CHAPTER 1

Introduction - What is a Resilient System?

Different scientific communities take different viewpoints of what it means for a dynamical system to be *resilient*. Engineers often use the term to describe systems that are *fault-tolerant* or *robust* to disturbances. Ecologists, on the other hand, use the term *ecological resilience* [**55**] to describe ecosystems that have the capacity to spring back after a catastrophic collapse. This monograph focuses on ecological resilience; where system collapse is taken as being inevitable, though the seeds for eventual system restoration are contained within the wreckage of that collapse. The dictionary states that resilience is the capacity to quickly recover from difficulties. This definition focuses on the ability to restore lost system functions and seems to be in greater harmony with how ecologists use the term. This monograph, therefore, adopts the ecologist's view of system resilience.

Ecological resilience may be graphically illustrated by the bouncing ball in Fig. 1. Hitting the floor is the collapse experienced by the falling ball. The rubber ball is resilient because it flies back into the air after hitting the floor. On the other hand, if the ball had been a wad of wet dough, it would have hit the floor and stuck there, thereby illustrating that a wet ball of dough is not resilient. The bouncing rubber ball can also be used to illustrate another aspect of resilient systems; namely

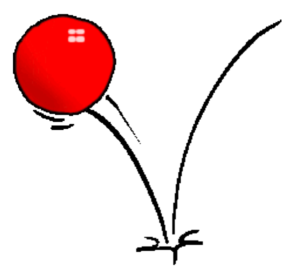

FIGURE 1. bouncing ball

that their behaviors are *recurrent*. After the rubber ball hits the ground, it bounces back into the air and then falls back to earth where it hits the ground again. This generates a sequence of bounces that continue for several cycles. In other words, resilient systems traverse a cycle of modes in which the system function (i.e. the ball's height above ground) declines until, upon hitting the floor, it redirects its kinetic energy and bounces back up. For the ball this decline is inevitable. What makes the ball resilient is that it can redirect or reorganize itself in a manner that allows recovery of some of its original height. This means that resilience refers not only to a system's ability to *withstand* collapse, it also refers to the ability of that system to *restore* lost system function.

Ecologically resilient systems therefore cycle back and forth between a *nominal* regime and an *alternative* regime. The transition between regimes is called a *regime shift*. Fig. 2 illustrates these shifts graphically by plotting an energy (Lyapunov) function, $V(x)$, for a multi-stable system whose two stable equilibria are local minima of V. The region of attraction for one of the equilibria is the nominal regime and the region of attraction for the alternative equilibrium is the alternative regime. A regime shift occurs if an external disturbance causes the system state to jump from the nominal regime into the alternative regime. Once the system state enters the alternative regime, it remains in that regime until a future disturbance forces it back across the energy barrier separating the two regimes. We refer to this first shift as a *collapse* since the alternative regime is usually treated as being undesirable. System restoration occurs when the system state shifts back to the nominal

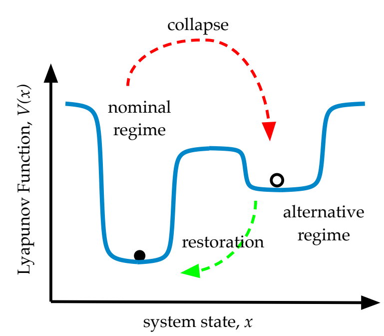

FIGURE 2. Regime Shifts in a Multi-stable System

regime. This restoration of system function will always occur in an ecologically resilient system, though it may take quite some time before the restoration is complete. For this reason, one may *intervene* by intentionally triggering regime shifts that force the system to jump back more quickly. Achieving ecological resilience for the system in Fig. 2 therefore requires the *active management of regime shifts*. With regard to system collapse, regime shift management seeks to reduce the likelihood of an impending collapse. This management policy may also be called a *conservation* policy. For a collapsed system, regime shift management seeks to hasten the restoration of lost system function. This is done through a sequence of intentionally triggered regime shifts that form a restoration plan for the system.

This monograph introduces a mathematical framework for regime shift management that can be used to enhance a system's ecological resilience. There are at least three challenges to be overcome in the development of a framework for ecological resilience. The first challenge stems from the equilibrium-based view of regime shifts that was popularized in [**97**]. Real life ecosystems are non-equilibrium processes for which an equilibrium based notion of a regime is too restrictive. So our first problem involves finding a precise formalization of the regime shift concept that extends the equilibrium-based notion in [**97**] to non-equilibrium processes. In the second place we need to identify specific mechanisms driving a system state to jump into the alternative regime. A good understanding of the mechanisms triggering a regime shift will allow one to develop measures characterizing how close a system is to collapse. These measures play an important role in developing conservation policies seeking to forestall an impending shift. Finally, while system collapse is often sudden and catastrophic, restoration is often more complicated. It is usually impossible to jump back to the nominal regime with a single regime shift. In many cases, the system's resources need to be released and re-organized before a shift back to the nominal regime can be realized. This means that complex systems may need to follow a sequence of intermediate regimes (a.k.a. order of succession) before restoring full system function. So the third problem involves identifying a sequence of intermediate regimes leading back to full restoration of system function.

The following chapters address these three challenges with regard to compartmental systems used in modeling ecosystem food webs. These system models are sometimes referred to as *consumer-resource systems* because their dynamics are governed by the flow of resources to consumers. In particular, we take the equilibrium-based notion of a regime and generalize it to define regimes in terms of the basic sets of a Morse decomposition for the system's chain recurrent set. A regime shift then occurs when the system state jumps between basic sets. This formalization allows us to identify two distinct regime shift mechanisms. There is a *bifurcation-induced* regime shift that is generated by local bifurcations of the system and there is a *shock-induced* regime shift that is triggered by impulsive shock like disturbances. For each of these regime shift mechanisms we show how semidefinite programming tools can be used to measure how close a system is to an impending regime shift. The monograph then turns to the problem of system restoration. In particular it uses a discrete abstraction of the system to characterize sequences of shock-induced regime shifts that the system can generate. This abstraction is then used to develop plans that realize the full restoration of a collapsed system. through active management of its regime shifts.

The remainder of the monograph is organized as follows. Chapter 2 defines regime shifts in terms of the system's basic sets. Chapter 3 introduces a measure of how close a system is to triggering a bifurcation-induced regime shifts, also called the D2B or distance-to-bifurcation. Chapter 4 uses a stochastic reachability problem to estimate the likelihood of a shock-induced regime shift. This likelihood provides a probabilistic measure of a system's susceptibility to shock-induced regime shifts. Chapter 5 uses algorithmic analysis of dynamical systems to construct a discrete abstraction of a system's regime shift sequences and then shows how this abstraction could be used to formulate restoration plans for a collapsed system. Chapter 6 closes with some comments on future directions for this work.

Further Readings: Control engineering approaches to resilience are frequently confined to fault detection and accommodation [**86**] and fault tolerant control [**6**]. This view of resilience has been important in reconfigurable flight control systems [**7, 32**]. The ecological view of resilience, on the other hand, emphases renewal over an engineering desire for predictability. This perspective was raised in C.S. Holling's review essay [**55**] and was based on insights drawn from his earlier work concerning the resilience of terrestrial (forest) systems [**54**]. This ecological viewpoint stresses the recurrent nature of resilient systems and later work sought to apply this notion to complex socio-ecological systems. [**56**]. A popularized extension of this recurrence idea was referred to as *panarchy* in [**34, 45**] and has motivated the founding of international research organizations (*Resilience Alliance* and *Stockholm Resilience Centre*) that study the ecological resilience of real-life socio-ecological systems.

Note on Mathematical Notation: The set of integers, real numbers, and complex numbers are denoted as $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$, respectively. The vector space of n-dimensional real valued vectors (a.k.a. Euclidean n-space) is denoted as $\mathbb{R}^n$. The set of n by m real valued matrices is $\mathbb{R}^{n\times m}$. The set of non-negative n-dimensional real vectors is denoted as $\mathbb{R}_{\geq 0}$. A function $f : X \to Y$ is a rule that associates each element of the set X to at most one element of the set Y. The value that f takes at $x \in X$ is denoted as $f(x)$. A function with n continuous derivatives is said to be C^n.