Efficiently Attentive Quantized Event-Triggered Systems

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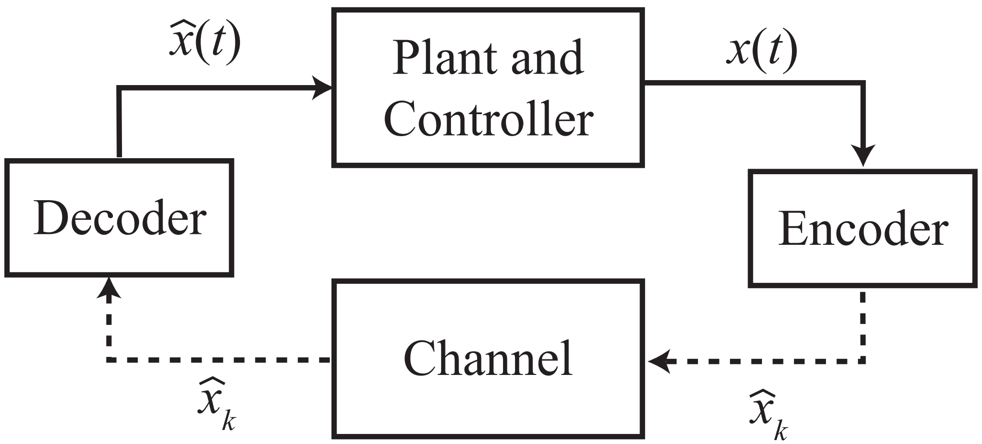
Sampled Data Control System



- $\{s_k\}_{k=0}^{\infty}$ = sampling instants
- Plant/Controller state, $x(t)$ satisfies

$$\dot{x}(t) = f(x(t), u(t), w(t)); \quad x(0) = x_0$$

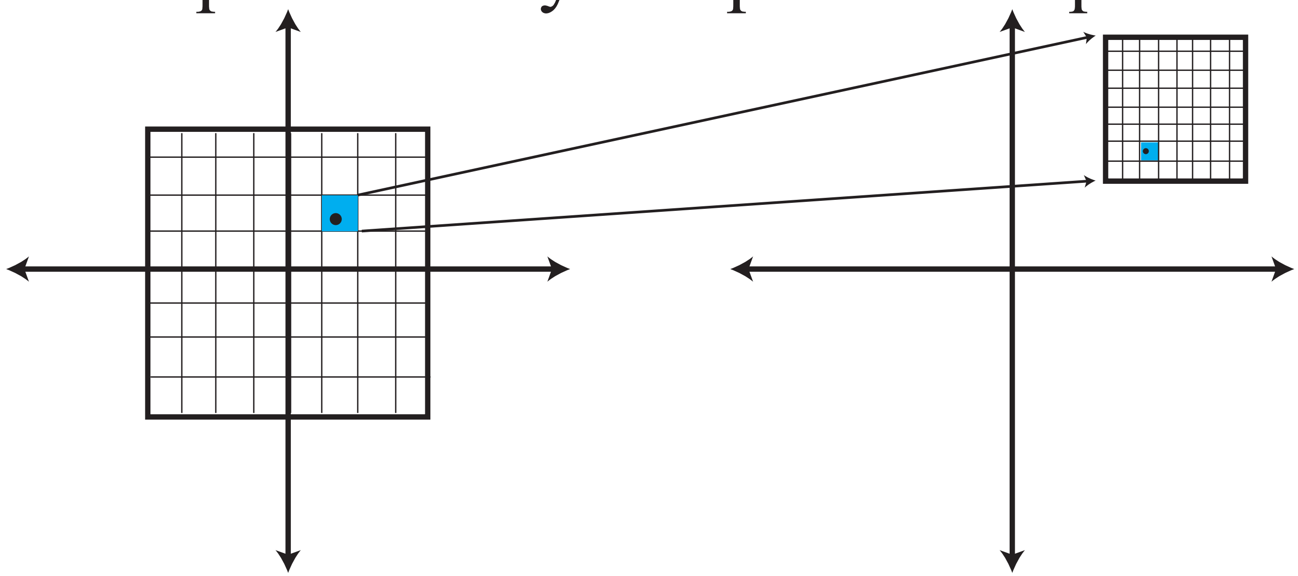
- Sampled and Quantized state, $\hat{x}_k = Q(x(s_k))$
- Reconstructed state $\hat{x}(t) = \hat{x}_k$ for $t \in [s_k, s_{k+1})$.

- Encoder - quantizes and samples system state
- Channel - Lossy with finite bit-rate
- Decoder - zero-order hold
- Plant/Controller - stable under perfect state feedback

Minimum Information for Stabilization?

Dynamically Quantized Feedback

- Discrete-time Linear System with one sample delay
  - state is periodically sampled with quantization[1,2]



Uncertainty sets at time instant k

Uncertainty sets at time instant $k+1$

- Necessary and sufficient bit rate for asymptotic stability

$$R \geq \underline{R} = \sum_{i=1}^{n} \max(0, \log_2 |\lambda_i|)$$

Emulation Method

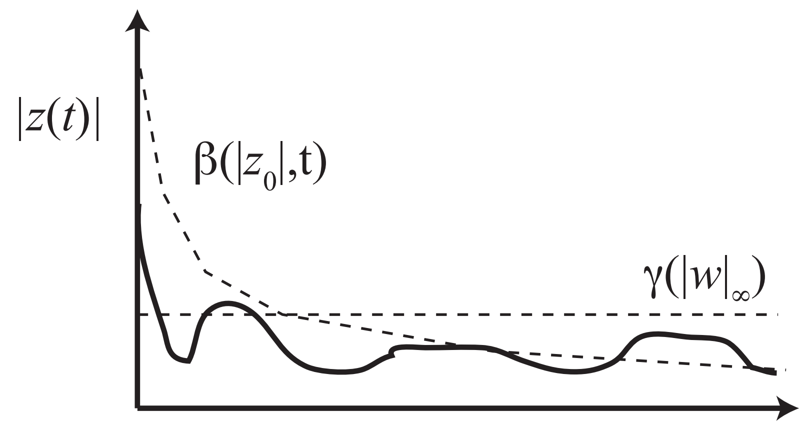
- "Emulation Method" for Sampled System Design
  - design controller, K, that leaves "continuously" sampled system input-to-state stable (ISS)[3].

There exist functions $\beta \in \mathcal{K}L$ and $\gamma \in \mathcal{K}$ such that

$$|z(t)| < \max\{\beta(|z_0|, t), \gamma(|e|_{\mathcal{L}_\infty})$$

$e \in \mathcal{L}_\infty$ and $z(\cdot) : \mathbb{R}^+ \to \mathbb{R}^n$ satisfies

$$\dot{z}(t) = f(z(t), K(z(t) + e(t))$$



- Select Sampling Instants to guarantee that the sporadically sampled system is also ISS.

Event-Triggered Feedback[4]

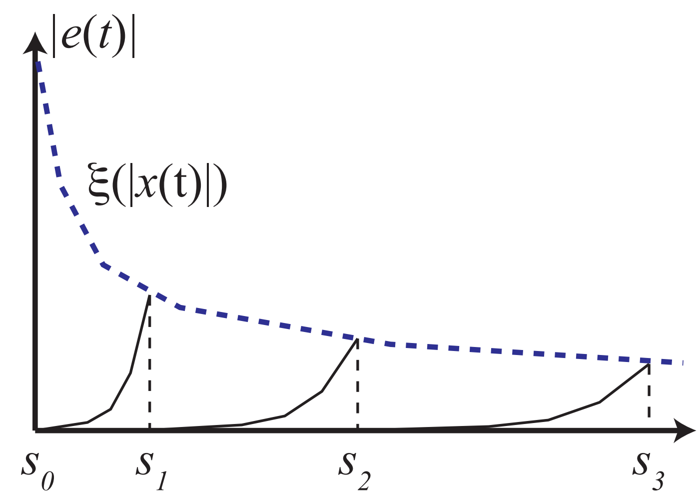
- State is sporadically sampled with no quantization
- Since the "continuously" sampled system is ISS, there exists a positive definite function $V(\cdot) : \mathbb{R}^n \to \mathbb{R}^+$ and functions $\alpha, \gamma \in \mathcal{K}$ such that

$$\frac{\partial V}{\partial x} f(x, K(x+e)) \leq -\alpha(|x|) + \gamma(|e|)$$

- Select sampling instants, $\{s_k\}_{k=0}^{\infty}$ so that for all time, t,

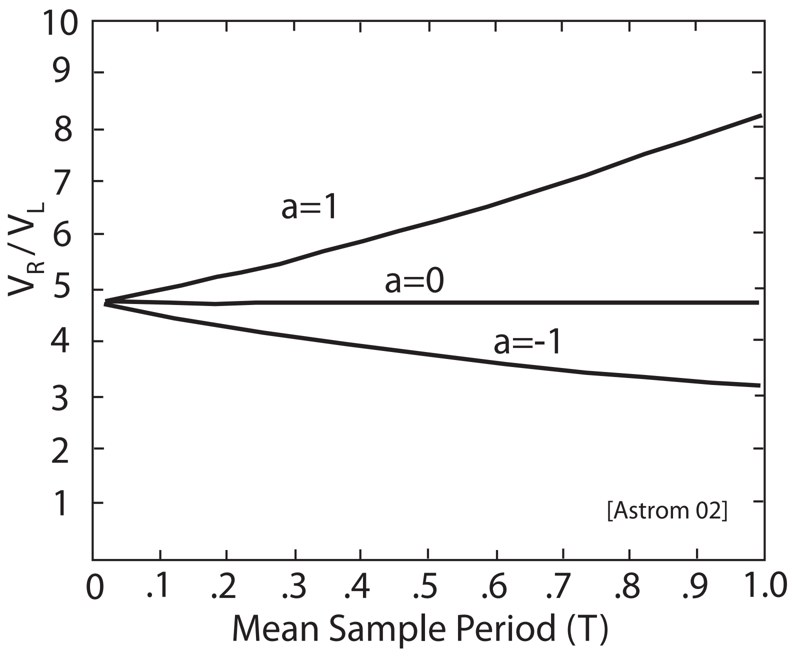
$$|e(t)| \leq \gamma^{-1}\left((1-\sigma)\alpha(|x(t)|)\right) \equiv \xi(|x(t)|)$$

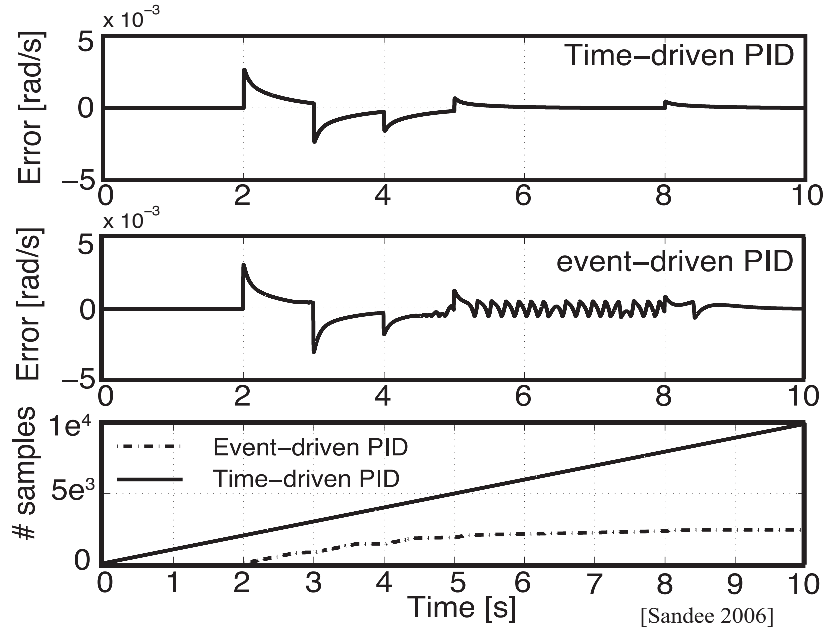
where $0 < \sigma < 1$



Resource Utilization in Event-Triggered Systems

Prior work has suggested that event-triggered systems have lower resource utilization than comparable periodically sampled systems[5,6,7,8].





But this may not always be the case.

Zeno Sampling and Efficient Attentiveness[9,10]

- Consider the following system

$$\dot{x}(t) = f(x(t)) + u(t) \text{ with } u(t) = -2f(\hat{x}_k)$$

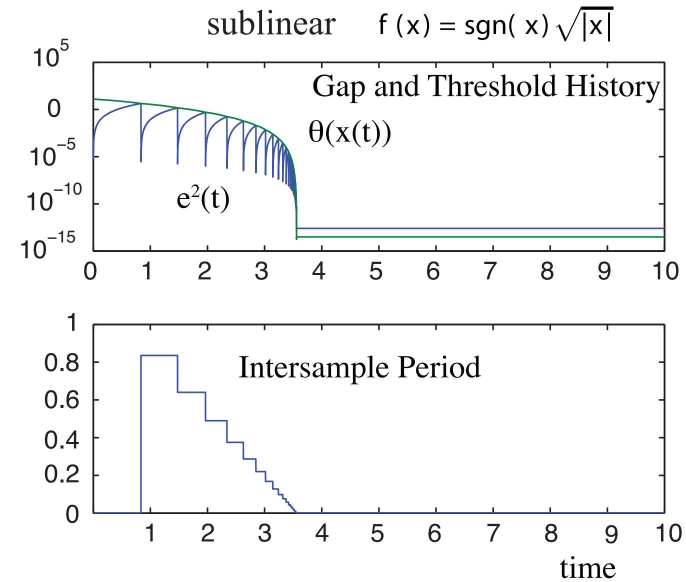
where

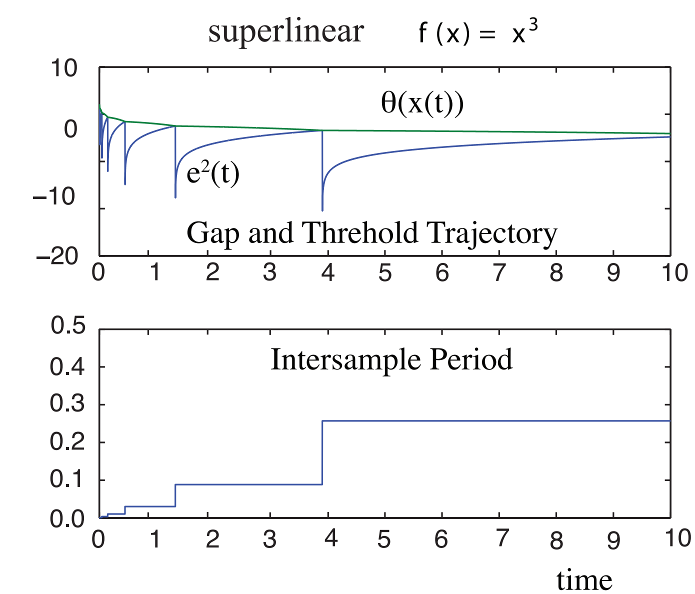
sublinear	superlinear
$f(x) = \text{sgn}(x)\sqrt{\lvert x \rvert}$	$f(x) = x^3$

- Given sampling instant s_k, the next sampling instant s_{k+1} is the first time when the state leaves

$$\Omega_k = \left\{ x \in \mathbb{R}^n \;:\; (\hat{x}_k - x)^2 \le \lvert x \rvert^2 \equiv \theta(\lvert x \rvert) \right\}$$

- **Zeno-sampling** :
  infinite samples over a finite interval
- **Efficiently Attentive :**
  Intersampling Intervals get larger as system state approach origin.

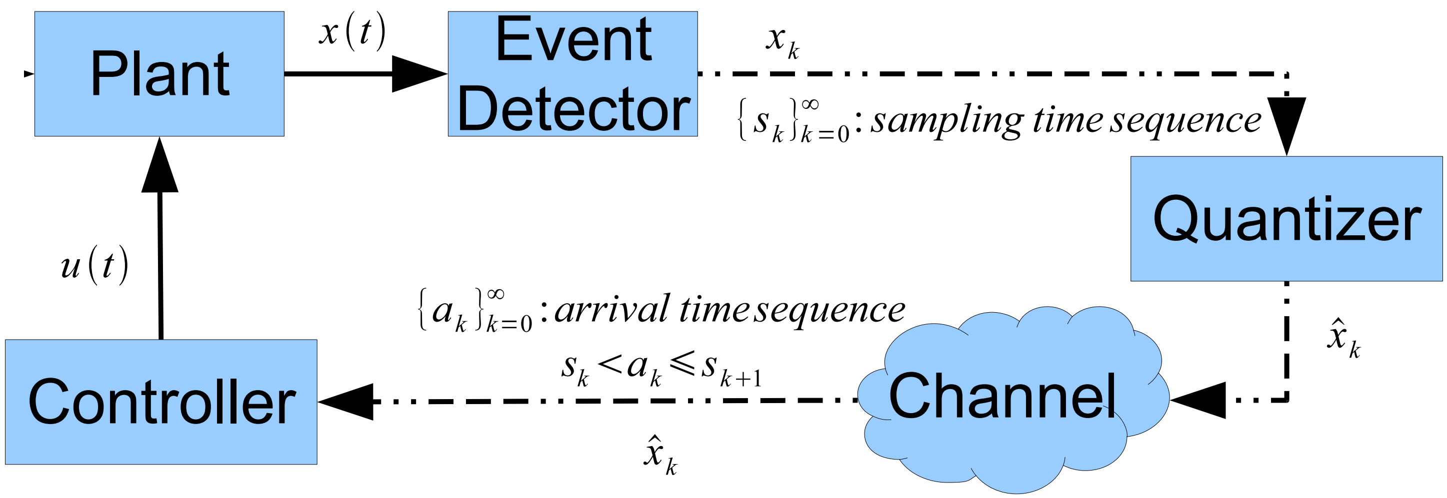




Dynamic Quantization versus Event-Triggering

- **Differences**
  - Single sample delay (Q) versus small delay (ET)
  - Utilization measured by bit rate versus inter-sampling time
  - Dynamic quantizers achieve minimum stabilizing bit rate, but lower resource utilization not guaranteed for event-triggered systems
- **Similarities**
  - both attempt to reduce "information" over channel.
  - quantization in "time" versus "space.
  - both discretize a continuous-time control system
- **Objectives:**
  - Unification quantization and event-triggering
  - Design Event-triggers to control resource utilization by ensuring efficient attentiveness

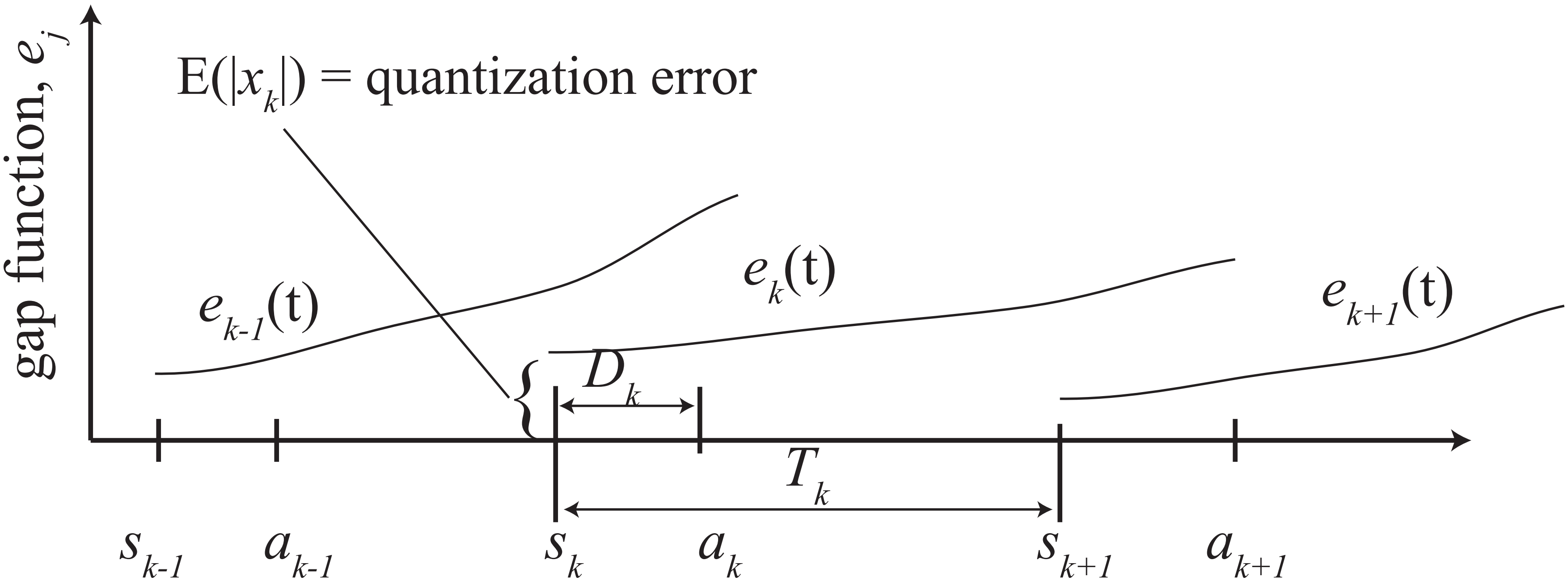
Quantized Event-Triggered Networked System[11,12]



- **Sampling Instants**, $\{s_k\}_{k=0}^{\infty}$, and **Arrival Instants** $\{a_k\}_{k=0}^{\infty}$.
- **State Equation** is $\dot{x}(t) = f(x(t), k(\hat{x}_k))$ where $\hat{x}_k$ is the quantized state and $t \in [a_k, a_{k+1})$.
- **Quantized state**, $\hat{x}_k$, satisfies $|x(s_k) - \hat{x}_k| < E(|x(s_k)|)$ where $E \in \mathcal{K}$ is the **Quantization Error**.
- **Controller**, K, ensures "continuously" sampled system is ISS with respect to the **gap** $e_k(t) = x(t) - \hat{x}_k$.

Modeling of Channel Delay and Quantization[13,14,15]

- Quantization Error, $E(|x_k|)$
  Delay = $D_k = a_k - s_k$
  Inter-sampling Interval = $T_k = s_{k+1} - s_k$
  Admissible Sampling: $s_k < a_k < s_{k+1}$



- What are dynamics of gap function, $e_k(t) = x(t) - \hat{x}_k$?

Dynamics of the Gap Function[11,12]

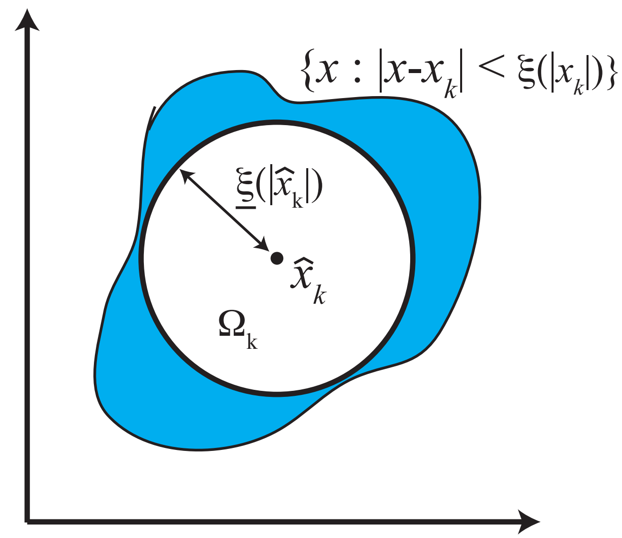
Lipschitz on Compacts: $L_k > 0$ such that

$$|f(x(t), K(\hat{x}_k))| \leq |f(\hat{x}_k, K(\hat{x}_k))| + L_k |e_k(t)|$$

for all $x(t) = \hat{x}_k + e_k(t) \in \Omega_k$ where

$$\Omega_k = \left\{ x \in \mathbb{R}^n \,:\, |x| \leq |\hat{x}_k| + \underline{\xi}(|\hat{x}_k|) \right\}$$

and $\underline{\xi}(s) = \sup\{r \,:\, \xi(s-r), s < r\}$

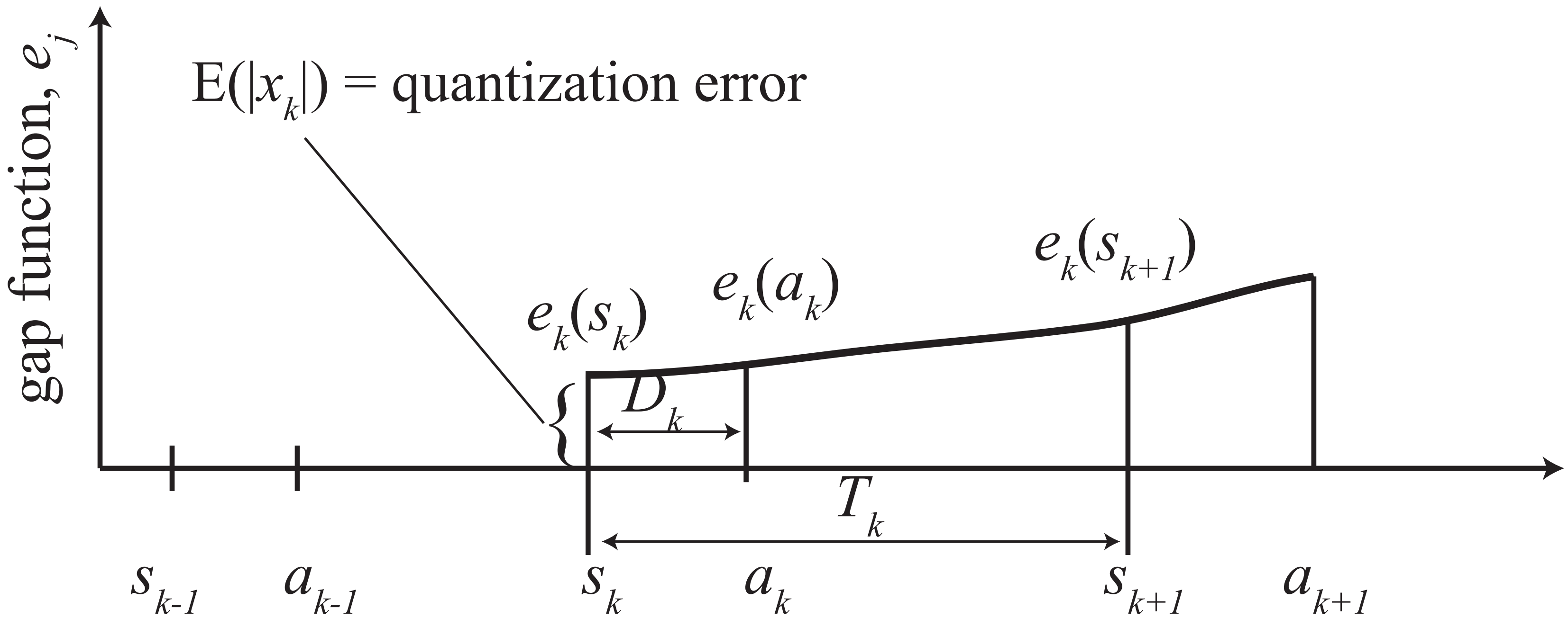


- Since $x(t) \in \Omega_k$, the gap, $e_k(t)$, satisfies the following differential inequality for $t \in [s_k, a_{k+1})$

$$\frac{d|e_k|}{dt} \leq |\dot{e}_k(t)| \leq |f(\hat{x}_k, K(\hat{x}_k))| + L_k |e_k(t)|, \quad e_k(s_k) < E(|x_k|)$$

- This is a linear differential inequality and we can use the comparison principle to bound the gap $|e_k(t)|$

Lower Bound on Inter-sampling Interval[10,11,13]

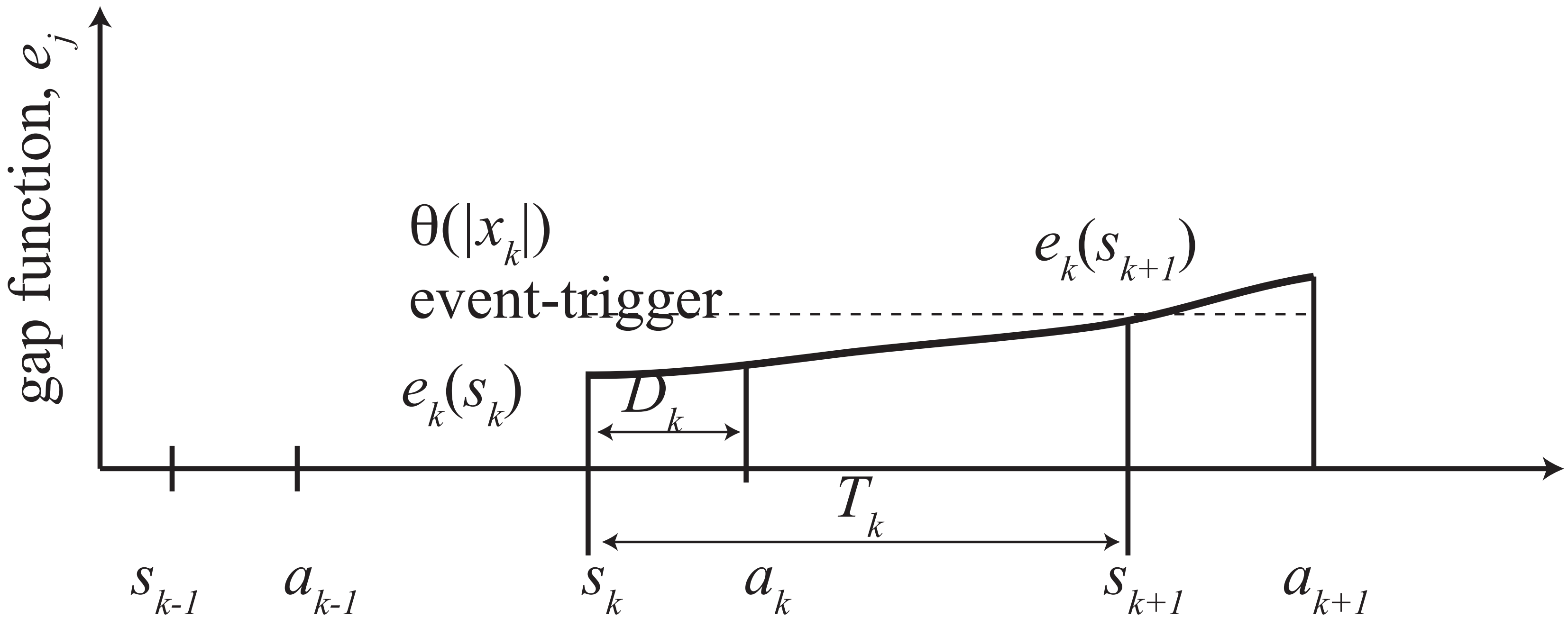


- kth Gap at kth transmission time is $|e_k(s_k)| < E(|x_k|)$
- kth Gap at $k+1$st transmission time is

$$|e_k(s_{k+1})| \leq E(|x_k|)e^{L_k T_k} + \frac{\Psi(\hat{x}_k, \hat{x}_{k-1})}{L_k}\left(e^{L_k T_k} - 1\right)$$

where $\Psi(\hat{x}_, \hat{x}_{k-1}) = |f(\hat{x}_k, K(\hat{x}_k))| + 2|f(\hat{x}_k, K(\hat{x}_{k-1})|$

Lower Bound on Inter-sampling Interval

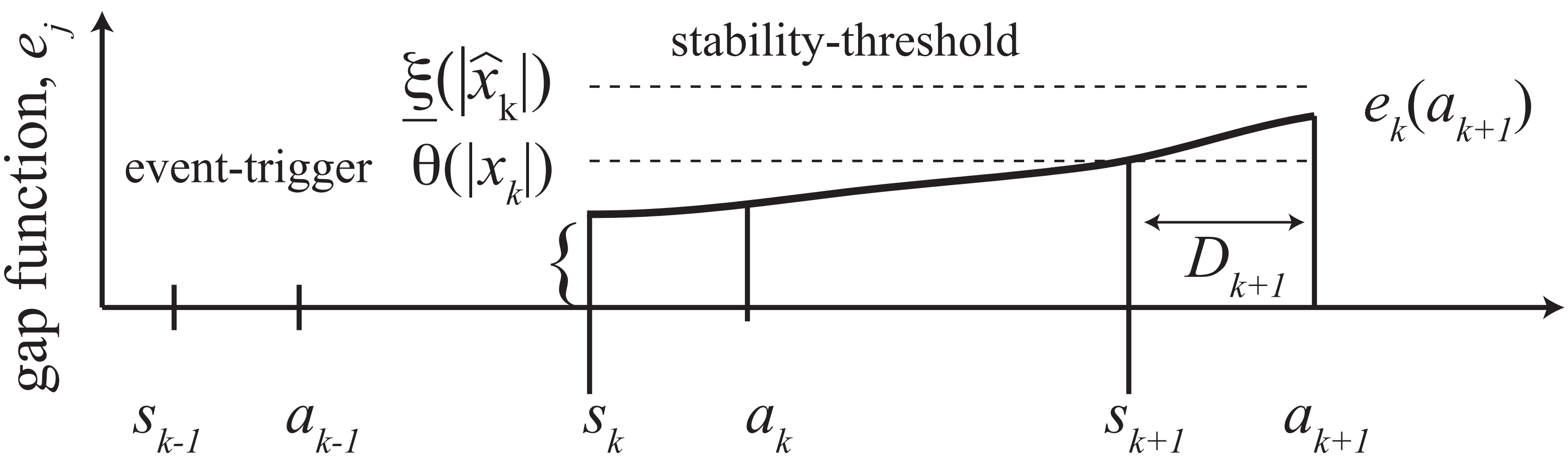


- s_{k+1}, is the first time when $|e_k(t)| = \theta(|\hat{x}_k|)$. We refer to $\theta(\cdot) : \mathbb{R} \to \mathbb{R}$ as the *event-triggering function*.

- With $|e_k(s_{k+1})| = \theta(|\hat{x}_k|)$, our earlier constraint becomes

$$T_k > \frac{1}{L_k}\left(\ln\left(1 + \frac{L_k\theta(|\hat{x}_k|)}{\Psi(\hat{x}_k, \hat{x}_{k-1})}\right) - \ln\left(1 + \frac{L_k E(|x_k|)}{\Psi(\hat{x}_k, \hat{x}_{k-1})}\right)\right)$$

Upper Bound on Stabilizing Delay[11,13]



- Using similar techniques, we find

$$|e_k(a_{k+1})| \leq \theta(|\hat{x}_k|)e^{L_k D_{k+1}} + \frac{|f(\hat{x}_k, K(\hat{x}_k))|}{L_k}\left(e^{L_k D_{k+1}} - 1\right)$$

- Asymptotic stability requires $|e_k(a_{k+1})| \leq \underline{\xi}(|\hat{x}_k|)$, so the stabilizing delay satisfies

$$D_{k+1} \leq \frac{1}{L_k}\left(\ln\left(1 + L_k \frac{\underline{\xi}(|\hat{x}_k|)}{|f(\hat{x}_k, K(\hat{x}_k))|}\right) - \ln\left(1 + L_k \frac{\theta(|\hat{x}_k|)}{|f(\hat{x}_k, K(\hat{x}_k))|}\right)\right)$$

Asymptotic Stability[11,12]

Assume that the **event-triggering threshold** θ satisfies

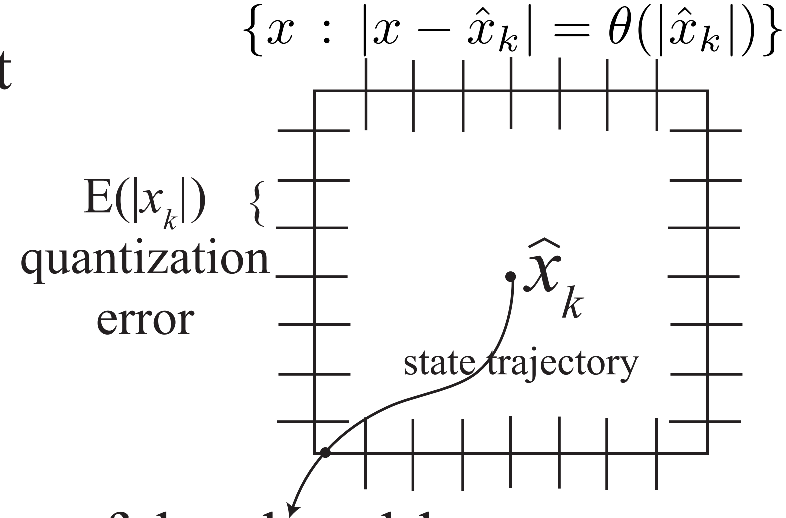
Quantization Error $E(s) \leq \theta(s) \leq \underline{\xi}(s)$ **Stability Threshold**

for any $s \in \mathbb{R}^+$ and assume the delay $D_k < \min\{\underline{T}_k, \overline{D}_k\}$ where

$$
\begin{aligned}
\underline{T}_k &= \frac{1}{L_k}\left(\ln\left(1+\frac{L_k\theta(|\hat{x}_k|)}{\Psi(\hat{x}_k,\hat{x}_{k-1})}\right)-\ln\left(1+\frac{L_kE(|x_k|)}{\Psi(\hat{x}_k,\hat{x}_{k-1})}\right)\right)\\
\overline{D}_k &= \frac{1}{L_{k-1}}\left(\ln\left(1+L_{k-1}\frac{\underline{\xi}(|\hat{x}_{k-1}|)}{|f(\hat{x}_{k-1},K(\hat{x}_{k-1}))|}\right)\right.\\
&\qquad \left.-\ln\left(1+L_{k-1}\frac{\theta(|\hat{x}_{k-1}|)}{|f(\hat{x}_{k-1},K(\hat{x}_{k-1}))|}\right)\right)
\end{aligned}
$$

Then the closed-loop quantized event-triggered system is asymptotically stable with an inter-sampling interval T_k which is always bounded below by $\underline{T}_k > 0$.

Stabilizing Bit Rate[11,12]



- N_k=Number of Bits used to represent sampled state $\hat{x}_k$
- Assuming $|x|$ is the sup-norm, then

$$N_k = \lceil \log_2 2n \rceil + (n-1) \left\lceil \log_2 \left\lceil \frac{\theta(|\hat{x}_{k-1}|)}{E(|x_k|)} \right\rceil \right\rceil \quad \text{bits}$$

- To guarantee the asymptotic stability of the closed-loop system, these bits must be delivered within the delay

$$D_k \leq \overline{D}_k \equiv \frac{1}{L_{k-1}} \left(\ln \left(1 + L_{k-1} \frac{\underline{\xi}(|\hat{x}_{k-1}|)}{|f(\hat{x}_{k-1}, K(\hat{x}_{k-1}))|} \right) - \ln \left(1 + L_{k-1} \frac{\theta(|\hat{x}_{k-1}|}{|f(\hat{x}_{k-1}, K(\hat{x}_{k-1}))|} \right) \right)$$

- A bit rate stabilizing this system must therefore be

$$R_k = \frac{N_k}{D_k} > \underline{R}_k \equiv \frac{L_k}{\ln 2} \left(A(|\hat{x}_k|)(n-1) + B(|\hat{x}_k|) \right)$$

Conditions for Zero Bit Rate[11]

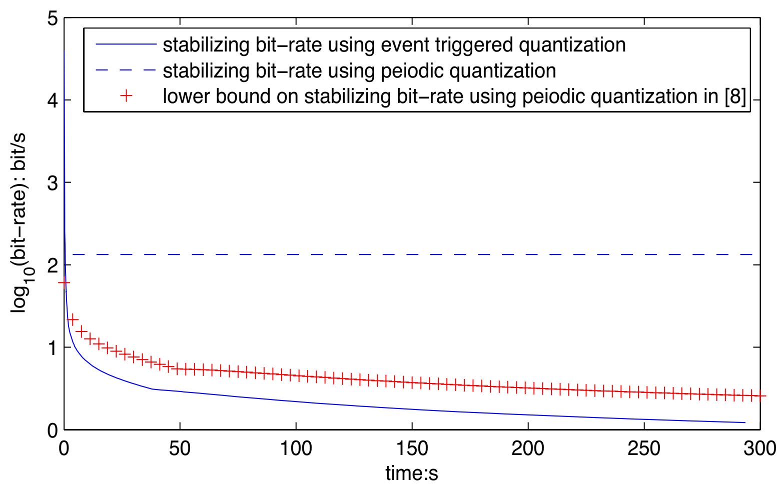
- In some cases, we can show that the stabilizing bit rate goes to zero as the system approaches its equilibrium point.
- System Equations

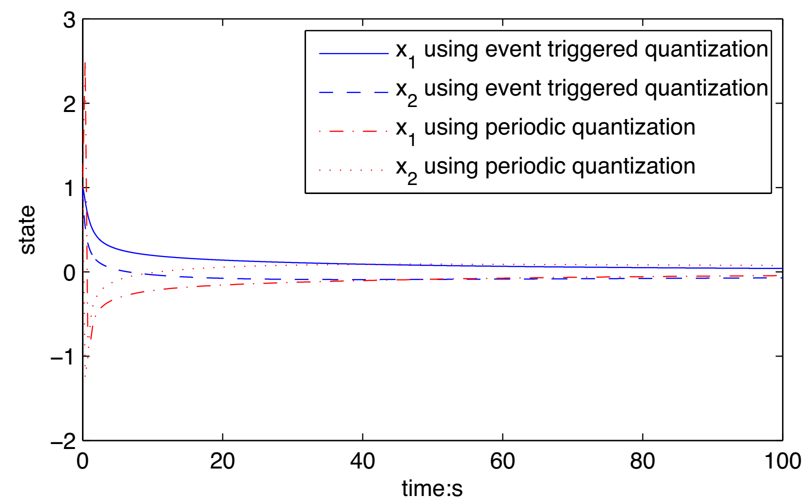
$$\begin{aligned}\dot{x}_1 &= x_1^3 + 2x_2^3 + u \\ \dot{x}_2 &= -x_1^3 - x_2^3\end{aligned}$$

Feedback Control

$$u = -2\hat{x}_1^3 - \hat{x}_2^3$$

Switching Condition: $|e_k(t)| = \theta(|\hat{x}_k|) = 0.015|\hat{x}_k|$





Efficiently Attentive Stabilizing Bit Rates[10,12]

- It can be very difficult to determine the minimal stabilizing bit rate. In such cases, a reasonable option is to require the bit rate to be **efficiently attentive**
- A bit rate is **efficiently attentive** if it is an increasing function of state

Assume that the delay $D_k < \overline{D}_k$, and the event-triggering satisfies

$$E(s) \leq \theta(s) \leq \underline{\xi}(s)$$

Let $\phi_c, \phi_u \in \mathcal{K}$ such that $|f(x, K(x))| \leq \phi_c(|x|)$ and $|K(x)| \leq \phi_u(|x|)$.
If we also know that

$$\lim_{s \to 0} \frac{\theta(s)}{E(s)} < \infty, \quad \lim_{s \to 0} \frac{\phi_c(s)}{\theta(s)} < \infty, \quad \lim_{s \to 0} \frac{\phi_u(s)}{\theta(s)} < \infty$$

then there exists a continuous, positive definite, increasing function $\underline{R}(|\hat{x}_k|)$ such that if the actual bit rate is greater than $\underline{R}$, then the system is asymptotically stable.

Simulation Example[12,18]

- This example extends prior work to essentially bounded disturbances
- System Equations

$$\dot{x}_1 = x_1^3 + 2x_2^3 + u_1 + w_1$$
$$\dot{x}_2 = -x_1^3 - x_2^3 + u_2 + w_2$$

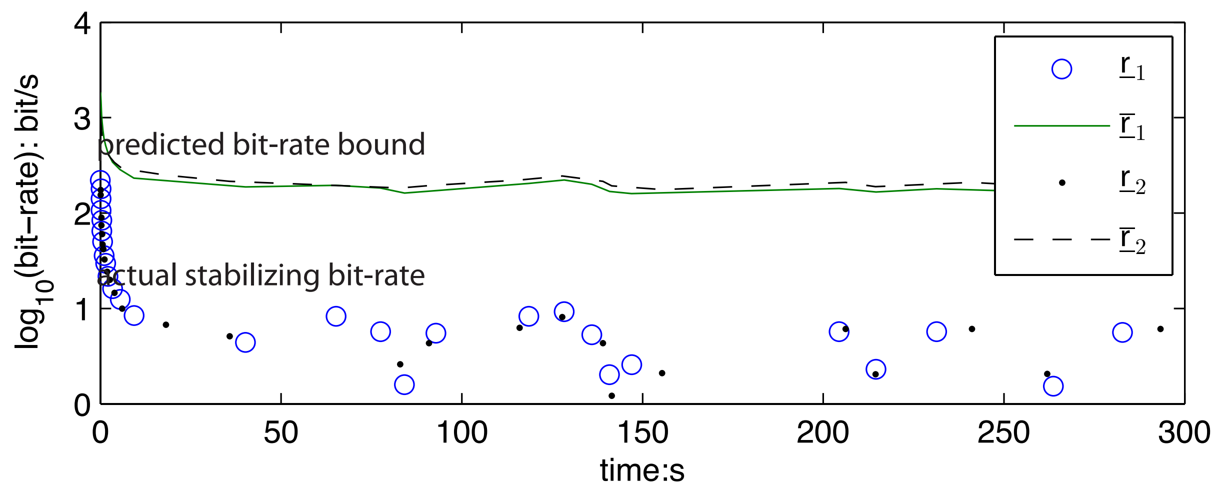
Controller

$$u_1 = -3\hat{x}_1^3, \quad u_2 = -3\hat{x}_2^3$$

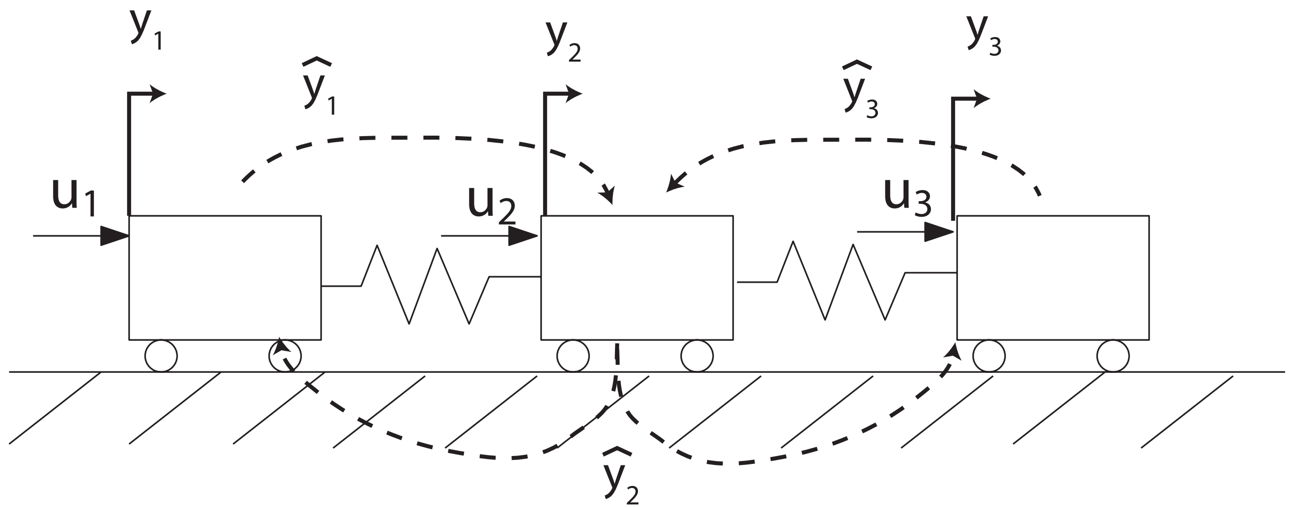
- Event Trigger and Quantization Map

$$\theta_1(s, \overline{w}) = 0.075 s^{1.5} + 0.05\overline{w}$$
$$E_1(s, \overline{w}) = 0.025 s^{1.5} + 0.017\overline{w}$$

- Predicted bit-rates are conservative bound on actual bit-rates



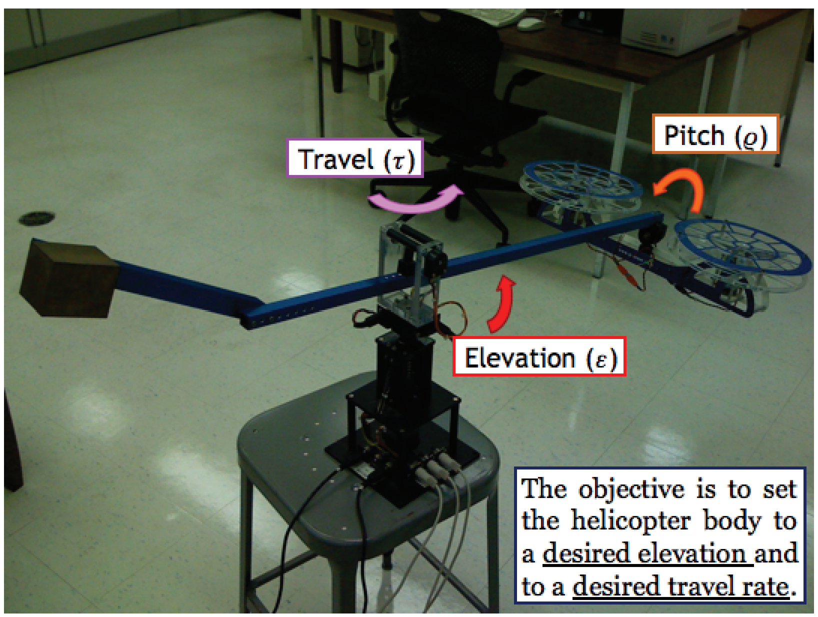
Wireless Networked Control Systems[15]



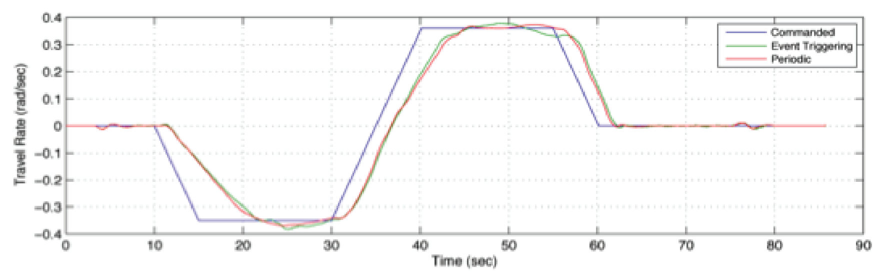
- Event-triggering gives rise to sporadic message streams in wireless networked control systems.
- After an impulsive disturbance is applied to middle cart, one can bound the future inter-sampling times and bit-rate requirements of all controllers.
- Can we use this information to reschedule controller transmission to maintain both overall physical system performance while staying within communication network's capacity limits?

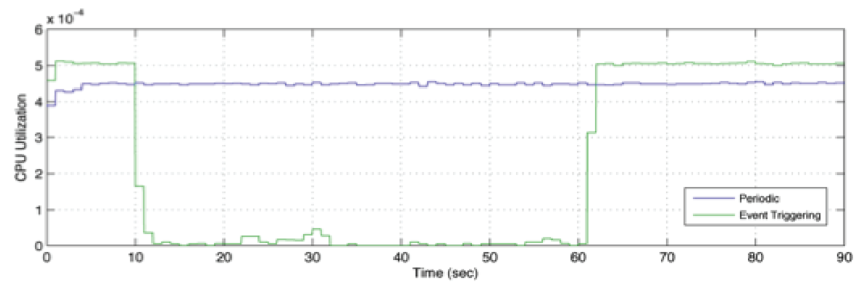
Safety-Critical Systems

- One concern with event-triggered systems is that they are ill suited for safety-critical systems.
- In the presence of disturbances, however, event-triggered solutions must be implemented with a minimum sampling frequency whose size is determined by the "disturbance".
- With efficiently attentive systems, event frequency increases in the presence of impulsive disturbances.



http://www.nd.edu/~lemmon/projects/NSF-05-1518/heli-movie/





Future Directions

- Relax **conservativeness** of bounds[19].
- Event-trigger design is based on existence of ISS controller (input disturbances). This may not always be possible. One solution may be to extend framework to **iISS controllers**[16].
- These results provide some guidance on the selection of controllers, quantizers, and event-triggers. We still need to formalize this guidance into a **design procedure**.
- Similarities between dynamic quantization and the quantized event-triggers. When do we achieve the known **necessary and sufficient stabilizing bit rates for linear systems**?
- Efficiently attentive systems provide a basis for co-design of communication/controller in a deterministic setting. Is it possible to extend these ideas to a stochastic setting? One possible approach would involve the use of stochastic ISS concepts[17].

References

1.) R.W. Brockett and D. Liberzon. Quantized feedback stabilization of linear systems. Automatic Control, IEEE Transactions on, 45(7):1279–1289, 2000.

2.) S. Tatikonda and S. Mitter. Control under communication constraints. Automatic Control, IEEE Transactions on, 49(7):1056–1068, 2004.

3.) D. Nesic and A.R. Teel. Input-output stability properties of networked control systems. IEEE Transactions on Automatic Control, 49(10):1650–1667, 2004.

4.) P. Tabuada. Event-triggered real-time scheduling of stabilizing control tasks. IEEE Transactions on Automatic Control, 52(9):1680–1685, September 2007.

5.) K-E Arzen. A simple event-based PID controller. In Procedingsof the 14th World Congress of the International Federation of Automatic Control (IFAC), Beijing, P.R. China, 1999.

6.) K.J. Astrom and B.M. Bernhardsson. Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In Proceedings of the 41st IEEE Conference on Decision and Control, volume 2, pages 2011–2016, Las Vegas, Nevada, USA, December 10-13 2002.

7.) J.H. Sandee, P.M. Visser, and W.P.M.H. Heemels. Analysis and experimental validation of processor load for event-driven controllers. In IEEE Conference on Control and Applications (CCA), pages 1879–1884, Munich, Germany, 2006

8.) T. Henningsson, E. Johannesson, and A. Cervin. Sporadic event-based control of first-order linear stochastic systems. Automatica, 44(11):2890–2895, November 2008.

9.) M.D. Lemmon (2010), Event-triggered Feedback in Control, Estimation, and Optimization, in Networked Control Systems, editors A. Bemporad, M. Heemels, M. Johansson, Volume 405 Lecture Notes in Control and Information Sciences,pages 293-358, Springer-Verlag Berlin Heidelburg, 2010.

10.) X. Wang and M.D. Lemmon (2011), Attentively Efficient Controllers for Event-triggered Feedback Systems, IEEE Conference on Decision and Control, Orlando Florida, December 2011.

11.) L. Li, M.D.Lemmon, and X. Wang (2012), Stabilizing Bit-Rates in Quantized Event Triggered Control Systems, submitted to Hybrid Systems: computation and control, Beijing, China, April 2012.

12.) L. Li, M.D.Lemmon, and X. Wang (2012), Stabilizing Bit-Rates in Perturbed Event Triggered Control Systems, submitted to IFAC Conference on Analysis and Design of Hybrid Systems, Eindhoven, Neitherlands, 2012.

13.) X. Wang and M.D. Lemmon. Self-triggered feedback control systems with finite-gain l2 stability. IEEE Transactions on Automatic Control, 54(3):452–467, March 2009

14.) X. Wang and M.D. Lemmon (2010), Self-triggering under state-independent disturbances, IEEE Transactions of Automatic Control, vol 55, no. 6, pages 1494-1500, 2010.

15.) X. Wang and M.D. Lemmon (2011), Event-triggering in distributed networked control systems, IEEE Transactions on Automatic Control, volume 56, number 3, pages 586-601, March 2011.

16) D. Angeli, E.D. Sontag, and Y. Wang. A characterization of integral input-to-state stability. IEEE Transactions on Automatic Control, 45(6):1082–1097, 2000.

17) H. Deng, M. Krstic, and R. Williams, “Stabilization of stochastic nonlinear systems driven by noise of unknown covariance,” IEEE Transactions on Automatic Control, vol. 46, no. 8, pp. 1237–1253, 2001.

18) D. Liberzon and J.P. Hespanha. Stabilization of nonlinear systems with limited information feedback. Automatic Control, IEEE Transactions on, 50(6):910–915, 2005.

19) Anta, A.; Tabuada, P.; , "Isochronous manifolds in self-triggered control," Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on , vol., no., pp.3194-3199, 15-18 Dec. 2009