pp. 292-311. **The Class NP** (Sec. 7.3)

- Issue: many interesting problems seem to have only "brute force" algorithms of exponential time
- (p. 292) **HAMPATH = {(G,s,t)|G is graph with Hamiltonian path from s to t}**
  - **Hamiltonian Path** from s to t goes thru every other vertex
  - Easy decider by variant of algorithm for PATH
    - Modify PATH to generate all possible paths
    - With test after each one to **verify** if path is Hamiltonian
    - **Verifier** runs in polynomial time
      - Keep a list of vertices
      - Follow path
      - Cross off matching vertex as each step
      - At end, if all vertices crossed off, accept; else reject
    - But the generator from PATH is *exponential*!
  - ***No known polynomial HAMPATH algorithm!***
- (p. 293) **COMPOSITES = {x|x=pq, for p,q>1}**
  - Verifier is trivial
  - No known polynomial generator
- (p. 293) Not all problems have polynomial verifiers
  - e.g. not(HAMPATH)

- (p. 293) Definition 7.18. **A verifier for language A is an algorithm V, where A = {w|V accepts <w,c> for some string c}**
  - For all w in A there is some c where V accepts <w,c>
  - c is “extra information” called a **certificate** or **proof**
  - e.g. for above problems, c is a “guess” of answer
    - HAMPATH: a path that is a Hamiltonian
    - COMPOSITES: a divisor
  - The ones that work are solutions to problem
  - Equivalent to stating “a solution exists”
  - Time for V expressed as a function of w
    - **Polynomial Time Verifier** for V runs in polynomial time
  - Language A is **polynomially verifiable** if it has a polynomial time verifier
- p. 294: example of NTM N_1 for HAMPATH that works in “nondeterministic polynomial time”
  - Remember time of NTM is time used by longest branch
  - Step 1 “generates” a solution (magically) as a series of vertex #s
  - Step 2 ensures no repeats
  - Step 3 ensures starts at s and ends at p
  - Step 4 is the **polynomial verifier** that checks edges exist

(p. 294) Definition 7.19: **<u>NP</u> is class of languages that have polynomial time verifiers**

- NP stands for "Non deterministic Polynomial"
- HAMPATH and COMPOSITES both in class NP

- (p, 294) **Theorem 7.20 Language A is in NP iff it is decided by some polynomial time NTM**
  - Proof: if A in NP then decided by NTM in polynomial time
    - Let V be matching polynomial verifier for A of $O(n^k)$
    - Define NTM N as follows: For input w of length n,
      - Nondeterministically select string c of length $\leq n^k$
        - c is "solution"
      - Run V on <w,c>
      - If V accepts, accept, else reject
  - Proof: if Poly time NTM N exists, then A is in NP
    - V constructed on <w,c> as follows
      - Simulate N on input w, treating each symbol of c as description of NTM choice to make at each step
      - If this branch accepts, accept, else reject
  - For HAMPATH
    - W is <G,s,t>
    - c is a path

- (p. 293) **NTIME(t(n)) = {L|L is language decided by some O(t(n)) time NTM}**
- **NP = U_k NTIME(n^k) for all k**
- (p. 295) **CLIQUE = {<G,k>|G undirected graph with k-clique}** in in NP
  - k-clique = set of k vertices with edges between each pair of vertices in set
  - (p. 296) Proof by demonstrating polynomial time verifier
- (p. 297) **SUBSET-SUM = {<S,t>|S = {x_1, …x_k}** and for some {y_1, …, y_l} subset of S and Σy_i = t}
- (p. 299) **SAT** ={<wff>|wff a satisfiable Boolean formula}
  - **wff** is well-formed-formula constructed from
    - Boolean variables
    - Boolean operations AND, OR, NOT
  - **Satisfiability**: test if there is a substitution of 0s and 1s to variables that makes the wff true
- Summary:
  - ***P = class that can be decided quickly***
  - ***NP = class that can be verified quickly***
- **Biggest question in CS: Is P = NP, or P a subset of NP**?
  - Is there a language in NP that is not in P?