Event-Triggered Networked Control Systems

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Many of the results for event-triggered control of embedded systems can be extended to networked control systems. A networked control system or NCS is a set of controllers that coordinate their actions over a communication network. For NCS, event-triggering is used to decide when to *transmit* or *broadcast* the system state to a local controller's neighbors. Using events to trigger communication actually provides a much stronger motivation for event-triggered control. The reason for this is that in many cases, the energy or cost associated with the transmission of a bit of information is much more than the energy associated with using that bit to compute the control law. Event-triggering, therefore, provides a realistic way of reducing traffic congestion in communication networks used by NCS. The objective of this section is to show how the earlier results from event-triggered control of embedded systems can be extended to networked control systems. The NCS architecture under study is first discussed and then we derive event-triggered triggers that assure input-to-state stability of the NCS. As in the case of embedded systems, the NCS implementation introduces a number of so-called *network artifacts* that complicate the analysis of the idealized model. These network models include delays in the transmission of information as well as dropped information packets. An important issue that should be addressed in the study of such systems is the impact that such network artifacts have on the overall performance of the NCS.

While there is a great deal of literature [1] [2] [4][5] [9] examining networked control systems, there is relatively little work pertaining to event-triggered NCS. Most of the results in this section are drawn from [7] and [6]. Related work will be found in [3].

Model of Networked Control System: Let's first describe a model of a networked control system or NCS. Consider a distributed NCS containing N agents. Figure 1 provides a graphic illustration of an NCS with three agents. Each agent consists of a *physical* component and a *cyber* component. The physical components are interconnected as shown by the solid lines in the figure. The cyber components are also interconnected through a communication network whose links are shown by the dashed lines in the figure.

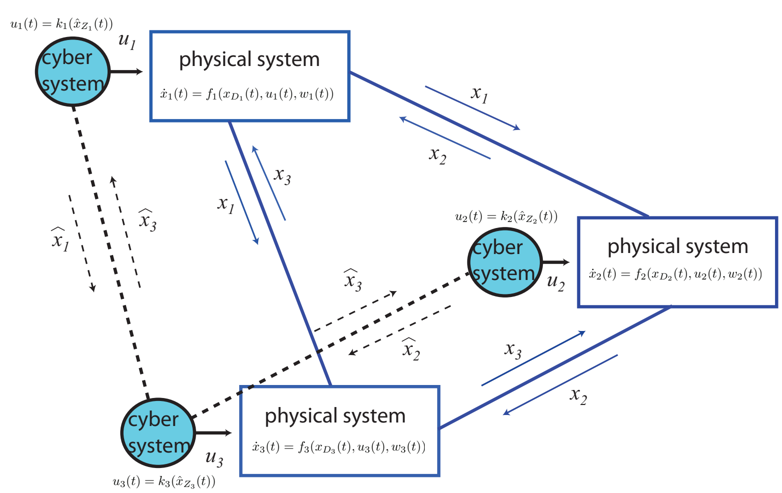

Figure 1: Model of Event-Triggered Networked Control Systems

This system may be more formally characterized using graph theoretic notation. In particular, let $\mathscr{N} = \{1,2,\cdots,N\}$ denote the set of agents. A graph $\mathscr{G}_{cp} = (\mathscr{N}, E_{cp})$ represents the physical coupling between the agents. $\mathscr{N}$ denotes the vertices of the graph and $E_{cp} \subset \mathscr{N} \times \mathscr{N}$ denotes the set of edges in the graph. In this case an edge (i,j) is in E_{cp} if the dynamics of agent j's physical component are directly driven by agent i's local state. The graph $\mathscr{G}_{cm} = (\mathscr{N}, E_{cm})$ is models the interconnections between the cyber-components of the agents. As before $\mathscr{N}$ denotes the vertices (nodes) of the graph and $\mathscr{E}_{cm} \subset \mathscr{N} \times \mathscr{N}$ represents the edges of the graph.

In this section, the graphs for the physical and cyber interconnections need not be the same. This requires us to define a number of special neighborhoods in the graph. In particular, we let

- $Z_i = \{j \in \mathscr{N} \mid (j,i) \in E_{cm}\}$ represents those agents whose cyber-components can send information to agent i's cyber-component.
- $U_i = \{j \in \mathscr{N} \mid (i,j) \in E_{cm}\}$ denotes those agents whose cyber-components can receive information from agent i's cyber-component.
- $D_i = \left\{j \in \mathscr{N} \mid (j,i) \in E_{cp}\right\}$ represents those agents whose physical components directly drive the dynamics of agent i's physical component.
- $S_i = \left\{j \in \mathscr{N} \mid (i,j) \in E_{cp}\right\}$ denotes those agents whose physical components are directly driven by the physical component of agent i.

For any set $\Sigma \subset \mathscr{N}$, we let $|\Sigma|$ denote the number of elements in that set and we let $\overline{\Sigma} = \Sigma \cup \{i\}$.

The physical component of agent i is characterized by a *local state* $x_i : \Re \to \Re^n$ where x_i satisfies the differential equation

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_{\overline{D}_i}(t), u_i(t), w_i(t)) \\ x_i(t_0) &= x_{i0} \end{aligned}$$

where $x_{\overline{D}_i} = \{x_j\}_{j \in \overline{D}_i}$ are the local states of agent i's neighbors that are physically connected to it. The system dynamics are characterized by the function $f_i : \Re^{n|\overline{D}_i|} \times \Re^m \times \Re^\ell \to \Re^n$ is continuous and locally Lipschitz satisfying $f_i(0,0,0) = 0$. $u_i : \Re \to \Re^m$ is a control input generated by the cyber-component of the agent and $w_i : \Re \to \Re^\ell$ is an external disturbance.

The control u_i is generated by agent i's cyber-component. Since these cyber components exchange information over a digital communication network, local states are transmitted in a discrete manner. In particular, we let $\{r^i_j\}_{j=1}^\infty$ denote the sequence of broadcast release times for the ith agent. So the *transmitted* state from agent j is denoted as

$$\hat{x}_j(t) = x_j(r_j)$$

for $t \in [r_j, r_{j+1})$ and $j = 0, \ldots, \infty$. Agent i's cyber component uses the local state information received from all its neighbors in the set Z_i to compute the control u_i. So let $k_i : \Re^{n|\overline{Z}_i|} \to \Re^m$ denote the i'th agent's local controller so that

$$u_i(t) = g_i(\hat{x}_{\overline{Z}_i}(t))$$

Following the same notational conventions as before, $\hat{x}_{\overline{Z}_i}$ denotes the broadcast states of all neighbors of agent i whose cyber-components send information directly to agent i.

ISS Event-Triggered Networked Control: We can now derive ISS event-triggers for the NCS described above. In particular, let $e_i(t) = \hat{x}_i(t) - x_i(t)$ denote the local *gap* between agent i's current state and its last broadcast state. We assume there exists a positive definite function $V : \Re^{nN} \to \Re$ and class $\mathscr{K}$ functions γ_i, ψ_i, and β_i (for $i = 1, \ldots, N$) such that

$$\dot{V} = \sum_{i=1}^N \frac{\partial V}{\partial x_i} f_i(x_{\overline{D}_i}, k_i(x_{Z_i} + e_{Z_i}), w_i) \leq \sum_{i=1}^N \left(-\gamma_i(\|x_i\|) + \psi(\|e_i\|) + \beta_i(\|w_i\|)\right) \tag{1}$$

where e_{Z_i} is the gap of all agent i's cyber-neighbors. This assumption means that V is an ISS-Lyapunov function when the the gap $e_i = 0$. In view of our earlier discussion, this is sufficient to imply that the local controllers k_i leave the original "continuously" sampled version of the networked control system input-to-state stable.

So again, we select a user parameter $\sigma_i \in (0,1)$ and note that if the local state and gap trajectories satisfy the inequality

$$-\sigma_i \gamma_i(\|x_i(t)\|) + \psi(\|e_i(t)\|) \leq 0 \tag{2}$$

for all $t \in \Re$ and all $i = 1,\ldots,N$, then the bound on $\dot{V}$ becomes

$$\dot{V} \leq \sum_{i=1}^{N} \left(-(1-\sigma_i)\gamma_i(\|x_i\|) + \beta_i(\|w_i\|)\right)$$

This is, of course, a dissipative inequality which we saw earlier was sufficient to show that the event-triggered NCS is input-to-state stable with respect to the external input w_i.

As before in our study of the embedded event-triggered controllers, the inequality in equation (2) can be used as the basis of a state-dependent threshold test. In particular, the ith agent would check the validity of the following threshold test on the gap,

$$\psi_i(\|e_i(t)\|) \leq \sigma_i \gamma_i(\|x_i(t)\|)$$

At the broadcast time r_j^i, the local gap, $e_i = 0$. This gap then grows until it exceeds the state dependent threshold $\gamma_i(\|x_i(t)\|)$. The violation of that threshold triggers agent i to broadcast its state again. Note that this is a "cooperative" broadcast mechanism in that the violation of the threshold results in an agent sharing its local state information with its neighbors. In other words, the success of such an event-triggered broadcast scheme relies on all agent's agreeing to work in the same manner.

Note that the ISS event trigger given above is only a *local* function of the agent's state. This is important, for it means each agent is able to trigger its broadcast without relying directly on its neighbors. A key part of the prior analysis is the assumption that there exists a ISS Lyapunov function that is "separable" in the sense specified by the bounds in equation (1). Such a Lyapunov function may be constructed if we can identify a set of N positive definite functions $V_i : \Re^n \to \Re$ for $i = 1,\ldots,N$ with class $\mathscr{K}$ functions γ_i, η_i, ψ_i, and β_i such that

$$\frac{\partial V_i}{\partial x_i} f_i(x_{\overline{D}_i}, k_i(x_{\overline{Z}_i} + e_{\overline{Z}_i}), w_i) \leq -\gamma_i \|x_i\|^2 + \sum_{j \in D_i cup Z_i} \eta_j \|x_j\|^2 + \sum_{j \in \overline{Z}_i} \psi_j \|e_j\|^2 + \beta_i^2 \|w_i\|^2 \tag{3}$$

In this case, we can then see that by choosing $V = \sum_{i=1}^{N} V_i$, then we can easily see that

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{N} \left(-\gamma_i \|x_i\|^2 + \sum_{j \in D_i \cup Z_i} \eta_j \|x_j\|^2 + \sum_{j \in \overline{Z}_i} \psi_j \|e_j\|^2 + \beta_i^2 \|w_i\|^2 \right) \\ &= \sum_{i=1}^{N} \left(-\left(\gamma_i - |S_i \cup U_i| \eta_i\right) \|x_i\|^2 + \psi_i |\overline{U}_i| \|e_i\|^2 + \beta_i^2 \|w_i\|^2 \right) \end{aligned}$$

Note that this matches the conditions in equation (1) provided the first term on the righthand side is negative definite. This term will be negative definite if we require

$$\gamma_i - |S_i \cup U_i| \eta_i > 0$$

This condition places a restriction on the amount of coupling between physically interconnected physical systems. In particular, it says that if we can appropriately bound this physical coupling and if there exist candidate ISS-Lyapunov functions satisfying the bounds in equation (3), then we can always construct a global V that is an ISS-Lyapunov function for the entire networked system. In this case, the associated ISS event-trigger is easily shown to have the form

$$\|e_i(t)\| \leq \sigma_i \sqrt{\frac{\gamma_i - |S_i \cup D_i| \eta_i}{|\overline{U}_i| \psi_i}} \|x_i(t)\| = \frac{\sigma_i}{\alpha_i} \|x_i(t)\|$$

which would ensure the $\mathscr{L}_2$ stability of the entire system.

The ability to construct V from smaller "local" candidate ISS-Lyapunov functions is important, for it allows us to distribute the design of the ISS event-triggers. This is particularly important in large-scale networked systems where agent subsystems may be added and modified in an ad hoc manner. One particularly good example where we can exploit this "distributed" strategy for constructing the ISS event triggers occurs when the underlying networked system is linear. In this case, the parameters in the triggering conditions can be computed using linear matrix inequalities [8].

Simulation results for this approach to event-triggered broadcasting are shown in figure 2. This example was taken from [7]. It consists of several carts that are interconnected as shown through soft springs. The local state of the ith cart is $x_i = \left[\begin{array}{cc} y_i & \dot{y}_i \end{array} \right]^T$ where y_i is the position of the ith cart with respect to the system's equilibrium point. Assuming soft spring coupling between the carts, we can see that the state equation for those carts with springs on both sides are

$$\dot{x}_i(t) = \frac{d}{dt}\left[\begin{array}{c} y_i \\ \dot{y}_i \end{array} \right] = \left[\begin{array}{c} \dot{y}_i(t) \\ u_i(t) + k_i^1 \tanh(y_{i+1}(t) - y_i(t)) + k_i^2 \tanh(y_{i-1}(t) - y_i(t)) + w_i(t) \end{array} \right]$$

for all $t \in \Re$. In this case k_i^1 denotes the spring constant for the spring on the right hand side of the ith cart and k_i^2 denotes the spring constant on the left hand side of the cart. The function $u_i : \Re \to \Re$ denotes the "control" applied to the cart by its local controller.

In this example the communication network's links mirror the physical interactions between the carts so that $Z_i = D_i$. The sampled state is denoted as $\hat{x}_i(t) = \left[\begin{array}{cc} \hat{y}_i(t) & \frac{d}{dt}\hat{y}_i(t) \end{array} \right]^T$ where $\hat{y}_i(t) = y_i(r_j)$ and $\frac{d}{dt}\hat{y}_i(t) = \dot{y}_i(r_j)$ for all $t \in [r_j, r_{j+1})$ and $j = 0, \ldots, \infty$. The local control is computed from these sampled measurements as

$$u_i(t) = K_i \hat{x}_i(t) - k_i^1(\tanh(\hat{y}_{i+1}(t) - \hat{y}_i(t)) - k_i^2 \tanh(\hat{y}_{i-1}(t) - \hat{y}_i(t))$$

In this case, the agents at the either end of the carts uses the ISS event-trigger $5.9\|e_j^i(t)\| < 0.2\|x_i(r_j^i)\|$ and the interior agents use the event trigger $10.3\|e_i^j(t)\| < 0.2\|x_i(r_j^i)\|$. The results from this simulation are shown in figure 2.

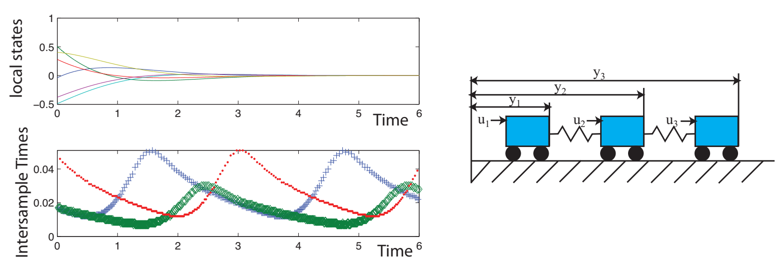

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Figure 2: Simulation Example of Event-Triggered Networked Control System consisting of three coupled carts.

The top plot on the left hand side of figure 2 plots the state trajectories for all three carts. As can be seen, this event-triggered system is asymptotically stable since all points asymptotically approach their equilibrium points at zero. The bottom plot on the left hand side of figure 2 plots the intersample time intervals that were generated by the proposed event triggers. As can be seen, these intersample time intervals vary over time in a regular manner.

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