted, because of a common teacher and the method or style of teaching used, and also because of the interaction that students have with one another throughout the class (Lissitz & Chardos (1975). The amount of dependency is probably not great, but it nonetheless often exists.

Method

A SAS (1996) program was written that allowed for a Monte Carlo simulation of two separate distributions both having a variance of one and a mean of zero. A specified population correlation coefficient, ρ, was a characteristic of the population from which the samples were randomly drawn. The program generated a multivariate normal distribution by first randomly selecting one number from a standard normal distribution. The next step in the program's functioning generated n random numbers (where *n* is equal to the group size) that were independent of one another; again from a standard normal distribution. The n numbers were then combined with the first random number (derived from the first step) and these numbers were correlated to one another by the extent specified in the program's instructions. This procedure was independently performed again for group two. Using the standard equation for a two-sample *t* test, a *t* value was calculated. This procedure was repeated 1,000,000 times for each value of ρ and each ν. Both groups had an equal number of "subjects" ($n_1 = n_2$) and ρ was the same within each group. However, the observations from group one were independent of the observations from group two. The variance-covariance matrix (which always had ones in the principal diagonal, that is $\sigma^2 = 1$) was manipulated by changing the off diagonals to a specified covariance. Since the variance of both groups had a value of one throughout the present study, the term covariance and correlation can be used interchangeably; this is because $\rho = \sigma_{xy}/\sigma_x\sigma_y$.

Procedure

A Monte Carlo simulation was conducted that randomly sampled from a specified distribution which had a predetermined ρ and degrees of freedom, ν. From this simulation 1,000,000 *t* tests were performed for each ρ and ν used in the study in order to obtain the empirically generated critical values of the particular distribution. The "critical values" as defined here were determined by finding the point that divided the rejection region from the nonrejection region for both the negative and the positive sides of the distribution. The mean of the absolute value for the lower value and the upper value was found, and this mean value is what will be referred to as the empirically generated critical value in the remainder of the present study. The formula that was used to calculate the *t* values was the standard equation for a two-sample *t* test.

To demonstrate the program's proper functioning, independent observations were used within each of the two groups so that $\rho = 0$, therefore, the off diagonals in the variance-covariance matrix were all zero. Because the observations were independent, the distribution of empirically generated *t*'s should have distributed as the *t* distribution does. Table 1 is a comparison sample using the critical values from four ν from the empirically generated *t* values compared to the critical values of the *t* distribution.

Insert Table 1 about here

As is evident from Table 1, the critical values of the *t* distribution and those of the empirically generated distribution are virtually the same. The absolute differences between the critical values of the *t* distribution and the empirically generated distribution displayed in Table 1 ranged from 0 to .047 with a mean of .005 and a standard deviation of .009. Table 1 demonstrates the program's capabilities by closely replicating the *t* distribution when the assumptions of the *t* test were met.

Results

Table 2 gives the findings of the Monte Carlo simulations for varying degrees of nonindependence. These data are consistent with previous studies regarding the inflation of the nominal alpha level when a positive correlation is introduced to the observations within groups. Located in Table 2 are the critical values for the empirically generated *t* distributions (as well as the theoretical *t* distribution for comparative purposes) when various degrees of nonindependence amongst observations existed within groups.

Insert Table 2 about here

This table reports ρ of .05, .20, .40, .50, and .80. The ν that was used in this table are even integers from 2-30, 40, 50, 60 and 120 which provided for 760 empirically generated critical values. The mean difference between the absolute negative *t* value and the positive *t* value obtained for the critical values displayed in Table 2 was .0229 with a standard deviation of .0764.

Table 2 resembles the results from the Lissitz & Chardos (1975) study when ν = 60, the only ν used in their study. Lissitz and Chardos also used only 10,000 *t* replications whereas the present study used 1,000,000 replications. Accordingly, the empirical values of Table 1 are closer to the values in the *t* table than are those obtained by Lissitz and Chardos. Furthermore, their study gave the percent of *t*'s beyond the tabled critical values, while Table 2 in the present study gives proposed critical values.

A peculiar relationship was found to exist between the amount of nonindependence and the degrees of freedom. In a study by Scariano and Davenport (1987), it was shown that in a one-way analysis of variance having a positive correlation within groups, the greater ν in the F test the more the Type I error rate became inflated (as cited in Stevens, 1996). However, when the critical values obtained in the present study were plotted, a surprising curvilinear relationship was found to exist. Figure 1 shows the critical values plotted for the empirically generated *t* distributions as a function of sample size and degree of correlation.

Insert Figure 1 about here

Figure 1 shows that for small degrees of freedom the empirical critical values start off high and fall sharply (note that this is also true of the *t* distribution as well). Although the critical values of the *t* distribution decrease as ν increases, the empirically generated critical values fall for very small ν (2 and 4), but they all increase after four, the greatest increase being for $\rho = .80$.

Using 10 and 20 degrees of freedom as an example, the *t* distribution's critical value of $\alpha = .05$, two-tailed, are 2.228 and 2.086 respectively. Contrast this with the case that $\rho = .20$. Using the same degrees of freedom and significance level, the critical values are 3.528 and 4.041. The *t* distribution's value *drops* .142 units but the correlated distribution's difference *rose* by .513 units. This is the case when the proportion of variance that can be accounted for is a mere four percent. Another example is when eight and 60 degrees of freedom are compared. Whereas the *t* distribution's critical values are 2.306 and 2.000, using the same significance level as before, the critical values drop .306 units. However, when there is a slight correlation amongst observations within groups, $\rho = .05$, the critical values are 2.594 and 3.246. Therefore, when $\rho = .05$ (when only one quarter of a percent of the variance is accounted for), the critical values *rise* .652 units. This demonstrates that the greater the degrees of freedom with nonindependent groups, the less robust the *t* test is and the more likely there will be a Type I error. Even a seemingly insignificant degree of nonindependence, such as $\rho = .05$, can cause an inflation of the alpha level and could lead to misleading conclusions.

Discussion

The results of the present computer simulation are consistent with other similar studies (Lissitz & Chardos, 1975; Zimmerman & et. al., 1992; Zimmerman, 1997). However, the curvilinear relationships of the critical values plotted as a function of the number of degrees of freedom and amount of correlation within groups, has not been previously reported. These curvilinear characteristics of dependent groups show that unless $\nu \leq$ four, the greater the number of degrees of freedom the higher the critical value must be for the obtained *t* value to be significant, if in fact there is a degree of correlation within the groups.

When a researcher thinks that his/her observations are correlated, Stevens (1996) suggests using a more stringent alpha level. This suggestion by Stevens clearly has some validity to it, but what may now be more appropriate is to use the Empirically Generated Critical Value Table provided in Table 2 of this paper. These values are each based on 1,000,000 sample *t*'s drawn from specified distributions and are believed to be very stable with regards to the information that they convey. It is believed that a computed *t* that is greater than the critical value in Table 2 having the same ρ and ν as is used in the Empirically Generated Critical Value Table, may be appropriately viewed as statistically significant under the chosen significance level. That is, if the observations are nonindependent to the extent of $\rho = .20$, two groups of 11 participants each would require a *t* of 4.041 to be significant at the .05 significance level for a two-tailed test. Contrast this with the *t* distribution's critical value of 2.086. This 1.955 difference is substantial and could increase Type I errors substantially.

Perhaps the most difficult job of researchers is to maintain a bias free study. Box (1954) suggests using randomisation to control for nonindependence, but he realized that sometimes "data occur" in which there is no way to control for violating the independence assumption (p. 484). What many users of statistical tests fail to realize is just how easy it is to violate assumptions and how such crucial assumptions are often violated. Scheffé (1959) states that assumptions "can be violated in many more ways than they can be satisfied" (p. 331). Peckham, like Scheffé, says that assumptions of statistical tests are seldom met in empirical research (as cited in Papanastasion, 1982). This knowledge of the difficulty in meeting assumptions, coupled with the results of the effects of not meeting certain assumptions, as this study has shown, is a bit disturbing. Many decisions rest upon significant differences between means evaluated by the *t* test or related tests. One major assumption that cannot be violated if the *t* test is to remain valid is the assumption of independence. If this assumption is violated, the nominal α level in the *t* table can be much too small, which of course can lead to claiming significance when the null hypothesis is true. To use the critical values in the *t* table one must obtain independent samples, which is not always easy to do, or use a critical value table that takes into account nonindependence for the *t* distribution, such as the one provided in Table 2 of the present study. Use of this table will presumably reduce Type I errors by requiring a larger *t* for claiming significance at a given α level and return the Type I error rate to approximately the nominal α level chosen.

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Table 1

Comparison of the *t* Distribution's Critical Values with the Empirically Generated Critical Values when Observations are Independent ($\rho = 0$)

		0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001
	1Q=	**0.40**	**0.25**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**
	2Q=	**0.80**	**0.50**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**
ρ=0 and ν=4	*t Distribution*	*0.271*	*0.741*	*1.533*	*2.132*	*2.776*	*3.747*	*4.604*	*7.173*
	Empirically Generated Distribution	0.271	0.741	1.536	2.136	2.786	3.762	4.617	7.126
	Absolute Difference:	0.000	0.000	0.003	0.004	0.010	0.015	0.013	0.047
ρ=0 and ν=20	*t Distribution*	*0.257*	*0.687*	*1.325*	*1.725*	*2.086*	*2.528*	*2.845*	*3.552*
	Empirically Generated Distribution	0.256	0.687	1.329	1.726	2.085	2.529	2.847	3.546
	Absolute Difference:	0.001	0.000	0.004	0.001	0.001	0.001	0.002	0.006
ρ=0 and ν=60	*t Distribution*	*0.254*	*0.679*	*1.296*	*1.671*	*2.000*	*2.390*	*2.660*	*3.232*
	Empirically Generated Distribution	0.254	0.679	1.297	1.671	2.004	2.393	2.663	3.221
	Absolute Difference:	0.000	0.000	0.001	0.000	0.004	0.003	0.003	0.011
ρ=0 and ν=120	*t Distribution*	*0.254*	*0.677*	*1.289*	*1.658*	*1.980*	*2.358*	*2.617*	*3.160*
	Empirically Generated Distribution	0.254	0.678	1.292	1.662	1.984	2.361	2.619	3.161
	Absolute Difference:	0.000	0.001	0.003	0.004	0.004	0.003	0.002	0.001

Absolute differences ranged from 0 to .047 with a mean of .005 and a standard deviation of .009

Table 2

Critical *t* Values and Empirically Generated Critical *t* Values for Specified ρ and ν

Table 2

Critical t Values and Empirically Generated Critical t Values for Specified ρ and ν

	1Q=	**0.40**	**0.25**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**
	2Q=	**0.80**	**0.50**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**
ρ	**ν**	***Critical Values of the Theoretical t Distribution***							
0	**2**	*0.289*	*0.816*	*1.886*	*2.920*	*4.303*	*6.965*	*9.925*	*22.326*
0	**4**	*0.271*	*0.741*	*1.533*	*2.132*	*2.776*	*3.747*	*4.604*	*7.173*
0	**6**	*0.265*	*0.718*	*1.440*	*1.943*	*2.447*	*3.143*	*3.707*	*5.208*
0	**8**	*0.262*	*0.706*	*1.397*	*1.860*	*2.306*	*2.896*	*3.355*	*4.501*
0	**10**	*0.260*	*0.700*	*1.372*	*1.812*	*2.228*	*2.764*	*3.169*	*4.144*
0	**12**	*0.259*	*0.695*	*1.356*	*1.782*	*2.179*	*2.681*	*3.055*	*3.930*
0	**14**	*0.258*	*0.692*	*1.345*	*1.761*	*2.145*	*2.624*	*2.977*	*3.787*
0	**16**	*0.258*	*0.690*	*1.337*	*1.746*	*2.120*	*2.583*	*2.921*	*3.686*
0	**18**	*0.257*	*0.688*	*1.330*	*1.734*	*2.101*	*2.552*	*2.878*	*3.610*
0	**20**	*0.257*	*0.687*	*1.325*	*1.725*	*2.086*	*2.528*	*2.845*	*3.552*
0	**22**	*0.256*	*0.686*	*1.321*	*1.717*	*2.074*	*2.508*	*2.819*	*3.505*
0	**24**	*0.256*	*0.685*	*1.318*	*1.711*	*2.064*	*2.492*	*2.797*	*3.467*
0	**26**	*0.256*	*0.684*	*1.315*	*1.706*	*2.056*	*2.479*	*2.779*	*3.435*
0	**28**	*0.256*	*0.683*	*1.313*	*1.701*	*2.048*	*2.467*	*2.763*	*3.408*
0	**30**	*0.256*	*0.683*	*1.311*	*1.699*	*2.045*	*2.462*	*2.756*	*3.396*
0	**40**	*0.255*	*0.681*	*1.303*	*1.684*	*2.021*	*2.423*	*2.704*	*3.307*
0	**50**	*0.255*	*0.679*	*1.299*	*1.676*	*2.009*	*2.403*	*2.678*	*3.261*
0	**60**	*0.254*	*0.679*	*1.296*	*1.671*	*2.000*	*2.390*	*2.660*	*3.232*
0	**120**	*0.254*	*0.677*	*1.289*	*1.658*	*1.980*	*2.358*	*2.617*	*3.160*
0.05	**2**	0.304	0.859	1.987	3.077	4.532	7.344	10.485	23.563
0.05	**4**	0.292	0.798	1.650	2.293	2.986	4.048	4.974	7.740
0.05	**6**	0.292	0.789	1.583	2.136	2.696	3.457	4.078	5.746
0.05	**8**	0.295	0.794	1.572	2.091	2.594	3.261	3.780	5.099
0.05	**10**	0.298	0.802	1.575	2.079	2.559	3.175	3.639	4.760
0.05	**12**	0.303	0.814	1.586	2.087	2.552	3.139	3.569	4.609
0.05	**14**	0.307	0.826	1.605	2.102	2.561	3.129	3.543	4.509
0.05	**16**	0.311	0.837	1.623	2.120	2.575	3.135	3.543	4.491
0.05	**18**	0.318	0.851	1.646	2.144	2.599	3.157	3.566	4.457
0.05	**20**	0.323	0.863	1.666	2.167	2.621	3.176	3.580	4.469
0.05	**22**	0.323	0.875	1.687	2.193	2.650	3.203	3.601	4.476
0.05	**24**	0.332	0.889	1.712	2.220	2.682	3.239	3.639	4.501
0.05	**26**	0.336	0.901	1.733	2.246	2.707	3.264	3.660	4.527
0.05	**28**	0.342	0.915	1.760	2.281	2.744	3.304	3.698	4.576
0.05	**30**	0.346	0.926	1.781	2.303	2.772	3.335	3.732	4.603
0.05	**40**	0.371	0.989	1.895	2.447	2.935	3.522	3.933	4.797
0.05	**50**	0.392	1.045	2.000	2.581	3.094	3.704	4.127	5.016
0.05	**60**	0.413	1.101	2.104	2.712	3.246	3.880	4.324	5.239
0.05	**120**	0.521	1.390	2.646	3.408	4.069	4.843	5.381	6.459
0.20	**2**	0.354	1.002	2.312	3.585	5.287	8.543	12.175	27.326
0.20	**4**	0.357	0.981	2.028	2.821	3.675	4.975	6.092	9.441
0.20	**6**	0.375	1.014	2.034	2.745	3.459	4.448	5.241	7.382
0.20	**8**	0.394	1.059	2.097	2.789	3.459	4.357	5.058	6.799
0.20	**10**	0.417	1.108	2.172	2.869	3.528	4.376	5.018	6.529
0.20	**12**	0.431	1.153	2.251	2.961	3.624	4.464	5.079	6.521
0.20	**14**	0.448	1.200	2.330	3.053	3.718	4.543	5.159	6.579
0.20	**16**	0.465	1.244	2.414	3.147	3.820	4.659	5.265	6.644
0.20	**18**	0.481	1.289	2.491	3.250	3.941	4.776	5.401	6.753
0.20	**20**	0.495	1.328	2.566	3.341	4.041	4.896	5.513	6.902
0.20	**22**	0.512	1.370	2.641	3.434	4.143	5.015	5.64	7.009
0.20	**24**	0.527	1.411	2.721	3.532	4.260	5.149	5.777	7.179
0.20	**26**	0.544	1.451	2.793	3.616	4.351	5.247	5.873	7.290
0.20	**28**	0.555	1.490	2.865	3.715	4.467	5.391	6.035	7.450
0.20	**30**	0.572	1.528	2.932	3.797	4.571	5.491	6.149	7.595
0.20	**40**	0.639	1.703	3.262	4.213	5.056	6.070	6.780	8.279
0.20	**50**	0.697	1.856	3.553	4.593	5.507	6.592	7.350	8.970
0.20	**60**	0.753	2.005	3.835	4.942	5.912	7.077	7.874	9.565
0.20	**120**	1.022	2.727	5.201	6.689	7.986	8.750	10.559	12.706

Table 2

ρ	1Q= / 2Q= ν	0.40 / 0.80	0.25 / 0.50	0.10 / 0.20	0.05 / 0.10	0.025 / 0.05	0.01 / 0.02	0.005 / 0.01	0.001 / 0.002
0.40	**2**	0.441	1.250	2.885	4.470	6.586	10.640	15.182	33.941
0.40	**4**	0.468	1.284	2.658	3.697	4.821	6.504	7.987	12.350
0.40	**6**	0.507	1.372	2.753	3.717	4.678	6.016	7.080	9.940
0.40	**8**	0.546	1.469	2.909	3.870	4.802	6.046	7.013	9.385
0.40	**10**	0.582	1.568	3.073	4.060	4.993	6.167	7.098	9.256
0.40	**12**	0.618	1.656	3.229	4.252	5.200	6.406	7.306	9.367
0.40	**14**	0.648	1.745	3.386	4.430	5.400	6.610	7.499	9.555
0.40	**16**	0.681	1.823	3.540	4.619	5.608	6.841	7.727	9.760
0.40	**18**	0.711	1.906	3.686	4.809	5.825	7.077	7.984	10.001
0.40	**20**	0.737	1.980	3.827	4.978	6.027	7.301	8.227	10.315
0.40	**22**	0.769	2.056	3.961	5.147	6.221	7.519	8.466	10.544
0.40	**24**	0.796	2.131	4.101	5.329	6.428	7.758	8.713	10.822
0.40	**26**	0.824	2.200	4.228	5.476	6.596	7.943	8.909	11.059
0.40	**28**	0.848	2.267	4.360	5.654	6.798	8.196	9.181	11.305
0.40	**30**	0.873	2.334	4.478	5.801	6.978	8.397	9.380	11.577
0.40	**40**	0.989	2.637	5.051	6.521	7.832	9.404	10.486	12.825
0.40	**50**	1.092	2.902	5.556	7.174	8.609	10.312	11.505	14.047
0.40	**60**	1.186	3.154	6.035	7.778	9.308	11.123	12.390	15.078
0.40	**120**	1.638	4.368	8.325	10.710	12.787	15.236	16.907	20.350
0.50	**2**	0.4991	1.418	3.274	5.067	7.478	12.069	17.242	38.733
0.50	**4**	0.541	1.482	3.069	4.270	5.567	7.498	9.226	14.343
0.50	**6**	0.592	1.602	3.216	4.340	5.462	7.021	8.269	11.635
0.50	**8**	0.642	1.729	3.423	4.555	5.651	7.110	8.247	11.027
0.50	**10**	0.689	1.856	3.636	4.806	5.907	7.335	8.401	10.940
0.50	**12**	0.735	1.968	3.838	5.054	6.177	7.607	8.676	11.146
0.50	**14**	0.774	2.079	4.036	5.278	6.436	7.886	8.943	11.406
0.50	**16**	0.813	2.187	4.233	5.520	6.701	8.176	9.242	11.655
0.50	**18**	0.852	2.280	4.416	5.757	6.975	8.485	9.566	11.996
0.50	**20**	0.887	2.377	4.589	5.972	7.231	8.761	9.872	12.358
0.50	**22**	0.923	2.470	4.763	6.187	7.474	9.044	10.168	12.678
0.50	**24**	0.958	2.565	4.941	6.413	7.732	9.346	10.483	13.029
0.50	**26**	0.991	2.652	5.096	6.598	7.945	9.570	10.733	13.313
0.50	**28**	1.023	2.735	5.261	6.818	8.201	9.884	11.066	13.651
0.50	**30**	1.054	2.817	5.408	7.001	8.420	10.132	11.367	13.965
0.50	**40**	1.198	3.193	6.118	7.899	9.483	11.391	12.701	15.530
0.50	**50**	1.326	3.522	6.743	8.707	10.445	12.522	13.963	17.049
0.50	**60**	1.439	3.836	7.332	9.449	11.312	13.516	15.059	18.340
0.50	**120**	1.999	5.328	10.158	13.063	15.601	15.580	20.619	24.832
0.80	**2**	0.868	2.458	5.674	8.770	12.903	20.982	29.85	66.730
0.80	**4**	0.978	2.673	5.530	7.701	10.032	13.534	16.600	26.037
0.80	**6**	1.093	2.954	5.930	8.004	10.073	12.902	15.225	21.391
0.80	**8**	1.201	3.236	6.398	8.521	10.575	13.310	15.415	20.578
0.80	**10**	1.305	3.504	6.873	9.082	11.162	13.855	15.872	20.651
0.80	**12**	1.402	3.748	7.308	9.616	11.767	14.505	16.518	21.221
0.80	**14**	1.484	3.979	7.719	10.109	12.322	15.094	17.131	21.799
0.80	**16**	1.566	4.205	8.143	10.614	12.884	15.732	17.750	22.433
0.80	**18**	1.645	4.402	8.519	11.112	13.465	16.372	18.485	23.163
0.80	**20**	1.720	4.604	8.883	11.560	13.993	16.972	19.094	23.986
0.80	**22**	1.790	4.802	9.253	12.013	14.495	17.558	19.739	24.630
0.80	**24**	1.866	4.986	9.614	12.471	15.054	18.174	20.388	25.336
0.80	**26**	1.935	5.173	9.926	12.867	15.487	18.664	20.961	25.945
0.80	**28**	1.999	5.334	10.266	13.311	16.014	19.293	21.593	26.671
0.80	**30**	2.065	5.514	10.575	13.692	16.460	19.805	22.150	27.288
0.80	**40**	2.351	6.275	12.026	15.513	18.636	22.377	24.978	30.490
0.80	**50**	2.610	6.942	13.292	17.163	20.599	24.691	27.537	33.622
0.80	**60**	2.844	7.581	14.484	18.671	22.362	26.692	29.741	36.289
0.80	**120**	3.968	10.592	20.194	25.959	31.009	36.914	40.967	49.349

Figure 1

Critical *t* Values Along with the Critical Values of the Empirically Generated Distributions Plotted as a Functions of ρ and ν

Figure 1

Critical *t* Values Along with the Critical Values of the Empirically Generated Distributions Plotted as a Function of ρ and ν