Final Exam : Due Before Noon December 11, 2017 The exam has four problems which are ordered approximately in ascending order of difficulty. The first problem looks at a simple system and compares feedback linearization control to passivity-based control. The second problem is an I/O feedback linearization problem in which the output is defined as the difference of two states. The third problem examines an extension of backstepping to non-affine systems. The fourth problem investigates the global uniform ultimate boundedness of a Luenberger observer for a nonlinear process. As I said in class, we will keep the recitation hours for next week and I'll discuss/answer questions related to the course material. I cannot, of course, answer direct questions regarding the exam.

Problem 1: Consider the following system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\rho(x_1) + u \end{aligned}$$

where $\rho(x_1)x_1 > 0$ when $x_1 \neq 0$ and $\rho(0) = 0$.

- When $u = 0$, show that the origin of this system is Lyapunov stable.
- Use I/S feedback linearization to determine a control input u that asymptotically stabilizes the origin of the system.
- Determine an output $y = h(x_1, x_2)$ such that the input-output system from u to y is passive and determine an output feedback control that asymptotically stabilizes the origin of the system.

Problem 2: Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1x_2 - x_2^2 + u \\ \dot{x}_2 &= x_1x_2 - x_2^2 + u \\ \dot{x}_3 &= x_1 + x_1x_2 - x_2^2 - (x_3 - x_1)^3 + u \\ y &= x_1 - x_2 \end{aligned}$$

- Determine the system's normal form.
- Find a state feedback controller that renders the input-output map finite gain $\mathcal{L}_2$ stable.
- Is the origin of the controlled system also asymptotically stable?

Problem 3:

Consider the following augmented system

$$\begin{aligned} \dot{z} &= f(z, \xi) \\ \dot{\xi} &= u \end{aligned}$$

where $f(0, 0) = 0$. Assume there exists a control $\phi(z)$ such that the function $V : \mathbb{R} \to \mathbb{R}$ satisfies $\frac{\partial V}{\partial x} f(z, \phi(z)) \leq -W(z)$ for some positive definite function W.

- Use the backstepping procedure to find a control input $u(z, \xi)$ that asymptotically stabilizes the origin of the augmented system.

Problem 4: Consider a scalar process whose state $x(\cdot) : \mathbb{R}^+ \to \mathbb{R}$ satisfies the differential equation

$$\dot{x}(t) = -x^3(t) + w(t), \quad x(0) = x_0$$

for all $t \geq 0$. Assume that $w \in \mathcal{L}_\infty$ with $\|w\|_{\mathcal{L}_\infty} \leq 1$. We're interested in building an *observer* whose internal state $\hat{x}(\cdot) : \mathbb{R}^+ \to \mathbb{R}$ satisfies the differential equation

$$\dot{\hat{x}}(t) = -\hat{x}^3(t) + L(x(t) - \hat{x}(t)), \quad \hat{x}_0 = \hat{x}(0)$$

for all $t \geq 0$ where $L > 0$ is a positive constant representing the *observer's gain*. Let the estimation error be defined as $e(t) = x(t) - \hat{x}(t)$.

- Prove that the estimation error is globally uniformly ultimately bounded and determine an upper bound on the steady state estimation error.