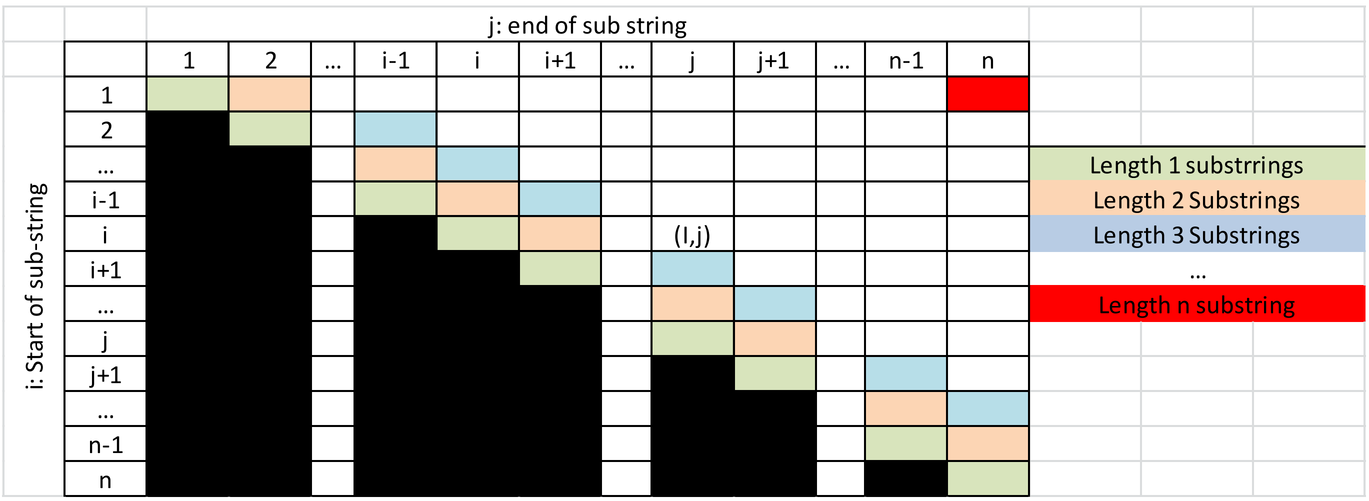
pp. 285-291. **The Class P** (Sec. 7.2)

- (p. 286) Definition: **Class P = class of all languages decidable by 1-tape TM in polynomial time**
  - P = union of all TIME(n^k) problems for all k
  - Key: if some fancy TM has polynomial time algorithm for some problem, then so does a simple 1-tape TM
  - Key: close match to problems solvable on real computers
- Approach to analyzing algorithms for membership in P
  - See if polynomial upper bound on number of stages
  - See if each stage solvable by polynomial time TM
- All the following are in P
  - (p. 287) PATH = {<G,s,t>| G is directed graph (V,E), with path from s to t}
    - O(N): Place mark on vertex s
    - O(|V||E|): Repeat until no more marked
      - If edge (a,b) leads from marked a to unmarked b, then mark b (at most |E| times per vertex)
    - O(|V|): If t is marked, accept, else reject
    - At most |V|+2 stages, totaling O(|V||E|) steps
  - (p. 289) RELPRIME = {<x,y>|x,y relatively prime}
  - (p. 323) Other languages in P: Ex. 7.8-11, 7.13, 7.14, 7.17

(p. 290) **Theorem 7.16. Every CFL has a decider is in P**

- i.e. if L expressible by a CFG, then there exists polynomial time decider
- Leads to (p. 322, Ex. 7.4) **closure of P under union, concatenation, and complement**
  - And Ex. 7.15 P **closed under star**
- Consider following as first notional proof of Theorem:
  - L = {w|w in a CFL from some CFG G}
  - Express G in **Chomsky Normal Form** (p. 109)
    - All rules of form A->BC or A->t
  - If w in L, |w|=n, any derivation has at most 2n-1 steps
  - Notionally, for particular w, decider for L tries all derivations with 2n-1 steps
  - But this is potentially exponential not polynomial

- Better algorithm uses **dynamic programming**:
  - Given a string w, record solution to smaller problems in nxn table (n=|w|) so don't need some terms to be recomputed over and over



- Cell(i,j) = set of variables that generate $w_i w_{i+1} \dots w_j$
  - Fill in for string lengths in order 1, 2, …
    - For length 1, look at A->b rules & record A in cell
  - Use entries for shorter strings in longer ones
  - To generate substring of length k-i+1, split $w_i \dots w_{k+1}$ into 2 pieces in k different ways:
    - $(w_i,\ w_{i+1} \dots w_{k+1})$, $(w_i w_{i+1},\ w_{i+2} \dots w_{k+1})$, $(w_i \dots w_{i+2},\ w_{i+3} \dots w_{k+1})$, … $(w_i \dots w_k,\ w_{k+1})$
  - For each split, examine each rule A->BC to see if B is generator for 1st part, & C a generator for 2nd part
  - If both, add A to Table(i,j)
- If S is in Table(1,n) then accept, else reject

- See page 291 for algorithm
- Algorithm executes in $O(n^3)$ time!
- Try Problem 7.4 on p. 322

w=baba		j: end of sub string				
		1	2	3	4	S->RT
i: Start of sub	1					R->TR|a
	2					T->TR|b
	3					
	4					