

# CSE 30151 Theory of Computing: Homework 7

## Decidability and Reducibility

Version 1: Nov. 21, 2017

### Instructions

- Unless otherwise specified, all problems from “the book” are from Version 3. When a problem in the International Edition is different from Version 3, the problem will be listed as V3:x.yy/IE:x.zz, where x.zz is the equivalent number. When Version 2 has a different number, it will be listed as V3:x.yy/V2:x.zz. If either IE or V2 does not have a matching number, the problem text will be duplicated.
- You can prepare your solutions however you like (handwriting, L<sup>A</sup>T<sub>E</sub>X, etc.), but you must submit them in **legible PDF**. You can scan written solutions in the library or using a smartphone (with a scanner app like CamScanner). It is up to you to ensure that submissions are legible. REMEMBER THAT IF WE CAN’T READ IT OR SCAN IS CUT OFF; YOU DON’T GET A GRADE FOR IT.
- Please give every PDF file a unique filename.
  - If you’re making a complete submission (all problems), name your PDF file `netid-hw5.pdf`, where `netid` is replaced with your `NetID`.
  - If you’re submitting some problems now and other problems later, name your file `netid-hw5-123.pdf`, where `123` is replaced with just the problems you are submitting now.
  - If you use the same filename twice, only the most recent version will be graded.
  - The time of submission is the time the most recent file was uploaded.
- If you use L<sup>A</sup>T<sub>E</sub>X and want to draw something like a state diagram, consider using the `tikz` package. A reference document is on the website under “Assignments”.
- You may also find the website <http://madebyevan.com/fsm/> a useful tool for drawing state diagrams via drop and drag. It will output both .png image files and latex in the tikz format.
- Submit your PDF file in Sakai. Don’t forget to click the Submit (or Resubmit) button!

### Practice Problems

These problems are from the book, and most have solutions listed for them. They are listed here for you to practice on as needed and any answers you generate **should not** be submitted. You are free to discuss these with others, but you are not allowed to post solutions to any public forum.

1. V3:4.1 Membership in decidable languages.
2. V3:4.5 Complement of  $E_{TM}$
3. V3:4.ba,d function properties
4. V3:4.10, 4.12, 4.14, 4.25 prove language decidability
5. V3:5.5 Reducibility
6. V3:5.7
7. V3:5.10 and 5.11 properties of TMs
8. V3:7.1c,d V3:7.2 c,d Big O notation
9. V3:7.8 Problems in P

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**Book Exercises**

These problems are found in the text book and are to be answered and submitted by each student. You are to solve them yourself. Use of solution manuals from any source or shared solutions is a violation of the ND Honor Code. You are also not allowed to show your solutions to another student.

Note: for these problems "high level" descriptions of TMs that are deciders or recognizers are fine (as in proofs of Theorem 4.2 on p. 195 or Theorem 4.3 on 196). Also when you are creating high-level descriptions and you are using algorithms as steps, refer to the theorem from the book or a prior language known to be decidable that guarantees the algorithm or machine you are using in the step exists. Be sure to include a step that ensures the input is formatted properly (e.g. "verify that  $\langle M \rangle$  is a validly formatted DFA description, and reject if not").

1. (5 points) 4.2 Show decidability
2. (5 points) V3:4.11, V2:4.10, IE:4.31. Show decidability. Hint: consider what the pumping lemma tells you about the size of languages and when they are infinite.
3. (5 points) 5.1 Show undecidability. Include a description of the language.
4. (10 points ) 5.2. Show co-Turing Recognizability. Remember co-Turing recognizable means that a TM can be built to recognize (not decide) the complement of a language. Include a description of the language being recognized.
5. (5 points) 5.3. Post Correspondence Problem. Note this does not need a proof, just solve the PCP puzzle using any (and only) number of tiles from the specified set.
6. (10 points) V3:5.12, V2:5.12, IE:5.28 Undecidability. Show the language. Hint: study the solutions to 5.10 and 5.11 (IE:5.26,27).
7. (5 points) 7.1a,b,e 7.2b,e Big O notation
8. (5 point) 7.9 membership in P