Problem 1: Formulate an optimal control problem that is relevant to your particular line of research. Clearly specify the problem in terms of the assumed dynamical control system, the target set, the admissible controls and the cost functional. Comment on the current status of this problem in your particular research area.

Problem 2: Given a piece of paper of area $A$ to make a rectangular box. Determine the maximum volume that can be obtained by formulating a constrained optimization problem in which none of the inequality constraints is active. Then use the necessary conditions for the unconstrained problem to identify a set of candidate solutions and then check to see which candidate solutions are feasible.

Problem 3: Consider the control system in Figure 1 and consider the following optimal control problem.

Find a finite control gain $k$ that minimizes $\int_0^\infty z^T(\tau)z(\tau)d\tau$ if the input signal $r$ is a unit step function.

Show that this problem has no solution if $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Determine if there is a solution when $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if so find the solution.

Problem 4: Consider the linear program

$$\text{maximize } 2x_1 + 3x_2$$
$$\text{subject to } x_1 + x_2 \leq 8$$
$$-x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

1. Write the KKT necessary conditions.

2. For each extreme point, verify whether or not the KKT conditions hold, both algebraically and geometrically. From this find the optimal solution.

Problem 5: Consider the optimization problem

$$\text{minimize: } f(x_1, x_2)$$
$$\text{subject to: } h_1(x_1, x_2) = x_1^2 - x_2$$
$$h_2(x) = x_2$$
Show that the optimal point $x^*$ is not a regular point of the constraints. Determine an objective function $f(x_1, x_2)$ for which there are no Lagrange multipliers $\lambda_1^*$ and $\lambda_2^*$ (not both zero) satisfying the first order necessary condition

$$\nabla f(x_1^*, x_2^*) + \lambda_1^* \nabla h_1(x_1^*, x_2^*) + \lambda_2^* \nabla h_2(x_1^*, x_2^*) = 0$$