Centrality

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Centrality

- Id “most important” vertices in a graph
- “Importance” has multiple possibilities
  - Importance to “flow” in graph as a network
  - Importance to “cohesiveness” of graph/subgraph
- Choice is app dependent

Network Flows

• “Something” “flows” from vertex to vertex over edges thru vertices
  – Indivisible transfers from one vertex to another
  – Duplication so source and destination have “copies”
  – Broadcast over all outgoing edges

• Options on constraining “path” of flow
  – Geodesics: shortest path
  – Paths: no vertex visited more than once
  – Trails: no edge traversed more than once
  – Walks: repeated vertices/edges possible

• Alternative on how centrality constructed
  – Radial: walks start/end on specific vertices
  – Medial: walks that “pass thru” some vertex
Vertex Centrality Metrics

• **Degree(v):** degree of edges incident on v
  – In-degree: measure of “friendship”
  – Out-degree: measure of “gregariousness”

• **Closeness(v):** reciprocal of sum of length of shortest path between v and all other vertices
  – Often normalized by dividing by N-1

• **Harmonic(v):** sum of reciprocal of length of shortest path between v and all other vertices
Vertex Betweenness Centrality

• Relates to “how important” vertex is to “shortest paths”

• For vertex \( v \), iterate over all vertex pairs \((s,t)\)
  - Compute shortest path \( s \) and \( t \) \( \sigma(s,t) \)
  - Count # that go thru \( v \)
  - Form fraction
  - Add to betweenness for \( v \)

• Variation: **Katz** centrality
  - Weight distant edges on paths lower

https://en.wikipedia.org/wiki/Centrality#/media/File:Graph_betweenness.svg
Eigenvector Centrality

- Assigns score $x[v]$ to each vertex $v$ based on scores of vertices to which it is connected
- Assume $A[u,v] = 1$ if edge from $u$ to $v$
- Then $Ax = \lambda x$ (an eigenvector)
- Example: PageRank
Other Centrality Metrics

- **Percolation PC$^t(v)$**: importance of a vertex as something “spreads” thru graph
  - Each “time step” advances the spread
- **Cross-clique X(v)**: # of cliques of which $v$ is a member
- **Freeman centralization**: Uses some other centrality metric to compare how “central” most central vertices are
- **Dissimilarity**: like eigenvector but with multiplication by a dissimilarity matrix