Centrality

Peter M. Kogge

Centrality

- Id "most important" vertices in a graph
- "Importance" has multiple possibilities
  - Importance to "flow" in graph as a network
  - Importance to "cohesiveness" of graph/subgraph
- Choice is app dependent

- https://en.wikipedia.org/wiki/Centrality

Network Flows

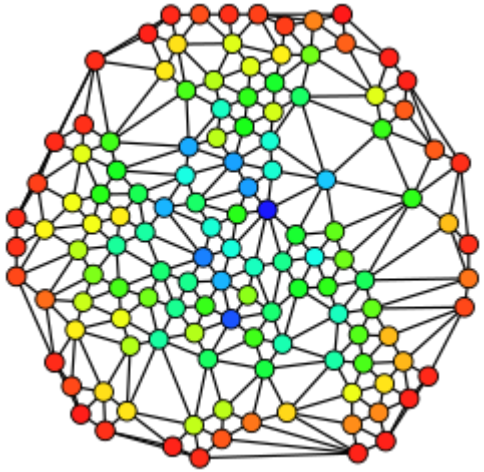
- "Something" "flows" from vertex to vertex over edges thru vertices
  - Indivisible transfers from one vertex to another
  - Duplication so source and destination have "copies"
  - Broadcast over all outgoing edges
- Options on constraining "path" of flow
  - **Geodesics**: shortest path
  - **Paths**: no vertex visited more than once
  - **Trails**: no edge traversed more than once
  - **Walks**: repeated vertices/edges possible
- Alternative on how centrality constructed
  - **Radial**: walks start/end on specific vertices
  - **Medial**: walks that "pass thru" some vertex

Vertex Centrality Metrics

- **Degree(v)**: degree of edges incident on v
  - In-degree: measure of "friendship"
  - Out-degree: measure of "gregariousness"
- **Closeness(v):** reciprocal of sum of length of shortest path between v and all other vertices
  - Often normalized by dividing by N-1
- **Harmonic(v):** sum of reciprocal of length of shortest path between v and all other vertices

Vertex Betweenness Centrality

- Relates to "how important" vertex is to "shortest paths"
- For vertex v, iterate over all vertex pairs (s,t)
  - Compute shortest path s and t σ(s,t)
  - Count # that go thru v
  - Form fraction
  - Add to betweenness for v
- Variation: **Katz** centrality
  - Weight distant edges on paths lower



https://en.wikipedia.org/wiki/Centrality#/media/File:Graph_betweenness.svg

Eigenvector Centrality

- Assigns score x[v] to each vertex v based on scores of vertices to which it is connected
- Assume A[u,v] = 1 if edge from u to v
- Then $Ax = \lambda x$ (an eigenvector)
- Example: PageRank

Other Centrality Metrics

- **Percolation $PC^t(v)$:** importance of a vertex as something "spreads" thru graph
  - Each "time step" advances the spread
- **Cross-clique X(v):** # of cliques of which v is a member
- **Freeman centralization**: Uses some other centrality metric to compare how "central" most central vertices are
- **Dissimilarity**: like eigenvector but with multiplication by a dissimilarity matrix