Outline: Circuits

- **Lecture A**
  - Physics 101
  - Semiconductors for Dummies
  - CMOS Transistors for logic designers
- **Lecture B**
  - NMOS Logic
  - CMOS Inverter and NAND Gate Operation
  - CMOS Gate Design
  - Adders
  - Multipliers
- **Lecture C**
  - Pass Transistors
  - Tri-states
  - Multiplexors
  - Latches
  - FlipFlops
  - Barrel Shifters
EE101

- **Charge (Q):** # of excess electrons beyond # of protons
  - Units of **Coulombs:** 1 coulomb = 6.24151 × 10^{18} electrons

- **Voltage (V):** electrical potential difference between two points in space
  - Point with lower potential called **negative**
  - Point with higher potential called **positive**

- **Current (I):** flow of electrons across a voltage potential
  - Electrons travel **from negative to positive**
  - Units of **Amperes** (A), where 1 A = 1 coulomb moving in 1 sec

- **Resistance (R):** property of material controlling amount of current: I = V/R or R = V/I or V = IR

- **Capacitance (C):** property of material to store a charge & form a voltage potential
  - V = Q/C = potential when Q charge is stored in a capacitance C
  - Units of capacitance are **Farads**, where 1 volt = 1 Coulomb / 1 Farad

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**Underlying Physics 101**

- **Atoms constructed from**
  - **Protons:** positively charged heavy particles
  - **Neutrons:** heavy particles with no charge
  - **Electrons:** very light negatively charged particles

- Protons & neutrons bind together in **nucleus**

- In neutral atom, # protons = # electrons
  - **Ion:** numbers not equal; atom is **charged**

- **Electrons circle nucleus in orbitals**
  - Energy of orbital higher the further from nucleus
  - Higher energy makes it easier to escape
  - Up to 2 electrons per orbital

- **Orbitals group into shells** (K, L, M, N, O, ...)
  - # electrons in shell n = 2n^2
  - Electrons fill lower shells first

- **Each shell consists of subshells s, p, d, f,...**
  - Different subshells have slightly different energies

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Electronic Band Structure

- **Valence Shell:** outermost shell with electrons
- **Covalent bonds:** Atoms near each other with incomplete valence shell “share” electrons that fill valence shell

- **Shells “above”** valence shell
  - Have higher energies
  - Allow electrons to escape much easier

- In solids, differences between energy levels in neighboring orbitals becomes small (but non 0), and group into **BANDS**
  - **Conduction Bands:** Those above valence
  - If conduction band overlaps valence band – electrons can move
    - I.E. “Current” can flow easily
Metals and Insulators

Metal:
- typically valence shell only partially filled
- may readily lose electrons from valence
- atoms become positively charged “holes”
- surrounded by “sea of free electrons”

Insulator (non-metal):
- valence shell is near full
- tough to pry out an electron
- when near a metal, loses an electron to form strong ionic bond

<table>
<thead>
<tr>
<th>Element</th>
<th>Symbol</th>
<th>Atomic #</th>
<th>Valence Shell</th>
<th>Electrons in Valence</th>
<th>Electrons to fill valence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>Al</td>
<td>13</td>
<td>M</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>Copper</td>
<td>Cu</td>
<td>29</td>
<td>N</td>
<td>1</td>
<td>31</td>
</tr>
<tr>
<td>Gold</td>
<td>Au</td>
<td>79</td>
<td>O</td>
<td>19</td>
<td>31</td>
</tr>
<tr>
<td>Boron</td>
<td>B</td>
<td>5</td>
<td>L</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>P</td>
<td>15</td>
<td>M</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Silicon</td>
<td>Si</td>
<td>14</td>
<td>M</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Germanium</td>
<td>Ge</td>
<td>32</td>
<td>N</td>
<td>4</td>
<td>28</td>
</tr>
</tbody>
</table>
What Makes Electrons Move?

- **Coulomb’s Law**: Force between two separated charged particles
  - \( \text{Force} = \frac{-q_i q_j}{4\pi \varepsilon r^2} \)
    - \( q_i, q_j \): charges of two particles in coulombs
    - \( r \): distance between charges
    - \( \varepsilon \) ("episilon"): permittivity of material
  - If signs of \( q \) the same, force is repulsive
  - If signs different, force is attractive

- Mass of electron ~ 1/1836'th of proton

- **Electric field around charge** \( i \) at radius \( r \): \( E = \frac{q_i}{4\pi \varepsilon r^2} \)
  - In units of Volts: “force per unit charge”
  - Force on particle \( j \) is thus \( Eq_j \)

- **Electric field between 2 parallel plates with equal & opposite charges**: \( E = \frac{V}{d} \)

**Resistance**

1. Conduction electrons drawn from region nearest +
2. This leaves positive “holes”
3. Which attract electrons from next region
4. …
5. At “-” end, electrons are drawn from potential

**Current flow** is hole flow from + into material and out towards negative end. *(OPPOSITE electrons)*

How much flow depends on **Voltage** and **Resistance**

**Current = Voltage / Resistance** or **Resistance = Voltage / Current**

Resistance in units of **ohms** (\( \Omega \)), or \( 1\Omega = 1 \text{ Volt} / 1 \text{ Amp} \)

**Resistance** \( R = \frac{\rho L}{A} \):
- \( A \): Cross section area
- \( L \): Length of material
- \( \rho \) (rho): Resistivity of material (in units of ohm meter)

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity, ( \rho ) (ohm-meter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals</td>
<td>10¹⁰</td>
</tr>
<tr>
<td>Semiconductors</td>
<td>variable</td>
</tr>
<tr>
<td>Electrolytes</td>
<td>variable</td>
</tr>
<tr>
<td>Insulators</td>
<td>10¹⁶</td>
</tr>
<tr>
<td>Superconductors</td>
<td>0 (exactly)</td>
</tr>
<tr>
<td></td>
<td>( 10²⁴ )</td>
</tr>
</tbody>
</table>
**Kirchhoff’s Laws**

**Resistance** \( R = \rho \frac{L}{A} \):
- \( A \) = cross section area
- \( L \) = length of material
- \( \rho \) = resistivity of material (in units of ohm meter)

\[ R = R_1 + R_2 \]

Why?

\[ R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{(R_1 + R_2)}{(R_1 \times R_2)} \]

Why?

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**Current Density (J)**

**Current Density** \( J = \) Current/Area = Amps/m\(^2\) = V/(\( \rho \)L)

(J is a “vector” with direction along path of electrons)

Why do we care? Too high a current density can cause:
- **Metal migration**
- **Burnout**
- **Skin effect**
Capacitors

Basic device:
• 2 conducting plates (of area A)
• Separated by a distance d
• With “dielectric” insulator material between
  • SiO₂ typical on chips

\[ C = \frac{\varepsilon A}{d} \]

where \( \varepsilon \) = “permittivity” of material, often written as \( \varepsilon_r \)
• \( \varepsilon_0 \) = permittivity of vacuum = 8.854x10⁻¹² F m⁻¹
• \( \varepsilon_r \) = permittivity of material relative to vacuum
• 1 F = 1 Coulomb / 1 Volt

What happens with large \( \varepsilon_r \)?

Capacitors in Action

(a) Both plates have no charge
(b) Switch closes
1. Electrons attracted off of left plate into battery
2. Left plate becomes positively charged
3. Atoms in dielectric have electrons attracted to left
4. This pushes positive charge to right
5. Electrons on right plate attracted to leftmost side
6. Leaving positive charge in right wire
7. Which is neutralized by electrons from – battery
8. Current stops when charge \( Q = CV \)

Which Direction is Current Flow?
Capacitor Circuits

\[ C = \varepsilon A/d \]

\[ C = 1/(1/C_1 + 1/C_2) = (C_1 \times C_2)/(C_1 + C_2) \]

Why?

\[ C = C_1 + C_2 \]

Why?

RC Circuits

What do we know?
1. \( I_R = I_C = I \)
2. \( V = V_C + V_R \) or \( V_R = V - V_C \)
3. \( I_R = I = V_R / R \)
4. \( I_C = I = C \frac{dV_C}{dt} \)
   or
   \( V_C = V - RC \frac{dV_C}{dt} \)
   or
   \( V_C = V(1 - e^{-t/RC}) \)

RC = “time constant”

What does a negative current flow mean?
Semiconductors

Key Materials: Si, B, P

<table>
<thead>
<tr>
<th>Material</th>
<th>Atomic #</th>
<th>Electrons per Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon Si</td>
<td>14</td>
<td>2, 8, 4</td>
</tr>
<tr>
<td>Boron B</td>
<td>5</td>
<td>2, 3</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>15</td>
<td>2, 8, 5</td>
</tr>
</tbody>
</table>
**Silicon**

\(\text{Si}:\) Silicon – has 4 electrons and space for 4 more in valence shell

\[ T = 0^\circ \text{K} \]

\[ T > 0^\circ \text{K} \]

thermal excitation

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**Doping: Mixing Into Pure Silicon**

P or Phosphorus

- One more electron in valence than Si
- Known as a **donor** atom

B or Boron

- One less electron in valence than Si
- Known as a **acceptor** atom

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PN Junctions

- A junction between p-type and n-type semiconductor forms a diode.
- Current flows only in one direction

\[
\begin{array}{c|c}
\text{Free holes} & \text{Free electrons} \\
p-type & n-type \\
\end{array}
\]

anode \hspace{1cm} cathode

\[
\begin{align*}
V & \leq 0, \text{ no current flows} \\
& \cdot V \text{ reinforces depletion zone} \\
& \cdot 0 < V < V_{bi} \\
& \cdot V \text{ cannot overcome attraction in junction} \\
& \cdot \text{Depletion zone shrinks} \\
& \cdot \text{No current flows} \\
V & \geq V_{bi}, \\
& \cdot \text{Electrons pulled left from P-junction} \\
& \cdot \text{Holes in N-type filled with electrons from } V \\
& \cdot \text{Current flows}
\end{align*}
\]

The “Why” of PN Junctions

- Electrons move from N to P materials
- With holes that “move right”
- Until charge near surface at PN repels more
nMOS Transistor

- Four terminals: gate, source, drain, body
- Gate – oxide – body stack looks like a capacitor
  - Gate and body are conductors
  - SiO₂ (oxide) is a very good insulator
  - Called metal – oxide – semiconductor (MOS) capacitor
    - Even though gate is often not metal

![nMOS Transistor Diagram]

nMOS Operation

- Body is commonly tied to ground (0 V)
- When the gate is at a low voltage:
  - P-type body is at low voltage
  - Source-body and drain-body diodes are OFF
  - No current flows, transistor is OFF
nMOS Operation Cont.

- When the gate is at a high voltage:
  - Positive charge on gate of MOS capacitor
  - Negative charge attracted to body
  - Inverts a channel under gate to n-type
  - **Now electrons** can flow through n-type silicon from **source** through channel to **drain**
  - Current flows from the drain to the source **(Why?)**
  - The transistor is **ON**

```
+---+    +---+    +---+
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
+---+    +---+    +---+
   Gate  Body  Source  Drain
```

pMOS Transistor

- Similar, but doping and voltages reversed
  - Body tied to **high voltage** \((V_{dd})\)
  - Gate low: transistor **ON**
  - Gate high: transistor **OFF**
  - **“Bubble”** on gate symbol indicates inverted behavior
  - Holes (and current) flow from the **source** to the **drain**

```
+---+    +---+    +---+
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
|   |    |   |    |   |
+---+    +---+    +---+
   Gate  Source  Drain
```

Body = 1
Circuit Symbols

**NMOS**
- Drain
- Gate
- Source

**PMOS**
- Source
- Drain
- Gate

- When not shown, standard is:
  - NMOS Body is tied to $V_{ss}$, the logic negative voltage supply.
  - PMOS Body is tied to $V_{cc}$, the logic positive voltage supply.

**Key Transistor Characteristics**

- **Threshold Voltage** on gate that causes current to flow
- **Resistance** of channel that conducts current
- **Capacitance** of Gate

- Key Controlling Physical Parameters
  - Length ($L$) of channel
  - Width ($W$) of Channel
  - Thickness ($t_{ox}$) of gate insulator
What Happens as We Make the Following Smaller?

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Width</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold Voltage</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The “Water” Analog

- Think of electrons as “drops of water”
- Water flows from high pressure (high voltage) to low pressure (low voltage)
- Flow of water can fill up a container (capacitor)
  - “Height” of water = voltage
- How high a certain amount of water fills a container depends on area of container
  - Wider area = higher capacitance => more water flow needed to raise level
- Transistors like toilet “flapper valves”
  - Turn “on” when water level reaches threshold
  - Assume threshold = 0.5 height in following
CMOS Switching Circuits

- Computing machines built from switches
- Encoding: voltage at points in circuit
- Operation: moving charge around

A Simple Inverter

PMOS conducts when charge (water) level is above switching threshold

PMOS conducts when level below

No conduction after output container is full

Signal Propagation (1)

$t < 0$
Vin = 0
Vout = 0

$t = 0$
Vin => 1
Vout = 0
Signal Propagation (2)

\[ t = 1 \]
\[ V_{in} = 1 \]
\[ V_{out} = 0 \]

\[ t = 2 \]
\[ V_{in} = 1 \]
\[ V_{out} = 1 \]

Delay and Energy Definitions

- **Propagation Delay**
  - time to fill output container to 50%
  - time to charge output capacitance to 50%

- **Switching Energy**
  - weight x height of water moved
  - charge x voltage of charge transferred
  - \( Q x V = CV x V = CV^2 \)