pp. 202. The Universal Turing Machine (Sec. 4.2)

- Remember A_{TM} = {<M,w>| M a TM that accepts w}
- KEY: we can write a recognizer U, but not a decider
- Let's design a Universal TM U which recognizes A_{TM}
 - Input is <M,w>
 - <M> is an "encoding" of M
 - <w> is an encoding of a string to be given to M
 - U *simulates* M processing w
 - If M ever enters its accept state, then U accepts,
 - If M ever enters its reject state, then U rejects
 - Note this loops on <M,w> if M loops on w
 - That's why it is NOT a decider
 - If we could determine if M never halts on w then U could halt in reject. That would make U a decider.

- What does U look like?
 - Long history of trying for "smallest" description
 - Typically defined in terms of "(m,n)"
 - m = # of states
 - n = # of tape symbols
- Turing's original 1936 machine
 - http://onlinelibrary.wiley.com/doi/10.1112/plms/s2-43.6.544/epdf
 - Turing used 7 symbols to encode M { A, C, D, R, L, N, ; }
 - M's State q_i encoded as DAⁱ
 - If M's $\Sigma = \{a_0, a_1, ...a_k\}$, then U's encoding of $a_i = DC^i$
 - a₀ is the blank
 - If M has $\delta(q_i, a_i) = (q_k, a_i, L)$ (similar for R)
 - Then encode as ;DAⁱDC^jDC^lLDA^k
 - Actually code spread out on every other tape cell
 - Termed the "F-cells" which are never changed
 - With intermediate "E-cells" used to "tag" where M is
 - Similar to the "*" we used earlier
 - Last transition has a ";;" following it
 - Additional symbols {e, ::, u, v, x, y, z} written into E-cells
 - Processing:
 - Find transition that corresponds to current state and w_i
 - Update w_i and mark next state. Test for accept/reject

 From: Neary, Turlough; Woods, Damien (2009), "Four Small Universal Turing Machines", Fundamenta Informaticae, 91

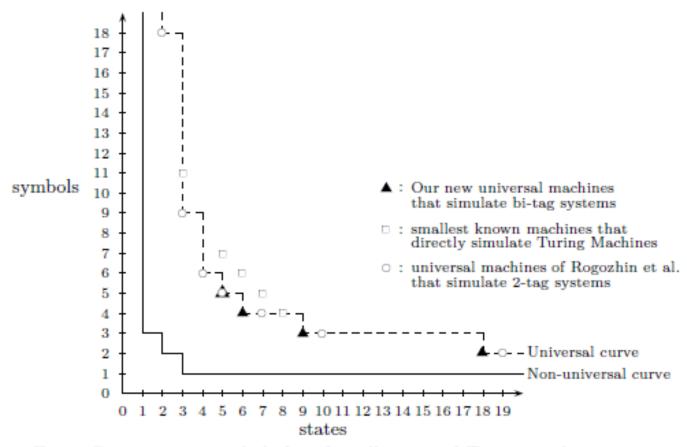


Fig. 1: Current state-symbol plot of small universal Turing machines.

- See following for more head-wall fun!!!
 - https://en.wikipedia.org/wiki/Universal Turing machine
 - A Business Card Universal Turing Machine:
 http://alvyray.com/CreativeCommons/BizCardUniversalTuringMachine v2.2.pdf
 - 4 Small UTMs: http://www.dna.caltech.edu/~woods/download/NearyWoodsMCU07.pdf
 - The World's smallest UTM: https://czyborra.com/thti/rogozhin1996.pdf