Intermittent Communication

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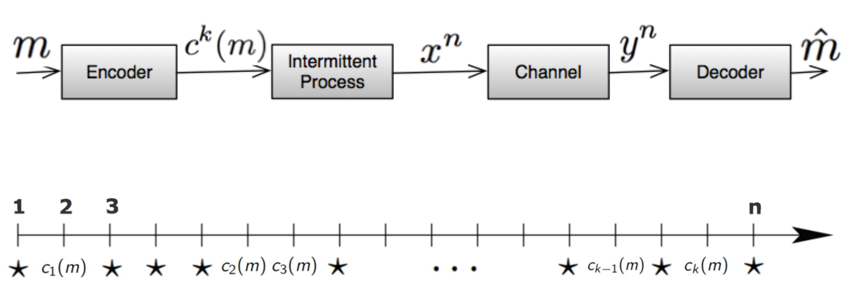
Motivation

- Bursty transmission of codeword symbols
  - Sporadically available channel: Insertion channels
  - Intermittent transmitter: Energy harvesting systems
  - Bursty multi-access communication: Random-access with/without collisions
- Receiver's task:
  - Detect the positions
  - Decode the message

System Model

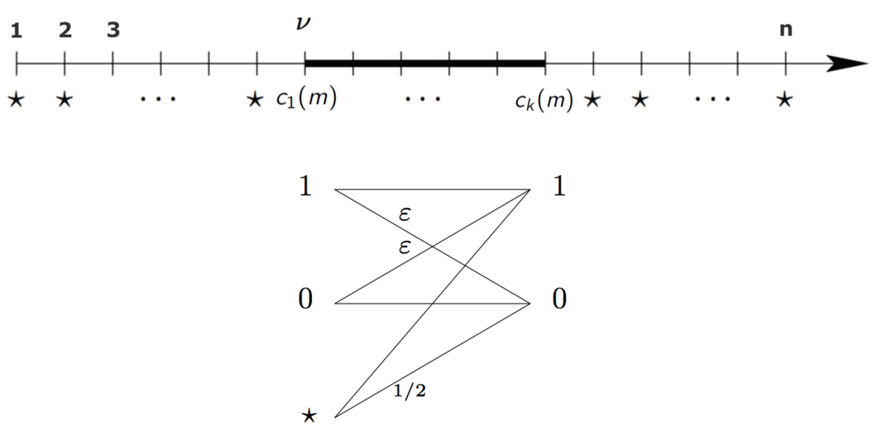
- x^n consists of c^k at k arbitrary time slots and is equal to the noise symbol $\star \in \mathcal{X}$ at the other $n-k$ time slots
- Intermittency rate $\alpha := n/k$: The larger α is, the more intermittent the system becomes
- Rate $R := \log M/k$ is achievable if $\mathbb{P}(\hat{m} \neq m) \to 0$ as $k \to \infty$
- Achievable rate region (R, α)



Comparison to Asynchronous Communication

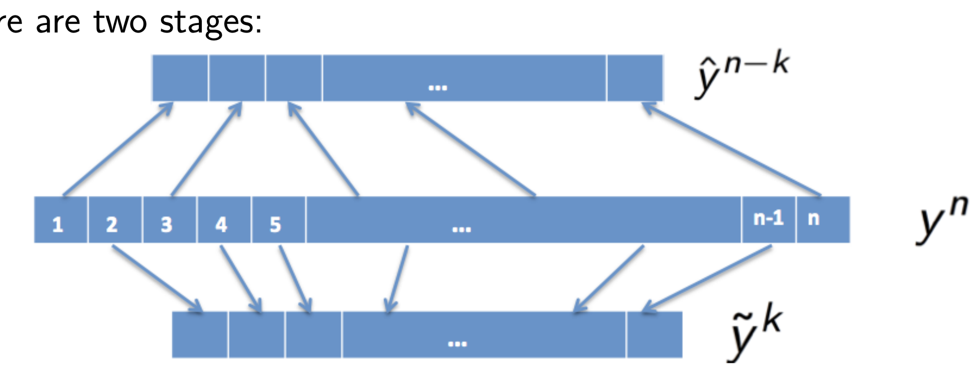
- Contiguous transmission of the codewords, but starts at a random time unknown to the receiver [Tchamkerten, Chandar, Wornell]
- $n = e^{\alpha k}$, $\alpha \to$ Asynchronous exponent
- $W_\star(\cdot) := W(\cdot|x = \star)$ the noise distribution of the channel
- Theorem:

$$C(\alpha) = \max_{P: D(PW||W_\star) \geq \alpha} \mathbb{I}(P, W)$$



Decoding from Pattern Detection

- The decoder chooses k of the n output symbols denoted by $\tilde{y}^k$ and the other symbols denoted by $\hat{y}^{n-k}$. For each choice there are two stages:



  - If $\tilde{y}^k \in T_{PW}$ and $\hat{y}^{n-k} \in T_{W_\star}$, then go to the second stage; otherwise, make another choice for the k symbols and repeat the two-stage procedure.
  - If $\tilde{y}^k \in T_{[W]_\mu}(c^k(m))$ for a unique index m and declares it as the message being sent; otherwise, make another choice for the k symbols and repeat the two-stage procedure.
- If the decoder does not declare any message as being sent by the end of all $\binom{n}{k}$ choices, then the decoder declares an error.

Partial Divergence

- Method of types: If a random sequence X^k is generated i.i.d. according to Q, then

$$\lim_{k\to\infty} -\frac{1}{k} \log \mathbb{P}(X^k \in T_P) = D(P||Q)$$

- Lemma: If a random sequence X^k is generated as follows: k_1 symbols are i.i.d. according to Q and k_2 symbols are i.i.d. according to P, where $k_1 + k_2 = k$ and $\rho := k_1/k$, then

$$\lim_{k\to\infty} -\frac{1}{k} \log \mathbb{P}(X^k \in T_P) := d_\rho(P||Q)$$

$$= D(P||Q) - \sum_{j=0}^{t-1} p_j \log(c^* + \frac{p_j}{q_j}) + \rho \log c^* + h(\rho),$$

where c^* is a function of ρ, P, and Q:

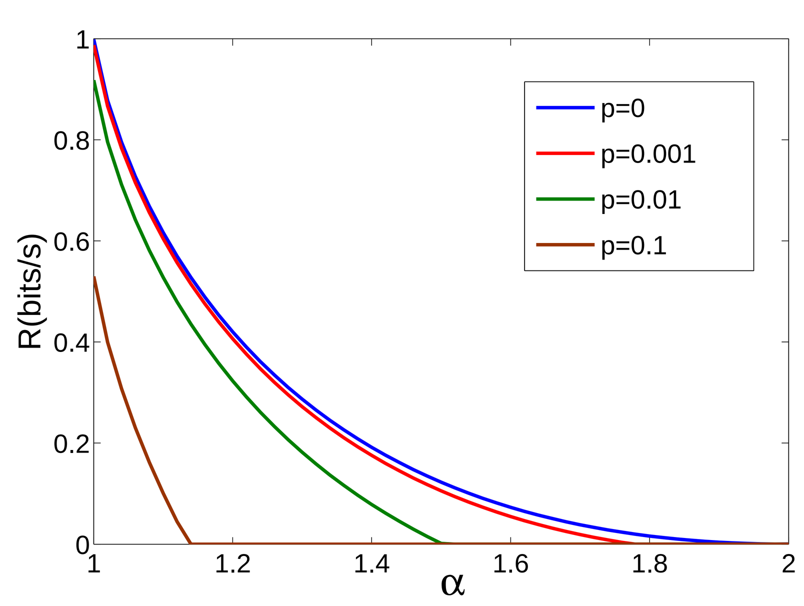
$$c^* \sum_{j=0}^{t-1} \frac{p_j q_j}{c^* q_j + p_j} = \rho.$$

Achievable Rates

Theorem: *Using decoding from pattern detection $R < \max_P\{\mathbb{I}(P, W) - f(P, W, \alpha)\}$ is achievable, where*

$$f(P, W, \alpha) := \max_{0\leq\beta\leq 1} \{(\alpha-1)h(\beta) + h((\alpha-1)\beta) - d_{(\alpha-1)\beta}(PW||W_\star) - (\alpha-1)d_\beta(W_\star||PW)\}.$$

Overhead cost $f(P, W, \alpha)$:
- How much we need to back off from the mutual information as the cost of intermittency
- $f(P, W, \alpha)$ is increasing in $\alpha \to$ increasing the intermittency rate decreases the achievable rate



Binary Intermittent Communication

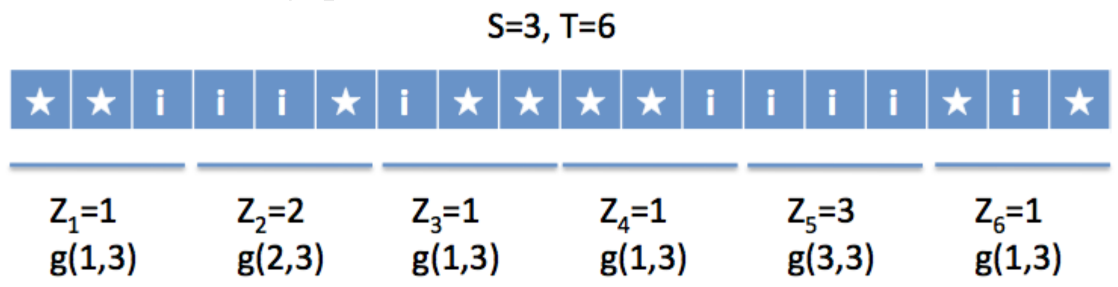
BSC with crossover probability 0 (noiseless) and $\star = 0$:
- Uniform insertion: $n-k$ zeroes are inserted uniformly over all $\binom{n}{k}$ possible realizations

$$g(k, n) := \max_{P(x^k)} \mathbb{I}(X^k; Y^n), \quad \phi(k, n) := k - g(k, n)$$

  - $g(k, n)$ for finite k and n can be numerically computed by evaluating the transition probabilities $P(Y^n|X^k)$ and using the Blahut algorithm
- IID insertion: At each time slot a codeword symbol is sent with probability $p_t := 1/\alpha$ and the insertion symbol 0 is sent with probability $1 - p_t$
  - At the decoder, there are N symbols, $N \sim$ negative binomial distribution
  - Information-stable: $C = \lim_{k\to\infty} g(k, N)/k$ [Dobrushin]
  - $N/k \xrightarrow{p} \alpha$ as $k \to \infty$

Upper Bounds for the IID Insertion Model

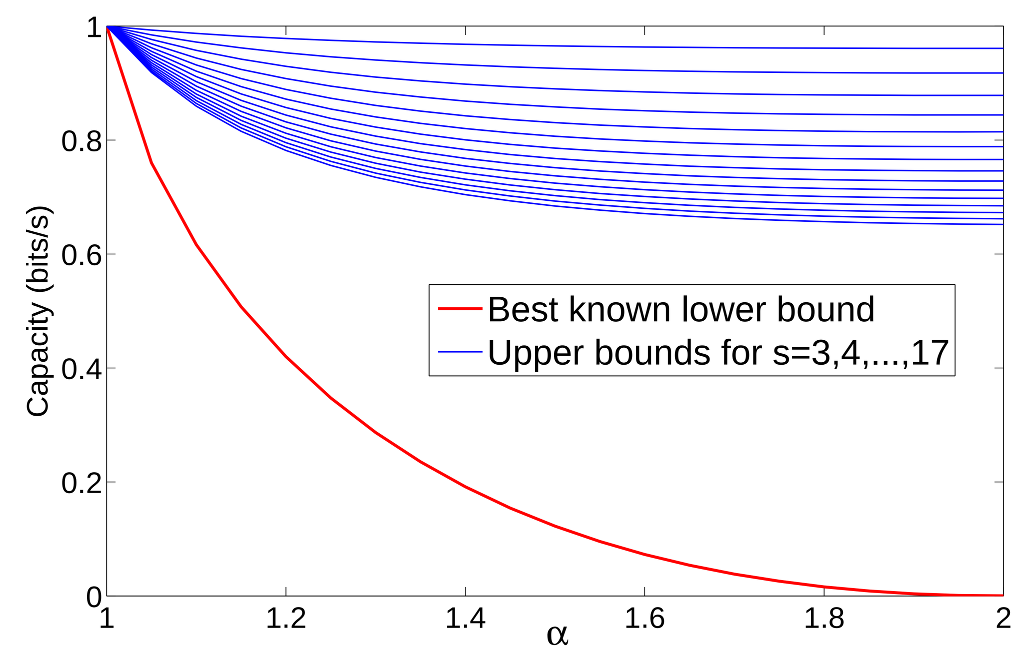
- Idea: Providing the decoder and encoder with side-information [Fertonani, Duman]
- Side-information: For consecutive blocks of length s of the output sequence, the number of information symbols $\to$ random sequence $\{Z_i\}_{i=1}^T$ i.i.d. $\sim B(s, p_t)$



- Let C_u be the capacity of this genie-aided system $\to$ an upper bound for the capacity of the original channel

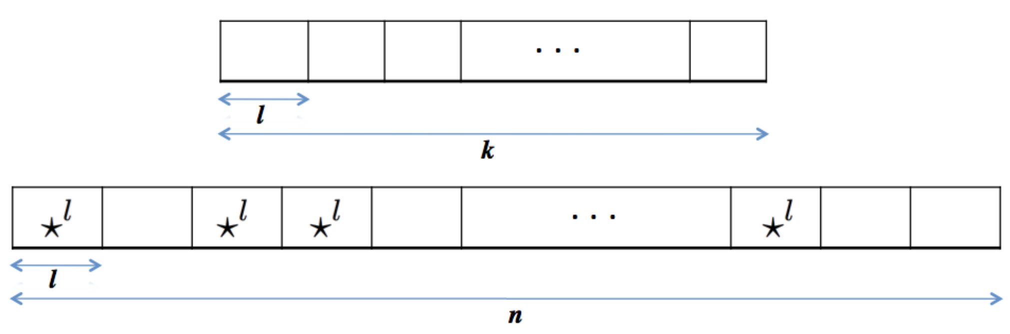
$$C_u = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{T} g(Z_i, s) = 1 - \frac{1}{sp_t} \sum_{a=0}^{s} \binom{s}{a} p_t^a (1-p_t)^{s-a} \phi(a, s)$$

- Maximum of s for which we could compute $\phi(a, s)$ is 17



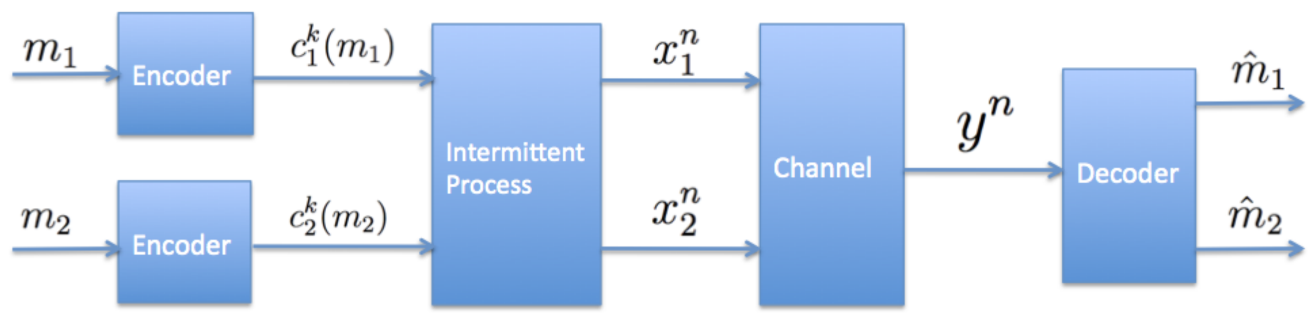
- Increasing $s \to$ less frequent side-information $\to$ better upper bound
- The plot suggests that the linear scaling is relevant
- The techniques can be used for non-binary and/or noisy i.i.d. insertions

Extension 1: Packet-Level Intermittent Communication



- *Small packet intermittent communication*: l is finite. If $l = 1$: Symbol-level intermittent communication
- *Medium packet intermittent communication*: $l = \lambda \log k$
- *Large packet intermittent communication*: $l = k^\lambda, \lambda \leq 1$. If $\lambda = 1$: Asynchronous communication

Extension 2: Intermittent Multi-Access Communication



$W(y|x_1, x_2)$, $W_{\cdot\star} := W(y|x_1, x_2 = \star)$, $W_{\star\cdot} := W(y|x_1 = \star, x_2)$, $W_{\star\star} := W(y|x_1 = \star, x_2 = \star)$
Rate region (R_1, R_2) is achievable if $\mathbb{P}((\hat{m}_1, \hat{m}_2) \neq (m_1, m_2)) \to 0$ as $k \to \infty$

- Random access without collisions: The output pair (x_1, x_2) of the intermittent process $(c_1, \star)$ or $(\star, c_2)$
  Theorem: Rates (R_1, R_2) satisfying

$$R_1 < \mathbb{I}(X_1; Y|x_2 = \star) - f_1(P_1, P_2, W)$$
$$R_2 < \mathbb{I}(X_2; Y|x_1 = \star) - f_1(P_1, P_2, W)$$

are achievable for any $(X_1, X_2) \sim P_1(x_1)P_2(x_2)$, where

$$f_1(P_1, P_2, W) := \max_{0\leq\beta\leq 1} \{2h(\beta) - d_\beta(P_1 W_{\cdot\star}||P_2 W_{\star\cdot}) - d_\beta(P_2 W_{\star\cdot}||P_1 W_{\cdot\star})\}.$$

Input distributions P_1 and P_2: trade-off between the two terms
- Random access with collisions: d collided symbols at the output of the intermittent process
  - Collisions as deletions: $k-d$ of the pair $(c_1, \star)$, $k-d$ of the pair $(\star, c_2)$, and d collided symbols that are deleted
  - Collisions as interference: $k-d$ of the pair $(c_1, \star)$, $k-d$ of the pair $(\star, c_2)$, and d of the pair (c_1, c_2)