CSE 30151 Theory of Computing: Homework 6
CFL Language, TMs

Version 1: Oct. 26, 2017

Instructions

- Unless otherwise specified, all problems from "the book" are from Version 3. When a problem in the International Edition is different from Version 3, the problem will be listed as V3:x.yy/IE:x.zz, where x.zz is the equivalent number. When Version 2 has a different number, it will be listed as V3:x.yy/V2:x.zz. If either IE or V2 does not have a matching number, the problem text will be duplicated.
- You can prepare your solutions however you like (handwriting, LaTeX, etc.), but you must submit them in **legible PDF**. You can scan written solutions in the library or using a smartphone (with a scanner app like CamScanner). It is up to you to ensure that submissions are legible. REMEMBER THAT IF WE CAN"T READ IT OR SCAN IS CUT OFF¡ YOU DON"T GET A GRADE FOR IT.
- Please give every PDF file a unique filename.
  - If you're making a complete submission (all problems), name your PDF file `netid-hw5.pdf`, where `netid` is replaced with your `NetID`.
  - If you're submitting some problems now and other problems later, name your file `netid-hw5-123.pdf`, where `123` is replaced with just the problems you are submitting now.
  - If you use the same filename twice, only the most recent version will be graded.
  - The time of submission is the time the most recent file was uploaded.
- If you use LaTeX and want to draw something like a state diagram, consider using the `tikz` package. A reference document is on the website under "Assignments".
- You may also find the website http://madebyevan.com/fsm/ a useful tool for drawing state diagrams via drop and drag. It will output both .png image files and latex in the tikz format.
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Practice Problems

These problems are from the book, and most have solutions listed for them. They are listed here for you to practice on as needed and any answers you generate **should not** be submitted. You are free to discuss these with others, but you are not allowed to post solutions to any public forum.

1. V3:2.33, V2:2.33, IE: 2.36 Using pumping lemma
2. 3.1 sequence of TM configurations
3. 3.5 Questions about TM capabilities
4. 3.8a design a TM
5. 3.8b design a TM
6. 3.10 A write-once TM
7. 3.15a decidable languages are closed under union
8. 3.16a recognizable languages are closed under union

Book Exercises

These problems are found in the text book and are to be answered and submitted by each student. You are to solve them yourself. Use of solution manuals from any source or shared solutions is a violation of the ND Honor Code. You are also not allowed to show your solutions to another student.

1. (5 points) V3:2.30a, V2:2.30a. Not in IE: Show $A = \{0^n1^n0^n1^n | n \geq 0\}$ is not context free
2. (5 points) V3:2.32, V2:2.32. Not in IE: Let $\Sigma = \{1, 2, 3, 4\}$. and $C = \{w|$ w has as many 1s as 2s, and as many 3s as 4s $\}$. Show C is not CF. Note the condition $|vxy| \leq p$ is valuable here.
3. (5 points) V3:2.34, V2:2.34, IE:2.46 Pumping value
4. (5 points) 3.4 Formal definition of an enumerator. Assume the second tape is where each string is generated before printout. Make sure you define some mechanism that signals when the string on the second tape is a "complete" string in the language, and a new string is to be started that is initially blank (like outputting a "carriage return" or using the "accept" state as a signal that second tape now includes a complete string in the language). Since there is no "input string" on the initial woek tape, for consistency let Σ be the alphabet of the strings that are printed out.

   The question "Include a definition of the enumerated language" means define what happens on a transition and exactly when the outside world knows a string is complete. How do you signal writing a character to print tape? Also ensure you discuss how the enumerator signals that for a finite language it has output the last string.
5. (5 points) 3.15e decidable languages closed under intersection
6. (5 points) 3.16b recognizable languages closed under concatenation

Non-book Problems

The following problems are not found in the text book. You are to solve them yourself. Use of any resource you used other than the text book or class notes must be cited. You are also not allowed to show your solutions to another student.

7. (5 points) Consider the language L = $\{a^n | n \geq 1$ and is prime$\}$. Show L is not context free using the pumping lemma.
8. (5 points) Show that the language from $\Sigma = \{a, b, c\}$ where the only constraint is that the number of a's in any string is less than the number of b's, and the number of b's is less than the number of c's, is not context free.
9. (5 points) Describe how to convert any FA into a TM, i.e. how to translate each component of the tuple of a FA into a TM. In particular discuss how to translate a transition, and then rewrite the DFA of Fig. 1.12 as a TM. Show all parts of the formal definition.
10. (5 points) Design a TM where the tape initially has two equal length substrings from $\{0, 1\}$, separated by a #. Change the first substring to one that represents the logical bit-by-bit AND of the two strings. You are free to destroy the second string as you wish. You can assume the second substring is always exactly the same length as the first. Show all parts of the formal definition.