On Event Design in Event-Triggered Feedback Systems ⋆

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Abstract

This paper studies event design in event-triggered feedback systems. A novel event-triggering scheme is presented to ensure exponential stability of the resulting sampled-data system. The scheme postpones the triggering of events over previously proposed methods and therefore enlarges the intersampling period. The resulting intersampling periods and deadlines are bounded strictly away from zero when the continuous time system is input-to-state stable with respect to measurement errors.

1 Introduction

This paper studies the event design in event-triggered feedback systems. Event-triggered systems update the control with new state information when the error between the previous sampled state and the current state exceeds a specified threshold. Prior work [1] on event design in such systems design state-dependent thresholds that ensure that a suitably chosen storage function decreases in a monotone manner over time. However, to guarantee the system satisfies the chosen stability concept, such monotonically decreasing behavior is not necessary. For switched systems, one may tolerate small increases in the storage function, provided the values that the storage function takes after each sampling instant are monotonically decreasing [2]. Based on this idea, this paper presents an event-triggering scheme for exponential stability in which the state is sampled with the storage function, $V(t)$, intersects suitably chosen exponentially decreasing functions.

2 Problem Formulation

Notation: We denote by $\mathbb{R}^n$ the n-dimension real vector space and by $\mathbb{R}^+$ the real positive numbers. We also use $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$. $\|\cdot\|$ denotes 2-norm of a vector or the induced matrix norm. We use $\vee$ to denote the logical operator OR and $\wedge$ to denote the logical operator AND. For a function $V(t)$, we denote the limit of V at t from the right by $V(t^+) = \lim_{s\to t^+} V(s)$.

Consider a sampled-data system. Let r_k denote the time when the kth control task is released for execution on the computer and f_k denotes the time when the kth task has finished executing. Then the system is

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ u(t) &= \gamma(x(r_k)), \quad x(0) = x_0 \end{aligned} \tag{1}$$

for any $t \in [f_k, f_{k+1})$ and any $k \in \mathbb{Z}_0^+$, where $x : \mathbb{R}_0^+ \to \mathbb{R}^n$ is the state trajectory, $x_0 \in \mathbb{R}^n$ is the non-zero initial state, and $u : \mathbb{R}_0^+ \to \mathbb{R}^m$ is a control input. $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $\gamma : \mathbb{R}^n \to \mathbb{R}^m$ are locally Lipschitz functions satisfying two assumptions:

Assumption 2.1 *There exist a positive definite $\mathbb{C}^1$ function $V : \mathbb{R}^n \to \mathbb{R}_0^+$, and class $\mathcal{K}$ functions α_1, α_2, $\bar{\alpha}$, $\bar{\beta}$, $\underline{\alpha}$, $\underline{\beta} : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ such that*

$$\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|) \tag{2}$$

$$\frac{\partial V(x)}{\partial x} f(x, \gamma(x-e)) \le -\bar{\alpha}(\|x\|) + \bar{\beta}(\|e\|) \tag{3}$$

$$\frac{\partial V(x)}{\partial x} f(x, \gamma(x-e)) \ge -\underline{\alpha}(\|x\|) - \underline{\beta}(\|e\|) \tag{4}$$

hold for all $x, e \in \mathbb{R}^n$.

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With Assumption 2.1, we can define a compact set

$$\Omega \triangleq \{x \in \mathbb{R}^n \mid V(x) \le V(x_0)\}. \tag{5}$$

With this set, we propose the second assumption.

Assumption 2.2 *Assumption 2.1 holds and f, α_1^{-1}, $\bar{\beta}$, $\underline{\beta}$, $\bar{\alpha} \circ \alpha_2^{-1}$, $\underline{\alpha} \circ \alpha_1^{-1}$ are locally Lipschitz, i.e. given the compact set $\Omega \subset \mathbb{R}^n$ in (5), there exist positive constants $L, L_1, L_2, \bar{a}, \bar{b}, \underline{a}, \underline{b} \in \mathbb{R}^+$ such that*

$$\|f(x, \gamma(\hat{x}))\| \le L_1\|\hat{x}\| + L_2\|x - \hat{x}\| \tag{6}$$
$$\alpha_1^{-1}(\|x\|) \le L\|x\| \tag{7}$$
$$\bar{\beta}(\|x\|) \le \bar{b}\|x\|, \;\; \underline{\beta}(\|x\|) \le \underline{b}\|x\| \tag{8}$$
$$\bar{\alpha} \circ \alpha_2^{-1}(\|x\|) \le \bar{a}\|x\|, \;\; \underline{\alpha} \circ \alpha_1^{-1}(\|x\|) \le \underline{a}\|x\| \tag{9}$$

hold for all $x, \hat{x} \in \Omega$.

Remark 2.1 *Equation* (3) *implies the continuous system $\dot{x} = f(x, \gamma(x - e))$ is input-to-state stable (ISS) with respect to e. Equations* (7) *–* (9) *further imply that the system is exponentially ISS with a linear gain. Further discussion on this assumption can be found in [1].*

Let $T_k = r_{k+1} - r_k$ denote the kth task period, $D_k = f_k - r_k$ denote the kth task delay, and $\eta_{k+1} = r_{k+1} - f_k = T_k - D_k$. We also define $e_k : \mathbb{R}_0^+ \to \mathbb{R}^n$ as $e_k(t) = x(t) - x(r_k)$ for $k \in \mathbb{Z}_0^+$, which is the measurement error. The control objective is to exponentially stabilize the sampled-data system with minimal computational resources, namely that we want to enlarge the sample period as much as possible.

3 Event-triggered Feedback Systems

This section introduces the event-triggered feedback scheme to ensure exponential stability of the sampled-data system. The main idea is to enforce that $V(t)$ is bounded by an exponentially decreasing function $h : \mathbb{R}_0^+ \times \mathbb{R}^n \to \mathbb{R}_0^+$. It is shown in Figure 1, where the horizontal axis is time, the vertical axis is the energy V, the solid curve is the trajectory of $V(x(t|x_0))$ (For simplicity, we sometimes denote it by $V(t)$ if it is clear in context), the dashed curves are the thresholds, and the dotted curve is the bounding function $h(t, x_0)$. It is easy to see from Figure 1 that $V(t)$ does not have to be always decreasing. Temporary increases in $V(t)$ are allowed as long as V is bounded by h. Such increases lead to longer task periods.

To enforce such bounded $V(t)$, we propose an event-triggering scheme. Given positive constants $\lambda \in \mathbb{R}^+$

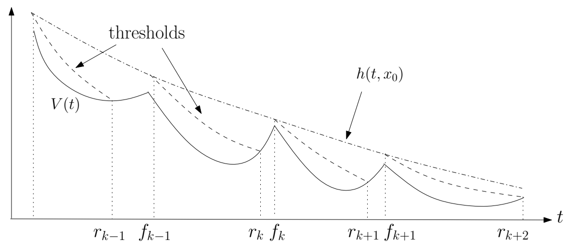

Fig. 1. The trajectory of V, the thresholds, and $h(t, x_0)$ in event-triggered systems with delays

and $\delta, \underline{\delta}, \hat{\delta}$ satisfying $0 < \hat{\delta} < \delta < \underline{\delta} < 1$, we define h by

$$h(t, x_0) = V(x_0)e^{-\hat{\delta}\bar{a}t}. \tag{10}$$

Let $h_k = h(f_k, x_0) = V(x_0)e^{-\hat{\delta}\bar{a}f_k}$. The $k+1$st task release time r_{k+1} is triggered when the logic rule

$$\left(E_1 \bigwedge E_2\right) \bigvee E_3, \tag{11}$$

is false, where

$$E_1 : V(t) \le h_k e^{-\delta\bar{a}(t - f_k)} \tag{12}$$
$$E_2 : t - f_k \le \lambda \tag{13}$$
$$E_3 : r_k \le t \le f_k. \tag{14}$$

E_1 is to ensure that $V(t)$ is always bounded by $h(t, x_0)$ over $[f_k, r_{k+1})$. E_2 is for the safety of the system. It requires the system to sample at least λ unit-time since the last finishing time. Therefore, even if E_1 malfunctions, the system is still monitored and the performance level can be preserved. Keeping E_1 in mind, E_2 also puts a bound on the ratio, $\frac{V(r_{k+1})}{h_k}$, that is essential to the bound on task delays. The purpose of introducing E_3 is simple. It implies that the system does not have to consider the behavior of $V(t)$ during delays.

In the event-triggering scheme, the $k+1$st task delay $D_{k+1} = f_{k+1} - r_{k+1}$ is required to satisfy

$$D_{k+1} \le \Delta_{k+1} = \min\{c, \Delta_1^{k+1}, \Delta_2^{k+1}, \Delta_3^{k+1}\} \tag{15}$$

where $c \in \mathbb{R}^+$ is an arbitrarily chosen positive constant and given any $\theta \in (0, 1)$,

$$\Delta_1^{k+1} = \frac{1}{\bar{a}} \ln\left(1 + \frac{\bar{a}h_k(e^{-\hat{\delta}\bar{a}\eta_{k+1}} - e^{-\delta\bar{a}\eta_{k+1}})}{\bar{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)}\right), \tag{16}$$

$$\Delta_2^{k+1} = \frac{1}{\underline{a}} \ln\left(1 + \frac{(1-\theta)\underline{a}V(r_{k+1})}{\underline{a}\theta V(r_{k+1}) + \underline{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)}\right), \tag{17}$$

$$\Delta_3^{k+1} = \frac{1}{L_2} \ln\left(1 + \frac{(1-\underline{\delta})\bar{a}\theta V(r_{k+1})L_2}{\bar{b}(L_2\|e_k(r_{k+1})\| + L_1\|x(r_k)\|)}\right). \tag{18}$$

The bound in (16) is to ensure that $V(t)$ is still bounded by $h(t, x_0)$ over $[r_k, f_k)$. Bounds in (17) and (18) are to ensure at the time instant $t = f_k$, the decreasing rate $\dot{V}(f_k^+)$ is larger than the decreasing rate of $h_k e^{-\delta\bar{a}(t-f_k)}$ that is $-\delta\bar{a}$. It is shown in Lemma 3.1. In this way, we know $V(t)$ is going below the threshold right after f_k, which means $r_{k+1} - f_k > 0$ always holds. The constant c can be arbitrarily large. It ensures bounded D_k.

Lemma 3.1 *Consider the sampled-data system in* (1) *with the event-triggering scheme in* (11). *Suppose that Assumptions 2.1 – 2.2 hold for a compact set* Ω *defined in* (5). *Given* $k \in \mathbb{Z}^+$ *and* δ, $\hat{\delta}$, $\underline{\delta}$ *satisfying* $0 < \hat{\delta} < \delta < \underline{\delta} < 1$, *assume that* $V(f_k) \leq h_k$ *holds and* $x(t) \in \Omega$ *for all* $t \in (f_k, f_{k+1})$. *If the delay* D_{k+1} *satisfies* (15), *then*

$$V(t) \leq h_k e^{-\hat{\delta}\bar{a}(\eta_{k+1}+t-r_{k+1})}, \forall t \in [r_{k+1}, f_{k+1}), \quad (19)$$

$$\dot{V}(f_{k+1}^+) \leq -\underline{\delta}\bar{a}V(f_{k+1}). \quad (20)$$

Lemma 3.1 assumes that $x(t)$ is inside Ω. The following lemma will relax this assumption by showing that $V(t)$ is always bounded by the function $h(t, x_0)$ and therefore bounded by $h(0, x_0) = V(0)$.

Lemma 3.2 *Consider the sampled-data system in* (1) *with the event-triggering scheme in* (11). *Suppose that Assumptions 2.1 – 2.2 hold for a compact set* Ω *defined in* (5) *and* $r_0 = f_0 = 0$. *Given positive constants* $c, \lambda \in \mathbb{R}^+$ *and* $\delta, \underline{\delta}, \hat{\delta}$ *satisfying* $0 < \hat{\delta} < \delta < \underline{\delta} < 1$, *if* D_{k+1} *satisfies* (15), *then*

$$V(t) \leq h(t, x_0) \quad (21)$$

holds for all $t \in [f_k, f_{k+1})$ *and all* $k \in \mathbb{Z}_0^+$.

With these lemmas, we present the main theorem:

Theorem 3.3 *If the hypotheses in Lemma 3.2 hold, the sampled-data system is exponentially stable and there exist* ϕ, $\psi \in \mathbb{R}^+$, *dependent of* x_0, *such that* $T_k \geq \phi$ *and* $\Delta_k \geq \psi$ *hold for any* $k \in \mathbb{Z}_0^+$.

Remark 3.1 *Based on the triggering mechanism, the proposed scheme can be robust with respect to the disturbances. It is because that when the state is far away from the origin, small disturbances do not dominate the growth rate of* $V(t)$. *Therefore the intersampling periods and deadlines will not be reduced dramatically. However, when the state is close to the origin, the disturbances dominate, which may lead to fast sampling. In that case, the sampling rate will be subject to the hardware limitation (or computational resource available) and the state will stay in a small neighborhood of the origin. The same situation may happen using the scheme proposed in [1].*

4 Conclusions

This paper proposed a new event-triggering scheme that ensures exponential stability of the system. We show that the intersampling periods and deadlines generated by our scheme are bounded strictly away from zero. Simulation examples can be found in [2].

5 Proofs

PROOF. [Proof of Lemma 3.1] We prove the satisfaction of (19) by contradiction. Suppose that it does not hold. Then since $V(r_{k+1}) = h_k e^{-\delta\bar{a}\eta_{k+1}} < h_k e^{-\hat{\delta}\bar{a}\eta_{k+1}}$, there must exist $t^* \in [r_{k+1}, f_{k+1})$ and a positive constant ϵ_1 such that

$$t^* + \epsilon_1 \leq f_{k+1}, \quad (22)$$

$$V(t) \leq h_k e^{-\hat{\delta}\bar{a}(\eta_{k+1}+t-r_{k+1})}, \forall t \in [r_{k+1}, t^*], \quad (23)$$

$$V(t) > h_k e^{-\hat{\delta}\bar{a}(\eta_{k+1}+t-r_{k+1})}, \forall t \in (t^*, t^* + \epsilon_1). \quad (24)$$

By (2) and (7), we have $\|x(t)\| \leq LV(t)$. Equation (23), therefore, implies $\|x(t)\| \leq h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}}$ for any $t \in [r_{k+1}, t^*]$. Consequently, for any $t \in [r_{k+1}, t^*]$, $\|e_k(t)\| \leq h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|$. Applying this inequality into (3), we have for any $t \in [r_{k+1}, t^*]$, $\dot{V} \leq -\bar{a}V(t) + \bar{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)$. Then solving this differential inequality yields for any $t \in [r_{k+1}, t^*]$

$$V(t) \leq V(r_{k+1})e^{-\bar{a}(t-r_{k+1})} - \frac{\bar{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)}{\bar{a}}\left(e^{-\bar{a}(t-r_{k+1})} - 1\right). \quad (25)$$

Note that r_{k+1} is triggered by the violation of E_1 or E_2, which means $V(r_{k+1}) \leq h_k e^{-\delta\bar{a}\eta_{k+1}}$ holds. Applying this inequality into (25) yields

$$V(t) \leq h_k e^{-\hat{\delta}\bar{a}\eta_{k+1} - \bar{a}(t-r_{k+1})} < h_k e^{-\hat{\delta}\bar{a}(\eta_{k+1}+t-r_{k+1})} \quad (26)$$

for any $t \in (r_{k+1}, t^*]$, where the first inequality is obtained because $t - r_{k+1} < D_{k+1} \leq \Delta_1^{k+1}$. Since $V(t)$ is continuous over $[r_{k+1}, f_{k+1})$, we know that there exists a positive constant $\epsilon_2 < \epsilon_1$ such that $V(t^* + \epsilon_2) \leq h_k e^{-\hat{\delta}\bar{a}(\eta_{k+1}+t^*+\epsilon_2-r_{k+1})}$, which is contradicted with (24). So we conclude that (19) holds.

We now show the satisfaction of (20). Since $\|x\| \leq LV(x)$, (26) means $\|x(t)\| \leq h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}}$ for any $t \in [r_{k+1}, f_{k+1})$. With this bound, we know for any $t \in [r_{k+1}, f_{k+1})$, $\|e_k(t)\| \leq h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|$. Applying this inequality into (4) implies $\dot{V} \geq -\underline{a}V(t) - \underline{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)$. Solving this differential inequality yields

$$
\begin{aligned}
&V(f_{k+1}) \geq V(r_{k+1})e^{-\underline{a}(t-r_{k+1})} \\
&+\frac{\underline{b}\left(h_k L e^{-\hat{\delta}\bar{a}\eta_{k+1}} + \|x(r_k)\|\right)\left(e^{-\underline{a}(t-r_{k+1})}-1\right)}{\underline{a}} \geq \theta V(r_{k+1})
\end{aligned} \tag{27}
$$

where the last inequality is obtained by applying $t - r_{k+1} \leq D_{k+1} \leq \Delta_2^{k+1}$.

Consider $\|e_{k+1}(t)\|$ for any $t \in [r_{k+1}, f_{k+1})$. By (6), $\frac{d}{dt}\|e_{k+1}(t)\| \leq \|\dot{x}(t)\| \leq L_2\|e_{k+1}(t)\| + L_2\|e_k(r_{k+1})\| + L_1\|x(r_k)\|$. Solving this, we have

$$
\begin{aligned}
\|e_{k+1}(t)\| &\leq \frac{L_2\|e_k(r_{k+1})\| + L_1\|x(r_k)\|}{L_2}\left(e^{L_2(t-r_{k+1})}-1\right) \\
&\leq \frac{(1-\underline{\delta})\bar{a}\theta V(r_{k+1})}{\bar{b}},
\end{aligned} \tag{28}
$$

where the last inequality is obtained by applying $t - r_{k+1} \leq D_{k+1} \leq \Delta_3^{k+1}$. Combining (27) and (28) implies $\bar{b}\|e_{k+1}(f_{k+1})\| \leq (1-\underline{\delta})\bar{a}V(f_{k+1})$, which implies (20) according to (3).

PROOF. [Proof of Lemma 3.2] We use mathematical induction to prove (21) holds for all $t \in [f_k, f_{k+1})$ and all $k \in \mathbb{Z}_0^+$. We first show that (21) holds over $[f_0, f_1)$. By the definition of h in (10), it is obvious that $V(0) = h(0, x_0)$ and $\dot{V}(0^+) \leq -\bar{a}V(0) < -\delta\bar{a}V(0)$. Therefore $V(t)$ is always below the threshold curve over $t \in [f_0, r_1]$, which means for any $t \in [f_0, r_1]$, (21) holds. Also by Lemma 3.1, (21) holds for any $t \in [r_1, f_1)$.

Assume that (21) holds over $[f_{k-1}, f_k)$ and we now show that it is also true for $[f_k, f_{k+1})$. Since (21) holds at $t = f_{k-1}$, we know that $V(f_{k-1}) \leq h(f_{k-1}, x_0) = h_{k-1}$. Then, by Lemma 3.1, $\dot{V}(f_k^+) < -\delta\bar{a}V(f_k)$ holds. It means that $V(t)$ is always below the threshold curve over $t \in [f_k, r_{k+1}]$, which implies that (21) holds for $t \in [f_k, r_{k+1}]$. Also by Lemma 3.1, (21) holds for any $t \in [r_{k+1}, f_{k+1})$. Therefore, we conclude that (21) holds for any $t \in [f_k, f_{k+1})$, which completes the proof.

PROOF. [Proof of Theorem 3.3] By Lemma 3.2, $V(t) \leq h(t, x_0)$ holds for any $t \geq 0$. Since $h(t, x_0)$ is exponentially decreasing to zero, we know the system is exponentially stable.

We now show the bounds on T_k and D_k. By (6), the derivative of $\|e_{k+1}(t)\|$ over $t \in [f_{k+1}, r_{k+2})$ satisfies $\frac{d}{dt}\|e_{k+1}(t)\| \leq L_2\|e_{k+1}(t)\| + L_1\|x(r_{k+1})\|$. Solving this differential inequality with the initial condition given by $\|e_{k+1}(f_{k+1})\| \leq \frac{(1-\underline{\delta})\bar{a}}{\bar{b}}V(f_{k+1})$ obtained based on a similar analysis to Lemma 3.1, we have

$$
\begin{aligned}
\|e_{k+1}(t)\| &\leq \frac{(1-\underline{\delta})\bar{a}}{\bar{b}}V(f_{k+1})e^{L_2(t-f_{k+1})} \\
&+\frac{L_1\|x(r_{k+1})\|}{L_2}\left(e^{L_2(t-f_{k+1})}-1\right) \triangleq \kappa(t)
\end{aligned}
$$

for any $t \in [f_{k+1}, r_{k+2})$. Applying the inequality above into (3) implies $\dot{V}(t) \leq -\bar{a}V(t) + \bar{b}\kappa(t)$ for all $t \in [f_{k+1}, r_{k+2})$. Solving this differential inequality with the initial condition $V(f_k)$ leads to $V(r_{k+2}) \leq V(f_{k+1})e^{-\bar{a}\eta_{k+2}} - \frac{\bar{b}\kappa(r_{k+2})\left(e^{-\bar{a}\eta_{k+2}}-1\right)}{\bar{a}}$.

Note that $\frac{\|x(r_{k+1})\|}{L} \leq V(r_{k+1}) \leq h_k e^{-\delta\bar{a}\eta_{k+1}}$, $V(r_{k+1}) \leq h_{k+1}e^{(\hat{\delta}-\delta)\bar{a}\eta_{k+1} + \hat{\delta}\bar{a}D_{k+1}}$, and $V(f_{k+1}) \leq h_{k+1}$. Applying these inequalities with $D_{k+1} \leq c$ into the preceding inequality yields

$$
\begin{aligned}
&p(\eta_{k+2}) \triangleq e^{-\delta\bar{a}\eta_{k+2}} \leq q(\eta_{k+2}) \triangleq e^{-\bar{a}\eta_{k+2}} - \\
&\left((1-\underline{\delta})e^{L_2\eta_{k+2}} + \frac{\bar{b}LL_1e^{\hat{\delta}\bar{a}c}\left(e^{L_2\eta_{k+2}}-1\right)}{\bar{a}L_2}\right)\left(e^{-\bar{a}\eta_{k+2}}-1\right).
\end{aligned}
$$

Note that $\frac{dp(\eta_{k+2})}{d\eta_{k+2}}|_{\eta_{k+2}=0} = -\delta\bar{a}$ and $\frac{dq(\eta_{k+2})}{d\eta_{k+2}}|_{\eta_{k+2}=0} = -\underline{\delta}\bar{a}$. By the continuity of p and q, there must exist a constant ξ_c such that $\eta_{k+2} \geq \xi_c > 0$. Then with the event E_2 in (13), we have $\lambda \geq \eta_k \geq \xi_c$. Also note that the scheme guarantees that for any $k \in \mathbb{Z}_0^+$

$$
\begin{aligned}
&h_{k+1}e^{-\delta\bar{a}\lambda} \leq V(r_{k+2}) \leq h_{k+1}e^{-\delta\bar{a}\eta_{k+2}} \\
&V(r_{k+1}) \leq h_{k+1}e^{(\hat{\delta}-\delta)\bar{a}\eta_{k+1} + \hat{\delta}\bar{a}D_{k+1}} \\
&\|x(r_{k+1})\| \leq LV(r_{k+1}).
\end{aligned} \tag{29}
$$

Applying these inequalities into Δ_1^{k+1}, Δ_2^{k+1}, Δ_3^{k+1} defined in (16)–(18) such that the state-related terms can be canceled, we can easily find the positive constant bounds on Δ_i^{k+1}. Then by the definition of Δ_{k+1} in (15), there exits a positive constant ψ such that $\Delta_{k+1} \geq \psi$. Also $T_{k+1} = r_{k+1} - f_k + f_k - r_k \geq \xi_c + \psi = \phi$.

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