# Acknowledgements

The author gratefully acknowledges the partial financial support of the National Science Foundation (CNS-1239222) and Notre Dame's Environmental Change Initiative during the early phases of this work. The author would like to acknowledge the contribution of Dr. Tua Augustinius Tamba for his dissertation work [112] on shock-induced regime shifts and the Fussmann predator-prey system [37], Dr. Jennifer Tank for discussions regarding stream ecology and nutrient enrichment, Dr. Stuart Jones for helping set up biological chemostat experiments, Dr. David Lodge for impressing on me the importance of non-equilibrium dynamics in biology, and finally Dr. Jessica Hellmann for numerous discussions regarding the role of resilience in engineering and ecology.

# Bibliography

- Ilan Adler, Mauricio GC Resende, Geraldo Veiga, and Narendra Karmarkar. An implementation of karmarkar's algorithm for linear programming. *Mathematical programming*, 44(1-3):297–335, 1989.
- [2] Panos J Antsaklis, James A Stiver, and Michael Lemmon. Hybrid system modeling and autonomous control systems. In *Hybrid systems*, pages 366–392. Springer, 1992.
- [3] Radhakisan Baheti and Helen Gill. Cyber-physical systems. *The impact of control technology*, 12(1):161–166, 2011.
- [4] BR Barmish and CL DeMarco. A new method for improvement of robustness bounds for linear state equations. In *Proceedings of the Princeton Conf. Inform. Sci. Syst*, 1986.
- [5] Reinette Biggs, Stephen R Carpenter, and William A Brock. Turning back from the brink: detecting an impending regime shift in time to avert it. *Proceedings of the National academy of Sciences*, 106(3):826–831, 2009.
- [6] Mogens Blanke, Michel Kinnaert, Jan Lunze, Marcel Staroswiecki, and J Schröder. *Diagnosis and fault-tolerant control*, volume 2. Springer, 2006.
- [7] Marc Bodson and Joseph E Groszkiewicz. Multivariable adaptive algorithms for reconfigurable flight control. *IEEE transactions on control systems technology*, 5(2):217–229, 1997.
- [8] Alice Boit, Neo D Martinez, Richard J Williams, and Ursula Gaedke. Mechanistic theory and modelling of complex food-web dynamics in lake constance. *Ecology letters*, 15(6):594–602, 2012.
- [9] Stephen Boyd, Laurent El Ghaoui, Eric Feron, and Venkataramanan Balakrishnan. *Linear matrix inequalities in system and control theory*, volume 15. Siam, 1994.
- [10] James H Brown, James F Gillooly, Andrew P Allen, Van M Savage, and Geoffrey B West. Toward a metabolic theory of ecology. *Ecology*, 85(7):1771–1789, 2004.

- [11] SR Carpenter, JJ Cole, ML Pace, R Batt, WA Brock, T Cline, J Coloso, JR Hodgson, JF Kitchell, DA Seekell, L Smith, and Weidel B. Early warnings of regime shifts: a whole-ecosystem experiment. *Science*, 332(6033):1079–1082, 2011.
- [12] Stephen R Carpenter, Donald Ludwig, and William A Brock. Management of eutrophication for lakes subject to potentially irreversible change. *Ecological applications*, 9(3):751–771, 1999.
- [13] Bruce L Clarke. *Stability of complex reaction networks*. John Wiley and Sons, Inc., 1980.
- [14] Charles C Conley. Isolated invariant sets and the Morse index. American Mathematical Soc., 1978.
- [15] Carsten Conradi, Dietrich Flockerzi, Jörg Raisch, and Jörg Stelling. Subnetwork analysis reveals dynamic features of complex (bio) chemical networks. *Proceedings* of the National Academy of Sciences, 104(49):19175–19180, 2007.
- [16] Robert Costanza, Ralph d'Arge, Rudolf de Groot, Stephen Farberk, Monica Grasso, Bruce Hannon, Karin Limburg, Shahid Naeem, Robert V O'Neill, Jose Paruelo, et al. The value of the world's ecosystem services and natural capital. *NATURE*, 387:253, 1997.
- [17] David Cox, John Little, and Donal O'shea. *Ideals, varieties, and algorithms*, volume 3. Springer, 1992.
- [18] Vasilis Dakos, Marten Scheffer, Egbert H van Nes, Victor Brovkin, Vladimir Petoukhov, and Hermann Held. Slowing down as an early warning signal for abrupt climate change. *Proceedings of the National Academy of Sciences*, 105(38):14308– 14312, 2008.
- [19] Georgi M Daskalov. Overfishing drives a trophic cascade in the black sea. Marine Ecology Progress Series, 225:53–63, 2002.
- [20] Georgi M Daskalov, Alexander N Grishin, Sergei Rodionov, and Vesselina Mihneva. Trophic cascades triggered by overfishing reveal possible mechanisms of ecosystem regime shifts. *Proceedings of the National Academy of Sciences*, 104(25):10518– 10523, 2007.
- [21] D.L. DeAngelis. Dynamics of nutrient cycling and food webs. CHAPMAN AND HALL, NEW YORK, NY(USA). 1992., 1992.
- [22] W. Decker and C. Lossen. *Computing in Algebraic Geometry : a quick start using SINGULAR*. Srpinger, 2006.
- [23] Peter Demenocal, Joseph Ortiz, Tom Guilderson, Jess Adkins, Michael Sarnthein, Linda Baker, and Martha Yarusinsky. Abrupt onset and termination of the african

humid period:: rapid climate responses to gradual insolation forcing. *Quaternary* science reviews, 19(1-5):347–361, 2000.

- [24] Robert J Diaz and Rutger Rosenberg. Spreading dead zones and consequences for marine ecosystems. *science*, 321(5891):926–929, 2008.
- [25] Ian Dobson. Distance to bifurcation in multidimensional parameter space: Margin sensitivity and closest bifurcations. In *Bifurcation Control*, pages 49–66. Springer, 2003.
- [26] Ian Dobson and Liming Lu. New methods for computing a closest saddle node bifurcation and worst case load power margin for voltage collapse. *Power Systems, IEEE Transactions on*, 8(3):905–913, 1993.
- [27] Eusebius J Doedel. Auto: A program for the automatic bifurcation analysis of autonomous systems. *Congr. Numer*, 30:265–284, 1981.
- [28] Eusebius J Doedel and Mark J Friedman. Numerical computation of heteroclinic orbits. In *Continuation Techniques and Bifurcation Problems*, pages 155–170. Springer, 1990.
- [29] K. L. S. Drury and D. M. Lodge. Using mean first passage times to quantify equilibrium resilience in perturbed intraguild predation systems. *Theoretical Ecology*, 2(1):41–51, 2009.
- [30] Robert W Easton. Geometric methods for discrete dynamical systems. Oxford University Press, Oxford, 1998.
- [31] Herbert Edelsbrunner and John Harer. *Computational topology: an introduction*. American Mathematical Soc., 2010.
- [32] L Egbert and I Halley. Stabilator reconfiguration flight testing on the f/a-18e/f. In *Proceedings of the SAE Control and Guidance Meeting*, March 2001.
- [33] Jonathan A Foley, Michael T Coe, Marten Scheffer, and Guiling Wang. Regime shifts in the sahara and sahel: interactions between ecological and climatic systems in northern africa. *Ecosystems*, 6(6):524–532, 2003.
- [34] Carl Folke. Resilience: The emergence of a perspective for social–ecological systems analyses. *Global environmental change*, 16(3):253–267, 2006.
- [35] Carl Folke, Steve Carpenter, Brian Walker, Marten Scheffer, Thomas Elmqvist, Lance Gunderson, and CS Holling. Regime shifts, resilience, and biodiversity in ecosystem management. *Annual Review of Ecology, Evolution, and Systematics*, pages 557–581, 2004.

- [36] Gary Froyland and Michael Dellnitz. Detecting and locating near-optimal almostinvariant sets and cycles. *SIAM Journal on Scientific Computing*, 24(6):1839–1863, 2003.
- [37] Gregor F Fussmann, Stephen P Ellner, Kyle W Shertzer, and Nelson G Hairston Jr. Crossing the hopf bifurcation in a live predator-prey system. *Science*, 290(5495):1358–1360, 2000.
- [38] Julien Gagneur and Steffen Klamt. Computation of elementary modes: a unifying framework and the new binary approach. *BMC bioinformatics*, 5(1):1, 2004.
- [39] Pascal Gahinet, Pierre Apkarian, and Mahmoud Chilali. Affine parameter-dependent lyapunov functions and real parametric uncertainty. *Automatic Control, IEEE Transactions on*, 41(3):436–442, 1996.
- [40] PM Gahinet, Arkadii Nemirovskii, Alan J Laub, and Mahmoud Chilali. The LMI control toolbox. In *IEEE Conference on Decision and Control*, volume 2, pages 2038–2038, 1994.
- [41] Robert Ghrist. Barcodes: the persistent topology of data. Bulletin of the American Mathematical Society, 45(1):61–75, 2008.
- [42] Gert-Martin Greuel, Gerhard Pfister, and Hans Schönemann. Singular—a computer algebra system for polynomial computations. In *Symbolic computation and automated reasoning*, pages 227–233. AK Peters, Ltd., 2001.
- [43] Peter M Groffman, Jill S Baron, Tamara Blett, Arthur J Gold, Iris Goodman, Lance H Gunderson, Barbara M Levinson, Margaret A Palmer, Hans W Paerl, Garry D Peterson, et al. Ecological thresholds: the key to successful environmental management or an important concept with no practical application? *Ecosystems*, 9(1):1–13, 2006.
- [44] Robert L Grossman, Anil Nerode, Anders P Ravn, and Hans Rischel. *Hybrid systems*, volume 736. Springer, 1993.
- [45] Lance H Gunderson. *Panarchy: understanding transformations in human and natural systems*. Island press, 2001.
- [46] Wassim M Haddad, VijaySekhar Chellaboina, and Qing Hui. Nonnegative and compartmental dynamical systems. Princeton University Press, 2010.
- [47] David P Hamilton, Cayelan C Carey, Lauri Arvola, Peter Arzberger, Carol Brewer, Jon J Cole, Evelyn Gaiser, Paul C Hanson, Bas W Ibelings, Eleanor Jennings, et al. A global lake ecological observatory network (gleon) for synthesising highfrequency sensor data for validation of deterministic ecological models. *Inland Waters*, 5(1):49–56, 2015.

- [48] Vera Hárs and János Tóth. On the inverse problem of reaction kinetics. In Colloquia Mathematica Societatis János Bolyai, (Szeged, Hungary, 1979) Qualitative Theory of Differential Equations (M. Farkas ed.), volume 30, pages 363–379, 1981.
- [49] Philip Hartman. Ordinary differential equations. Society for Industrial and Applied Mathematics, 2 edition, 2002.
- [50] Alan Hastings and Thomas Powell. Chaos in a three-species food chain. *Ecology*, 72(3):896–903, 1991.
- [51] A. Hatcher. Algebraic Topology. Cambridge University Press, 2002.
- [52] Richard J Hobbs. New models for ecosystem dynamics and restoration. Island Press, 2009.
- [53] Richard J Hobbs and David A Norton. Towards a conceptual framework for restoration ecology. *Restoration ecology*, 4(2):93–110, 1996.
- [54] Crawford S Holling. Resilience and stability of ecological systems. *Annual review* of ecology and systematics, pages 1–23, 1973.
- [55] Crawford S. Holling. Engineering resilience versus ecological resilience. In Peter Schulze, editor, *Engineering within ecological constraints*, pages 31–44. National Academy Press Washington, DC, 1996.
- [56] Crawford S Holling. Understanding the complexity of economic, ecological, and social systems. *Ecosystems*, 4(5):390–405, 2001.
- [57] Terence P Hughes. Catastrophes, phase shifts, and large-scale degradation of a caribbean coral reef. *Science*, 265(5178):1547–1551, 1994.
- [58] Terence P Hughes, Maria J Rodrigues, David R Bellwood, Daniela Ceccarelli, Ove Hoegh-Guldberg, Laurence McCook, Natalie Moltschaniwskyj, Morgan S Pratchett, Robert S Steneck, and Bette Willis. Phase shifts, herbivory, and the resilience of coral reefs to climate change. *Current Biology*, 17(4):360–365, 2007.
- [59] Hiroyuki Ichihara. Sum of squares based input-to-state stability analysis of polynomial nonlinear systems. SICE Journal of Control, Measurement, and System Integration, 5(4):218–225, 2012.
- [60] Michael Charles Irwin. *Smooth dynamical systems*, volume 17. World Scientific, 2001.
- [61] John A Jacquez et al. Compartmental analysis in biology and medicine. JSTOR, 1985.
- [62] John A Jacquez and Carl P Simon. Qualitative theory of compartmental systems. *Siam Review*, 35(1):43–79, 1993.

- [63] Holly P Jones and Oswald J Schmitz. Rapid recovery of damaged ecosystems. *PloS one*, 4(5):e5653, 2009.
- [64] Tomasz Kaczynski, Konstantin Mischaikow, and Marian Mrozek. Computational homology, volume 157. Springer Science & Business Media, 2006.
- [65] William D Kalies, Konstantin Mischaikow, and Robert CAM Vandervorst. An algorithmic approach to chain recurrence. *Foundations of Computational Mathematics*, 5(4):409–449, 2005.
- [66] Steffen Klamt, Julien Gagneur, and Axel von Kamp. Algorithmic approaches for computing elementary modes in large biochemical reaction networks. *IEE Proceedings-Systems Biology*, 152(4):249–255, 2005.
- [67] Xenofon D Koutsoukos, Panos J Antsaklis, James A Stiver, and Michael D Lemmon. Supervisory control of hybrid systems. *Proceedings of the IEEE*, 88(7):1026–1049, 2000.
- [68] Ratnesh Kumar and Vijay K Garg. *Modeling and control of logical discrete event systems*, volume 300. Springer Science & Business Media, 2012.
- [69] Harold Joseph Kushner. Stochastic stability. In *Stability of stochastic dynamical sys*tems, pages 97–124. Springer, 1972.
- [70] HJ Kushner. Finite time stochastic stability and the analysis of tracking systems. *Automatic Control, IEEE Transactions on*, 11(2):219–227, 1966.
- [71] Yuri A Kuznetsov. *Elements of applied bifurcation theory*, volume 112. Springer Science & Business Media, 2013.
- [72] MD Lemmon and TA Tamba. Using elementary flux modes to estimate the distance to regime shifts in kinetic systems. *IFAC-PapersOnLine*, 48(27):193–198, 2015.
- [73] Johan Lofberg. Yalmip: A toolbox for modeling and optimization in matlab. In Computer Aided Control Systems Design, 2004 IEEE International Symposium on, pages 284–289. IEEE, 2004.
- [74] Diego de S Madeira and Jürgen Adamy. Output feedback control of rational nonlinear systems: A new approach based on passivity indices. In 2016 IEEE 55th Conference on Decision and Control (CDC), pages 3880–3885. IEEE, 2016.
- [75] Audrey L Mayer, Christopher W Pawlowski, and Heriberto Cabezas. Fisher information and dynamic regime changes in ecological systems. *ecological modelling*, 195(1):72–82, 2006.
- [76] John Milnor. *Morse Theory*. Number 51 in Annals of mathematics studies. Princeton university press, 1963.

- [77] Konstantin Mischaikow. Conley index theory. In *Dynamical systems*, pages 119–207. Springer, 1995.
- [78] Konstantin Mischaikow. Topological techniques for efficient rigorous computation in dynamics. Acta Numerica, 11:435–477, 2002.
- [79] H Michael Möller. On decomposing systems of polynomial equations with finitely many solutions. *Applicable Algebra in Engineering, Communication and Computing*, 4(4):217–230, 1993.
- [80] Magnus Nyström, Carl Folke, and Fredrik Moberg. Coral reef disturbance and resilience in a human-dominated environment. *Trends in Ecology & Evolution*, 15(10):413–417, 2000.
- [81] Bernt Karsten Øksendal and Agnès Sulem. Applied stochastic control of jump diffusions, volume 498. Springer, 2005.
- [82] Jacob Palis and Stephen Smale. Structural stability theorems. In *Global Analysis* (*Proc. Sympos. Pure Math., Vol. XIV, Berkeley, Calif., 1968*), pages 223–231. World Scientific, 1970.
- [83] Antonis Papachristodoulou and Stephen Prajna. On the construction of lyapunov functions using the sum of squares decomposition. In *Proceedings of the 41st IEEE Conference on Decision and Control, 2002.*, volume 3, pages 3482–3487. IEEE, 2002.
- [84] P. A. Parrilo. Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization. PhD thesis, California Institute of Technology, Pasadena, CA, 2000.
- [85] Daniel Pauly, Villy Christensen, and Carl Walters. Ecopath, ecosim, and ecospace as tools for evaluating ecosystem impact of fisheries. *ICES Journal of Marine Science: Journal du Conseil*, 57(3):697–706, 2000.
- [86] Marios M Polycarpou and Arthur J Helmicki. Automated fault detection and accommodation: a learning systems approach. *IEEE Transactions on Systems, Man, and Cybernetics*, 25(11):1447–1458, 1995.
- [87] S. Prajna, A. Jadbabaie, and G. J. Pappas. A framework for worst-case and stochastic safety verification using barrier certificates. *Automatic Control, IEEE Transactions* on, 52(8):1415–1428, 2007.
- [88] S. Prajna, A. Papachristodoulou, and P.A. Parrilo. Introducing sostools: A general purpose sum of squares programming solver. In *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, volume 1, pages 741–746. IEEE, 2002.

- [89] P. Protter. Stochastic Integration and Differential Equations: Version 2.1, volume 21. Springer, 2005.
- [90] Mihai Putinar. Positive polynomials on compact semi-algebraic sets. *Indiana University Mathematics Journal*, 42(3):969–984, 1993.
- [91] Peter J Ramadge and W Murray Wonham. Supervisory control of a class of discrete event processes. *SIAM journal on control and optimization*, 25(1):206–230, 1987.
- [92] John Rinzel. A formal classification of bursting mechanisms in excitable systems. In *Mathematical topics in population biology, morphogenesis and neurosciences*, pages 267–281. Springer, 1987.
- [93] Michael L Rosenzweig and Robert H MacArthur. Graphical representation and stability conditions of predator-prey interactions. *The American Naturalist*, 97(895):209–223, 1963.
- [94] Marten Scheffer. Ecology of shallow lakes. Springer Science & Business Media, 2004.
- [95] Marten Scheffer, Jordi Bascompte, William A Brock, Victor Brovkin, Stephen R Carpenter, Vasilis Dakos, Hermann Held, Egbert H Van Nes, Max Rietkerk, and George Sugihara. Early-warning signals for critical transitions. *Nature*, 461(7260):53–59, 2009.
- [96] Marten Scheffer and Stephen R Carpenter. Catastrophic regime shifts in ecosystems: linking theory to observation. *Trends in ecology & evolution*, 18(12):648–656, 2003.
- [97] Marten Scheffer, Steve Carpenter, Jonathan A Foley, Carl Folke, and Brian Walker. Catastrophic shifts in ecosystems. *Nature*, 413(6856):591–596, 2001.
- [98] Marten Scheffer, SH Hosper, ML Meijer, Brian Moss, and Erik Jeppesen. Alternative equilibria in shallow lakes. *Trends in ecology & evolution*, 8(8):275–279, 1993.
- [99] Stefan Schuster and Claus Hilgetag. On elementary flux modes in biochemical reaction systems at steady state. *Journal of Biological Systems*, 2(02):165–182, 1994.
- [100] D.A. Seekell, S.R. Carpenter, T.J. Cline, and M.L. Pace. Conditional heteroskedasticity forecasts regime shift in a whole ecosystem experiment. *Ecosystems*, 15:741– 747, 2012.
- [101] Jianbo Shi and Jitendra Malik. Normalized cuts and image segmentation. Departmental Papers (CIS), page 107, 2000.
- [102] Richard M Sibly, James H Brown, and Astrid Kodric-Brown. *Metabolic ecology: a scaling approach*. John Wiley & Sons, 2012.
- [103] Stephen Smale. On gradient dynamical systems. Annals of Mathematics, pages 199– 206, 1961.

- [104] Hal L Smith and Paul Waltman. The theory of the chemostat: dynamics of microbial competition, volume 13. Cambridge university press, 1995.
- [105] Gilbert Stengle. A nullstellensatz and a positivstellensatz in semialgebraic geometry. Mathematische Annalen, 207(2):87–97, 1974.
- [106] James A Stiver, Panos J Antsaklis, and Michael D Lemmon. A logical des approach to the design of hybrid control systems. *Mathematical and computer modelling*, 23(11):55–76, 1996.
- [107] J. Sturm. Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones. *Optimization method & software*, 11:625–653, 1999.
- [108] Katharine N Suding, Katherine L Gross, and Gregory R Houseman. Alternative states and positive feedbacks in restoration ecology. *Trends in Ecology & Evolution*, 19(1):46–53, 2004.
- [109] Katharine N Suding and Richard J Hobbs. Threshold models in restoration and conservation: a developing framework. *Trends in ecology & evolution*, 24(5):271–279, 2009.
- [110] T.A Tamba and M.D. Lemmon. The distance-to-bifurcation problem in non-negative dynamical systems with kinetic realizations. IEEE International Conference on Control and Automation (ICCA), 2014.
- [111] Tua A Tamba and M. D Lemmon. Using first passage times to manage eco-system regime shifts. In *Decision and Control, 2013. CDC. IEEE Conference on*, pages 2697–2702, 2013.
- [112] Tua Agustinus Tamba. *Forecasting regime shifts in nonlinear dynamical processes*. PhD thesis, University of Notre Dame, 2015.
- [113] Weehong Tan and Andrew Packard. Stability region analysis using sum of squares programming. In 2006 American Control Conference, pages 6–pp. IEEE, 2006.
- [114] Robert Tarjan. Depth-first search and linear graph algorithms. SIAM journal on computing, 1(2):146–160, 1972.
- [115] Marco Terzer and Jörg Stelling. Large-scale computation of elementary flux modes with bit pattern trees. *Bioinformatics*, 24(19):2229–2235, 2008.
- [116] Kim-Chuan Toh, Michael J Todd, and Reha H Tütüncü. Sdpt3—a matlab software package for semidefinite programming, version 1.3. *Optimization methods and soft*ware, 11(1-4):545–581, 1999.
- [117] George AK Van Voorn, Lia Hemerik, Martin P Boer, and Bob W Kooi. Heteroclinic orbits indicate overexploitation in predator-prey systems with a strong allee effect. *Mathematical biosciences*, 209(2):451–469, 2007.

- [118] Lieven Vandenberghe and Stephen Boyd. Semidefinite programming. *SIAM review*, 38(1):49–95, 1996.
- [119] Ulrike Von Luxburg. A tutorial on spectral clustering. *Statistics and computing*, 17(4):395–416, 2007.
- [120] Joachim Von Zur Gathen and Jürgen Gerhard. *Modern computer algebra*. Cambridge university press, 2013.
- [121] S. Waldherr and F. Allgöwer. Searching bifurcations in high-dimensional parameter space via a feedback loop breaking approach. *International Journal of System Science*, 40:769–782, 2009.
- [122] Richard J Williams and Neo D Martinez. Simple rules yield complex food webs. *Nature*, 404(6774):180–183, 2000.
- [123] Hakan Yazarel, Stephen Prajna, and George J Pappas. Sos for safety. In 2004 43rd IEEE Conference on Decision and Control (CDC)(IEEE Cat. No. 04CH37601), volume 1, pages 461–466. IEEE, 2004.
- [124] Peter Yodzis and S Innes. Body size and consumer-resource dynamics. American Naturalist, pages 1151–1175, 1992.