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Structural controllability of switched linear systems[☆]

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ABSTRACT

This paper studies the structural controllability of a class of uncertain switched linear systems, where the parameters of subsystems' state matrices are either unknown or zero. The structural controllability is a generalization of the traditional controllability concept for dynamical systems and purely based on the interconnection relation between the state variables and inputs through non-zero elements in the state matrices. In order to illustrate such a relationship, two kinds of graphic representations of switched linear systems are proposed, based on which graph theory-based necessary and sufficient characterizations of the structural controllability for switched linear systems are presented. Finally, the paper concludes with discussions on the results and future work.

1. Introduction

As a special class of hybrid control systems, a switched linear system consists of several linear subsystems and a rule that orchestrates the switching among them. Switching between different subsystems or different controllers can greatly enrich the control strategies and may achieve better control performances than fixed (non-switching) controllers (Liberzon, 2003). Besides, switched linear systems also have promising applications in control of mechanical systems, aircrafts, satellites and swarming robots. Driven by its importance in both theoretical research and practical applications, the switched linear system has attracted considerable attention during the last decade; see e.g., Ji, Lin, and Lee (2009), Liberzon (2003), Lin and Antsakis (2007), Qiao and Cheng (2009), Sun, Ge, and Lee (2002), Xie and Wang (2003).

Much work has been done on the controllability of switched linear systems. For example, the controllability and reachability for low-order switched linear systems have been presented in Loparo, Aslanis, and Hajek (1987). Complete geometric criteria for controllability and reachability were established in Sun et al. (2002) and Xie and Wang (2003). However, all the previous work mentioned above has been based on the traditional controllability concept. In this paper, we investigate the structural controllability of a class of uncertain switched linear system, where the parameters of subsystems' state matrices are either unknown or zero. This is a reasonable assumption as many system parameters are difficult to identify and only known to certain approximations. On the other hand, we are usually pretty sure where zero elements are either by coordination transformation or by the absence of physical connections among components in the system. Thus structural properties that are independent of a specific value of unknown parameters are of particular interest. A switched linear system is said to be structurally controllable if one can find a set of values for the unknown parameters such that the corresponding switched linear system is controllable in the classical sense. For linear structured systems, generic properties including structural controllability have been studied extensively and it turns out that generic properties including structural controllability are true for almost all values of the parameters; see e.g., Blackhall and Hill (2010), Dion, Commault, and van der Woude (2003), Glover and Silverman (1976), Lin (1974), Mayeda (1981), Murota (1987), Reinschke (1988), Shields and Pearson (1976), van der Woude (1991). This also holds true for switched linear systems studied here and presents one of the reasons why this kind of structural controllability is of interest.

It turns out that the structural controllability of switched linear systems only depends on graphic topologies among state and input vertices of individual subsystems and their union. The paper aims to characterize such a relationship, and its contribution is twofold. First, two kinds of graphic representations of switched linear systems are proposed. Second, graph theory-based necessary and sufficient characterizations of the structural controllability for switched linear systems are presented. Graphic conditions can help to understand how the graphic topologies of dynamical systems influence the corresponding generic properties, here especially

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for the structural controllability. This would be helpful in many practical applications and motivates our pursuit on illuminating the structural controllability of switched linear systems from a graph theoretical point of view. Preliminary results of this paper appeared in Liu, Lin, and Chen (2010). An extended version of this paper with illustrative examples can be found in Liu, Lin, and Chen (2013) as a technical report.

The organization of this paper is as follows: in Section 2, we introduce some basic preliminaries and the problem formulation, followed by structural controllability study of switched linear systems in Section 3, where several graphic necessary and sufficient conditions for the structural controllability are given. Finally, some concluding remarks are drawn in Section 4.

2. Preliminaries and problem formulation

2.1. Graph theory preliminaries

A matrix P is said to be a structured matrix if its entries are either fixed zeros or independent free parameters. A numerical matrix $\tilde{P}$ is called admissible (with respect to P) if it can be obtained by fixing the free parameters of P at some particular values. In addition P_{ij} is adopted to represent the element of P from row i and column j.

Consider a linear control system, $\dot{x} = Ax(t) + Bu(t)$, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^r$. The matrices A and B are assumed to be structured matrices, which means that their elements are either fixed zeros or free parameters. This structured system given by matrix pair (A, B) can be described by a directed graph, denoted as $\mathcal{G}(A, B)$, with vertex set $\mathcal{V} = \mathcal{X} \cup \mathcal{U}$, where $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$ called *state vertex set* and $\mathcal{U} = \{u_1, u_2, \ldots, u_r\}$ called *input vertex set*, and edge set $\mathcal{I} = \mathcal{I}_{UX} \cup \mathcal{I}_{XX}$, where $\mathcal{I}_{UX} = \{(u_i, x_j) | B_{ji} \neq 0, 1 \leq i \leq r, 1 \leq j \leq n\}$ and $\mathcal{I}_{XX} = \{(x_i, x_j) | A_{ji} \neq 0, 1 \leq i \leq n, 1 \leq j \leq n\}$ are the oriented edges between inputs and states and between states defined by the interconnection matrices A and B above. The following notations from Lin (1974) are recalled.

Definition 1 (*Stem*)**.** An alternating sequence of distinct vertices and oriented edges is called a directed path, in which the terminal node of any edge never coincide to its initial node or the initial or the terminal nodes of the former edges. A stem is a directed path in the state vertex set $\mathcal{X}$ that begins in the input vertex set $\mathcal{U}$.

Definition 2 (*Accessibility*)**.** A vertex (other than the input vertices) is called *nonaccessible* if and only if there is no possibility of reaching this vertex through any stem of the graph $\mathcal{G}$.

Definition 3 (*Dilation*)**.** Consider one vertex set S formed by the vertices from the state vertices set $\mathcal{X}$ and determine another vertex set $T(S)$, which contains all the vertices v with the property that there exists an oriented edge from v to one vertex in S. Then the graph $\mathcal{G}$ contains a '*dilation*' if and only if there exist at least a set S of k vertices in the vertex set of the graph such that there are no more than $k - 1$ vertices in $T(S)$.

2.2. Switched linear system, controllability and structural controllability

In general, a switched linear system is composed of a family of subsystems and a rule that governs the switching among them and is mathematically described by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ are the states, $u(t) \in \mathbb{R}^r$ are piecewise continuous input, $\sigma : [0, \infty) \to M \triangleq \{1, \ldots, m\}$ is the switching signal. System (1) contains m subsystems (A_i, B_i), $i \in \{1, \ldots, m\}$ and $\sigma(t) = i$ implies that the ith subsystem (A_i, B_i) is active at time instance t.

In the sequel, the following definition of controllability for the system (1) will be adopted (Sun et al., 2002):

Definition 4. The switched linear system (1) is said to be (completely) controllable if for any initial state x_0 and final state x_f, there exist a time instance $t_f > 0$, a switching signal $\sigma : [0, t_f) \to M$ and an input $u : [0, t_f) \to \mathbb{R}^r$ such that $x(0) = x_0$ and $x(t_f) = x_f$.

For the controllability of switched linear systems, a matrix rank condition was given in Sun et al. (2002).

Lemma 1. *If the matrix:*

$$\begin{aligned}[&B_1, B_2, \ldots, B_m, A_1B_1, A_2B_1, \ldots, A_mB_1, A_1B_2, A_2B_2, \ldots, A_mB_2, \ldots,\\ &A_1B_m, A_2B_m, \ldots, A_mB_m, A_1^2B_1, A_2A_1B_1, \ldots, A_mA_1B_1, A_1A_2B_1,\\ &A_2^2B_1, \ldots, A_mA_2B_1, \ldots, A_1A_mB_m, A_2A_mB_m, \ldots, A_m^2B_m, \ldots,\\ &A_1^{n-1}B_1, \quad A_2A_1^{n-2}B_1, \ldots, A_mA_1^{n-2}B_1, A_1A_2A_1^{n-3}B_1, A_2^2A_1^{n-3}B_1, \ldots,\\ &A_1^{n-3}B_1, \ldots, A_1A_m^{n-2}B_m, A_2A_m^{n-2}B_m, \ldots, A_m^{n-1}B_m]\end{aligned}$$

has full row rank n, then the switched linear system (1) is controllable, and vice versa.

Remark 1. This matrix is called the controllability matrix of the switched linear system (1) and denoted as $\mathcal{C}$ $(A_1, \ldots, A_m, B_1, \ldots, B_m)$. If we use *Im P* to represent the range space of a matrix P, then *Im* $\mathcal{C}$ $(A_1, \ldots, A_m, B_1, \ldots, B_m)$ is the controllable subspace of the switched linear system (1) (Sun et al., 2002). The above lemma implies that the system (1) is controllable if and only if *Im* $\mathcal{C}$ $(A_1, \ldots, A_m, B_1, \ldots, B_m) = \mathcal{R}^n$. Besides, its controllable subspace can be expressed as $\langle A_1, \ldots, A_m | B_1, \ldots, B_m \rangle$, which is the smallest subspace containing *Im* B_i, $i = 1, \ldots, m$ and invariant under the transformations $A_1, \ldots, A_m$ (Qiao & Cheng, 2009).

For the structured system (1), elements of all the matrices $(A_1, B_1, \ldots, A_m, B_m)$ are either fixed zero or free parameters and free parameters. A numerically given matrices set $(\tilde{A}_1, \tilde{B}_1, \ldots, \tilde{A}_m, \tilde{B}_m)$ is called an admissible numerical realization (with respect to $(A_1, B_1, \ldots, A_m, B_m)$) if it can be obtained by fixing all free parameter entries of $(A_1, B_1, \ldots, A_m, B_m)$ at some particular values. As aforementioned, we are interested in the structural controllability of (1).

Definition 5. The switched linear system (1) is said to be structurally controllable if and only if there exists at least one admissible realization $(\tilde{A}_1, \tilde{B}_1, \ldots, \tilde{A}_m, \tilde{B}_m)$ such that the corresponding switched linear system is controllable in the usual numerical sense.

Remark 2. It turns out that once a structured system is controllable for one choice of system parameters, it is controllable for almost all system parameters, in which case the structured system then will be said to be structurally controllable (Dion et al., 2003; Lin, 1974).

Before proceeding further, we need to introduce the definition of g-rank.

Definition 6. The generic rank (g-rank) of a structured matrix P is defined to be the maximal rank that P achieves as a function of its free parameters.

Then, we have the following algebraic condition for structural controllability:

Lemma 2. *The switched linear system (1) is structurally controllable if and only if g-rank $\mathcal{C}$ $(A_1, \ldots, A_m, B_1, \ldots, B_m) = n$.*

3. Structural controllability of switched linear systems

3.1. Criteria based on the union graph

For the switched linear system (1), a digraph $\mathcal{G}_i = \mathcal{G}(A_i, B_i)$ with vertex set $\mathcal{V}_i$ and edge set $\mathcal{E}_i$ can be adopted as the representation graph of its subsystems $(A_i, B_i), i \in \{1, \dots, m\}$. The switched linear system (1) can be represented by a union graph $\mathcal{G}$ (actually a digraph) of these digraphs $\mathcal{G}_i$.

Definition 7. Given a collection of digraphs $\mathcal{G}_i = \{\mathcal{V}_i, \mathcal{E}_i\}$, their union graph is $\mathcal{G}_1 \cup \mathcal{G}_2 \cup \cdots \cup \mathcal{G}_m = \{\mathcal{V}_1 \cup \mathcal{V}_2 \cup \cdots \cup \mathcal{V}_m; \mathcal{E}_1 \cup \mathcal{E}_2 \cup \cdots \cup \mathcal{E}_m\}$.

Remark 3. It turns out that the union graph $\mathcal{G}$ is the representation graph of linear structured system: $(A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m)$. The reason is as follows: if the element a_{ji} (b_{ji}) in matrix $[A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m]$ is a free parameter, this implies that there exist some matrices $[A_p, B_p]$, $p = 1, \dots, m$ such that the element at the position a_{ji} (b_{ji}) is also a free parameter and in the corresponding subgraph $\mathcal{G}_p$, there is an edge from vertex i to vertex j. According to the definition of union graph, it follows that there is also an edge from vertex i to vertex j in the union graph $\mathcal{G}$. If the element at the position a_{ji} (b_{ji}) in $[A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + A_m]$ is zero, this implies that for every matrices $[A_p, B_p]$, $p = 1, \dots, m$, the element at the position a_{ji} (b_{ji}) is zero and in the corresponding subgraph $\mathcal{G}_p$, there is no edge from vertex i to vertex j. It follows that there is also no edge in the union graph $\mathcal{G}$ from vertex i to vertex j.

Definition 8 (*Lin, 1974*)**.** The matrix pair (A, B) is said to be reducible or of form I if there exists a permutation matrix P such that they can be written in the following form: $PAP^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$, $PB = \begin{bmatrix} 0 \\ B_{22} \end{bmatrix}$, where $A_{11} \in \mathbb{R}^{p\times p}$, $A_{21} \in \mathbb{R}^{(n-p)\times p}$, $A_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$ and $B_{22} \in \mathbb{R}^{(n-p)\times r}$.

Remark 4. Whenever the matrix pair (A, B) is of form I, the system is structurally uncontrollable (Lin, 1974) and meanwhile, the controllability matrix $\mathcal{C} \triangleq \left[B, AB, \dots, A^{n-1}B\right]$ will have at least one row which is identically zero for all parameter values (Glover & Silverman, 1976). If there is no such permutation matrix P, we say that the matrix pair (A, B) is irreducible.

Definition 9 (*Lin, 1974*)**.** The matrix pair (A, B) is said to be of form II if there exists a permutation matrix P such that they can be written in the following form: $\left[PAP^{-1}, PB\right] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$, where $P_2 \in \mathbb{R}^{(n-k)\times(n+r)}$, $P_1 \in \mathbb{R}^{k\times(n+r)}$ with no more than $k - 1$ nonzero columns (all the other columns of P_1 have only fixed zero entries).

The following lemma characterizes the structural controllability for linear system (A, B) (Lin, 1974; Reinschke, 1988).

Lemma 3 (*Lin, 1974; Reinschke, 1988*)**.** *For linear structured system, the following statements are equivalent:*

(a) *the pair (A, B) is structurally controllable;*
(b) (i) *$[A, B]$ is irreducible or not of form I,*
 (ii) *$[A, B]$ has g-rank $[A, B] = n$ or is not of form II;*
(c) (i) *there is no nonaccessible vertex in $\mathcal{G}(A, B)$,*
 (ii) *there is no 'dilation' in $\mathcal{G}(A, B)$.*

This lemma proposes interesting graphic conditions for structural controllability of linear systems and reveals that the structural controllability is totally determined by the underlying graph topology. Next, we turn to the switched linear system (1) and prove a graphic sufficient condition for its structural controllability.

Theorem 4. *The switched linear system (1) with graphic topologies $\mathcal{G}_i$, $i \in \{1, \dots, m\}$, is structurally controllable if its union graph $\mathcal{G}$ satisfies*

(i) *there is no nonaccessible vertex in $\mathcal{G}$,*
(ii) *there is no 'dilation' in $\mathcal{G}$.*

Proof. Assume the two conditions in this theorem are satisfied. According to Remark 3 and Lemma 3, the corresponding linear system $(A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m)$ is structurally controllable. It follows that there exist some scalars for the free parameters in matrices (A_i, B_i), $i = 1, 2, \dots, m$ such that controllability matrix

$$[B_1 + B_2 + \cdots + B_m, (A_1 + A_2 + \cdots + A_m)(B_1 + B_2 + \cdots + B_m),$$
$$(A_1 + A_2 + \cdots + A_m)^2(B_1 + B_2 + \cdots + B_m), \ \dots,$$
$$(A_1 + A_2 + \cdots + A_m)^{n-1}(B_1 + B_2 + \cdots + B_m)]$$

has full row rank n. Expanding the matrix, it follows that matrix

$$[B_1 + B_2 + \cdots + B_m, A_1B_1 + A_2B_1 + \cdots + \cdots$$
$$+ A_mB_m, \ \dots, \ A_1^{n-1}B_1 + A_2A_1^{n-2}B_1 + \cdots + A_m^{n-1}B_m]$$

has full rank n.

The following matrix can be got after adding some column vectors to the above matrix:

$$[B_1 + B_2 + \cdots + B_m, B_2, \dots, B_m, A_1B_1 + A_2B_1 + \cdots + A_mB_m,$$
$$A_2B_1, \dots, A_mB_m, \dots, A_1^{n-1}B_1 + A_2A_1^{n-2}B_1 + \cdots + A_m^{n-1}B_m,$$
$$A_2A_1^{n-2}B_1, \dots, A_1A_m^{n-2}B_1, \dots, A_m^{n-1}B_m].$$

Since this matrix still has n linear independent column vectors, it follows that it has full row rank n. Next, subtracting $B_2, \dots, B_m$ from $B_1 + B_2 + \cdots + B_m$; subtracting $A_2B_1, \dots, A_mB_m$ from $A_1B_1 + A_2B_1 + \cdots + A_mB_1 + \cdots + A_1B_m + \cdots + A_mB_m$ and subtracting $A_2A_1^{n-2}B_1, \dots, A_1A_m^{n-2}B_1, \dots, A_m^{n-1}B_m$ from $A_1^{n-1}B_1 + A_2A_1^{n-2}B_1 + \cdots + A_1A_m^{n-2}B_1 + \cdots + A_m^{n-1}B_m$, we can get the following matrix:

$$[B_1, B_2, \dots, B_m, A_1B_1, A_2B_1, \dots, A_mB_m, \dots,$$
$$A_1^{n-1}B_1, A_2A_1^{n-2}B_1, \dots, A_1A_m^{n-2}B_1, \dots, A_m^{n-1}B_m],$$

which is the controllability matrix for switched linear systems (1). Since column fundamental transformation does not change the matrix rank, this matrix still has full row rank n. Hence, the switched linear system (1) is structurally controllable.

Actually, from the proof, we can see that full rank of controllability matrix of linear system $(A_1+A_2+\cdots+A_m, B_1+B_2+\cdots+B_m)$ in Remark 3 implies the full rank of controllability matrix of system (1), which means that the structural controllability of this linear system implies structural controllability of system (1). It turns out that this criterion is not necessary for system (1) to be structurally controllable. This implies that the union graph does not contain enough information for determining structural controllability. This is because edges from different subsystems are not differentiated in the union graph. In the following subsection, another graphic representation of switched linear systems is proposed, from which necessary and sufficient conditions for structural controllability arise.

3.2. Criteria based on the colored union graph

In the union graph, there is no distinction made between the edges from different subsystems. To solve this issue, we introduce the following *'colored union graph'* as another graphic representation of switched systems.

Definition 10. Given a collection of digraphs $\mathcal{G}_i = \{\mathcal{V}_i, \mathcal{I}_i\}$, their colored union graph is $\tilde{\mathcal{G}}(\tilde{\mathcal{V}}, \tilde{\mathcal{I}})$, where its vertex set $\tilde{\mathcal{V}} = \{\mathcal{V}_1 \cup \mathcal{V}_2 \cup \cdots \cup \mathcal{V}_m\}$ and edge set $\tilde{\mathcal{I}} = \{e|e \in \mathcal{I}_i, i = 1, 2, \ldots, m\}$, i.e., for $i \in \{1, \ldots, m\}$.

Intuitively, each edge e in the colored union graph $\tilde{\mathcal{G}}$ is associated with an index i (color) to indicate that e comes from the ith subsystem (subgraph $\mathcal{G}_i$). With the colored union graph, several graphic properties are introduced in the following lemmas.

Lemma 5. *There is no nonaccessible vertex in the colored union graph $\tilde{\mathcal{G}}$ of the switched linear system (1) if and only if the matrix $[A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m]$ is irreducible or not of form I.*

Proof. One vertex is accessible if and only if it can be reached by a stem. From Definitions 7 and 10, it follows that there is no nonaccessible vertex in the colored union graph if and only if there is no nonaccessible vertex in the union graph. Besides, from Remark 3, it is clear that the matrix representation of the union graph is $[A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m]$. According to Lemma 3, there is no nonaccessible vertex in the union graph if and only if matrix is irreducible or not of form I. Consequently the equivalence between accessibility of the colored union graph and irreducibility of this matrix gets proved.

A new graphic property '*S-dilation*' in the colored union graph is introduced here.

Definition 11. In the colored union graph $\tilde{\mathcal{G}}$, which is composed of subgraphs $\mathcal{G}_i, i = 1, 2, \ldots, m$, consider one vertex set S formed by the vertices from the state vertex set $\mathcal{X}$ and determine another vertex set $T(S) = \{v|v \in T_i(S), i = 1, 2, \ldots, m\}$, where $T_i(S)$ is a vertex set in $\mathcal{G}_i$ which contains all the vertices w with the property that there exists an oriented edge from w to one vertex in S. Then $|T(S)| = \sum_{i=1}^{m} |T_i(S)|$. If $|T(S)| < |S|$, we say that there is a *S-dilation* in the colored union graph $\tilde{\mathcal{G}}$.

Based on this new graphic property, the following lemma can be introduced.

Lemma 6. *There is an S-dilation in the colored union graph $\tilde{\mathcal{G}}$ of the switched linear system (1) if and only if matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ is of form II. It means that this matrix can be written into $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$, where $P_1 \in \mathbb{R}^{p\times k}$ with no more than $p - 1$ nonzero columns (all the other columns of P_1 have only fixed zero entries).*

Proof. From Lin (1974) and Mayeda (1981) or Lemma 3, it is known that in linear systems, there is no 'dilation' in the corresponding graph if and only if the matrix pair $[A, B]$ cannot be of form II or have g-rank $= n$. From the explanation of this result in Lin (1974) and Definition 9, P_1 in $[A, B]$ has p rows, which actually represents the p vertices of vertex set S (defined for dilation), and each nonzero element of each row of P_1 represents that there is one vertex pointing to the vertex presented by this row. Therefore, the number of nonzero columns in P_1 is the number of vertices pointing to some vertex in S, and actually equals to $|T(S)|$. Furthermore, by the definition of *S-dilation*, $|T(S)|$ is now the summation of $|T_i(S)|, i \in \{1, \ldots, m\}$, in every subgraph. It follows that there is *S-dilation* in $\tilde{\mathcal{G}}$ if and only if matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ is of form II.

Before going further to give another algebraic explanation of *S-dilation*, one definition and lemma proposed in Shields and Pearson (1976) must be introduced first.

Definition 12 (*Shields & Pearson, 1976*)**.** A structured $n \times m'$ $(n \leq m')$ matrix A is of form (t) for some t, $1 \leq t \leq n$, if for some k in the range $m' - t < k \leq m'$, A contains a zero submatrix of order $(n + m' - t - k + 1) \times k$.

Lemma 7 (*Shields & Pearson, 1976*)**.** *g-rank of $A = t$*

(i) *for $t = n$ if and only if A is not of form (n);*
(ii) *for $1 \leq t < n$ if and only if A is of form $(t + 1)$ but not of form (t).*

From the above definition and lemma, another lemma is proposed here.

Lemma 8. *There is no S-dilation in the colored union graph $\tilde{\mathcal{G}}$ of the switched linear system (1) if and only if the matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ has g-rank n.*

Proof. *Necessity:* If this matrix has g-rank $< n$, from Lemma 7, it follows that this matrix is of form (n). Then referring to Definition 12, the matrix must have a zero submatrix of order $(n+m'-t-k+1) \times k$. Here, t can be chosen as n, then matrix has a zero submatrix of order $(m' - k + 1) \times k$. For this $(m' - k + 1)$ rows, there are only $(m' - k)$ nonzero columns. Consequently, the matrix is of form II and by Lemma 6, there is *S-dilation* in the colored union graph $\tilde{\mathcal{G}}$ of system (1).

Sufficiency: If there is *S-dilation* in the colored union graph $\tilde{\mathcal{G}}$, by Lemma 6, the matrix is of form II, then obviously P_1 in this matrix cannot have row rank equal to k and furthermore, this matrix cannot have g-rank $= n$.

With the above definitions and lemmas, a graphic necessary and sufficient condition for the switched linear system to be structurally controllable can be proposed.

Theorem 9. *The switched linear system (1) with graphic representations $\mathcal{G}_i, i \in \{1, \ldots, m\}$, is structurally controllable if and only if its colored union graph $\tilde{\mathcal{G}}$ satisfies the following two conditions:*

(i) *there is no nonaccessible vertex in the colored union graph $\tilde{\mathcal{G}}$,*
(ii) *there is no S-dilation in the colored union graph $\tilde{\mathcal{G}}$.*

Proof. *Necessity:* (i) If there exist nonaccessible vertices in $\tilde{\mathcal{G}}$, by Lemma 5, the matrix $[A_1 + A_2 + \cdots + A_m, B_1 + B_2 + \cdots + B_m]$ is reducible or of form I. It follows that the controllability matrix

$$[B_1 + B_2 + \cdots + B_m, (A_1 + A_2 + \cdots + A_m)(B_1 + B_2 + \cdots + B_m), (A_1 + A_2 + \cdots + A_m)^2(B_1 + B_2 + \cdots + B_m), \ldots, (A_1 + A_2 + \cdots + A_m)^{n-1}(B_1 + B_2 + \cdots + B_m)]$$

always has at least one row that is identically zero (Remark 4). It is clear that every component of the matrix, such as B_i, A_iB_j and $A_i^pA_j^qB_r$ has the same row always to be zero. As a result, the controllability matrix

$$[B_1, \ldots, B_m, A_1B_1, \ldots, A_mB_1, \ldots, A_mB_m, A_1^2B_1, \ldots, A_mA_1B_1, \ldots, A_1^2B_m, \ldots, A_mA_1B_m, \ldots, A_1^{n-1}B_1, \ldots, A_mA_1^{n-2}B_1, \ldots, A_1A_m^{n-2}B_m, \ldots, A_m^{n-1}B_m]$$

always has one zero row and cannot be of full rank n. Therefore, the switched linear system (1) is not structurally controllable.

(ii) Suppose that the switched linear system (1) is structurally controllable, i.e., the controllability matrix satisfies g-rank $\mathcal{C}$ $(A_1, \ldots, A_m, B_1, \ldots, B_m) = n$. Specifically, $Im[B_1, \ldots, B_m, A_1 B_1, \ldots, A_mB_m, A_1^2B_1, \ldots, A_m^{n-1}B_m] = \mathbb{R}^n$ Since $\forall P \in \mathbb{R}^{n\times r}$, $Im(A_iP) \subseteq Im(A_i)$, we have that $Im[B_1, \ldots, B_m, A_1B_1, \ldots, A_mB_m, A_1^2B_1, \ldots, A_m^{n-1}B_m] \subseteq Im[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m] \subseteq \mathbb{R}^n$. Thus condition g-rank $\mathcal{C}(A_1, \ldots, A_m, B_1, \ldots, B_m) = n$ requires that $Im[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m] = \mathbb{R}^n$ and therefore g-rank $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m] = n$. However, if there is *S-dilation* in the colored union graph $\tilde{\mathcal{G}}$, by Lemma 6, g-rank $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m] < n$. Consequently, the switched linear system (1) is not structurally controllable.

Sufficiency: The general idea in the sufficiency proof is that we will assume that the two graphical conditions in the theorem hold. Then a contradiction will be found such that it is impossible that the switched linear system (1) is structurally uncontrollable.

Before proceeding to the switched linear system (1), first, consider a structured linear system:

$$\dot{x}(t) = Ax(t) + Bu(t). \tag{2}$$

It is well known that system (2) is structurally controllable if and only if there exists a numerical realization $(\tilde{A}, \tilde{B})$, such that rank $(sI - \tilde{A}, \tilde{B}) = n, \forall s \in \mathbb{C}$. Otherwise, the PBH test (Kailath, 1980) states that system (2) is uncontrollable if and only if for every numerical realization, there exists a row vector $q \neq 0$ such that $q\tilde{A} = s_0 q$, $s_0 \in \mathbb{C}$ and $q\tilde{B} = 0$, where rank $(s_0 I - \tilde{A}, \tilde{B}) < n$.

On one hand, if for every numerical realization rank $(sI - \tilde{A}, \tilde{B}) = n, \forall s \in \mathbb{C}\backslash\{0\}$, then the uncontrollability of system (2) implies necessarily that for every numerical realization there exists a vector $q \neq 0$ such that $q\tilde{A} = 0$ and $q\tilde{B} = 0$.

On the other hand, Lemma 14.1 of Reinschke (1988) states that, if in the digraph associated with (2), every state vertex is an end vertex of a stem (accessible), then g-rank $(sI - A, B) = n, \forall s \in \mathbb{C}\backslash\{0\}$, which means that for almost all numerical realization $(\tilde{A}, \tilde{B})$, rank $(sI - \tilde{A}, \tilde{B}) = n, \forall s \in \mathbb{C} \setminus \{0\}$.

Now considering the switched linear system (1), assume that the two conditions in Theorem 9 are satisfied. Due to Lemma 14.1 of Reinschke (1988), as all the parameters of matrices $A_1, \ldots, A_m, B_1, \ldots, B_m$ are assumed to be free, the condition (i) of Theorem 9 implies that, for almost all vector values $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_m)$, we have g-rank $(sI - (\bar{u}_1 A_1 + \cdots + \bar{u}_m A_m), (\bar{u}_1 B_1 + \cdots + \bar{u}_m B_m)) = n, \forall s \neq 0$. On the other hand, if the switched linear system (1) is structurally uncontrollable, then for all constant values, $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_m)$, linear systems defined by matrices $(\bar{A}, \bar{B})$ are also uncontrollable, where $\bar{A} = \sum_{i=1}^m \bar{u}_i A_i$ and $\bar{B} = \sum_{i=1}^m \bar{u}_i B_i$. We write the numerical realization of $(\bar{A}, \bar{B})$ as $(\tilde{\bar{A}}, \tilde{\bar{B}})$. This is due to the fact that for all constant values $\bar{u}$, $Im(\mathcal{C}(\bar{A}, \bar{B}) \subseteq Im(\mathcal{C}(A_1, \ldots, A_m, B_1, \ldots, B_m))$. Therefore, if the switched linear system is structurally uncontrollable, since for almost all $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_m)$, g-rank $(sI - (\bar{u}_1 A_1 + \cdots + \bar{u}_m A_m), (\bar{u}_1 B_1 + \cdots + \bar{u}_m B_m)) = n, \forall s \neq 0$, we have that for every numerical realization matrix pair $(\tilde{\bar{A}}, \tilde{\bar{B}})$, there exists a nonzero vector q such that $q\tilde{\bar{A}} = 0$ and $q\tilde{\bar{B}} = 0$. Since this statement is true for almost all the values $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_m)$, we have that for almost all $n \cdot m$-tuple values $\bar{u}^j = (\bar{u}_1^j, \ldots, \bar{u}_m^j), j = 1, \ldots, n \cdot m$, we can find nonzero vectors q_j such that the following holds:

$$\begin{cases} \sum_{i=1}^{m} \bar{u}_i^j q_j \tilde{A}_i = 0, & j = 1, \ldots, n \cdot m \\ \sum_{i=1}^{m} \bar{u}_i^j q_j \tilde{B}_i = 0. & j = 1, \ldots, n \cdot m. \end{cases} \tag{3}$$

Obviously, there cannot exist more than n linear independent vectors q_j. Let us denote $q_1, q_2, \ldots, q_n$ the vectors such that *span* $(q_1, q_2, \ldots, q_{n \cdot m}) \subseteq$ *span*$(q_1, q_2, \ldots, q_n)$ (we can renumber the vectors if necessary). All the vectors $q_j, j = n+1, \ldots, n \cdot m$ are linear combinations of $q_1, q_2, \ldots, q_n$. Therefore, system (3) contains the following equations:

$$\begin{cases} \sum_{k=1}^{n} \sum_{i=1}^{m} a_{i,k}^j(\bar{u}) q_k \tilde{A}_i = 0 & j = 1, \ldots, n \cdot m \\ \sum_{k=1}^{n} \sum_{i=1}^{m} a_{i,k}^j(\bar{u}) q_k \tilde{B}_i = 0 & j = 1, \ldots, n \cdot m \end{cases} \tag{4}$$

where $a_{i,k}^j(\bar{u})$ are linear functions of $\bar{u}^j, j = 1, \ldots, n \cdot m$. Since system (3) is satisfied for almost all the values, we can find $\bar{u}^j, j = 1, \ldots, n \cdot m$ such that

$$det \begin{bmatrix} a_{1,1}^1(\bar{u}) & a_{1,2}^1(\bar{u}) & \ldots & a_{m,n}^1(\bar{u}) \\ a_{1,1}^2(\bar{u}) & a_{1,2}^2(\bar{u}) & \ldots & a_{m,n}^2(\bar{u}) \\ \vdots & \vdots & \vdots & \vdots \\ a_{1,1}^{n \cdot m}(\bar{u}) & a_{1,2}^{n \cdot m}(\bar{u}) & \ldots & a_{m,n}^{n \cdot m}(\bar{u}) \end{bmatrix} \neq 0.$$

In this case, the only solution of (4) is $q_k \tilde{A}_1 = \cdots = q_k \tilde{A}_m = q_k \tilde{B}_1 = \cdots = q_k \tilde{B}_m = 0, k = 1, \ldots, n$. Obviously, if the switched linear system is structurally uncontrollable, then vector $q_k, k = 1, \ldots, n$ is nonzero. Consequently, the switched linear system (1) is structurally uncontrollable only if for every numerical realization there exists at least one nonzero vector q such that $qA_1 = \cdots = qA_m = qB_1 = \cdots = qB_m = 0$. However, if condition ii of Theorem 9 is satisfied, then g-rank $[A_1, \ldots, A_m, B_1, \ldots, B_m] = n$ and therefore, for at least one numerical realization, there does not exist a vector $q \neq 0$ such that $qA_1 = \cdots = qA_m = qB_1 = \cdots = qB_m = 0$. Hence, the two conditions are sufficient to ensure the structural controllability of the switched linear system (1).

Actually, using the terminologies '*dilation*' and '*S-dilation*' as graphic criteria is not so numerically efficient. For example, to check the second condition of Theorem 9, we need to test for all possible vertex subsets to see whether there exist *S-dilation* in the colored union graph or not. Consequently, we will adopt another notion '*S-disjoint edges*' to form a more numerically efficient graphic interpretation of structural controllability.

Definition 13. In the colored union graph $\tilde{\mathcal{G}}$, consider k edges $e_1 = (v_1, v_1'), e_2 = (v_2, v_2'), \ldots, e_k = (v_k, v_k')$. We define for $i = 1, \ldots, k$, S_i as the set of integers j such that $v_j = v_i$, i.e., $S_i = \{1 \leq j \leq k | v_j = v_i\}$. These k edges $e_1, e_2, \ldots, e_k$ are *S-disjoint* if the following two conditions are satisfied:

(i) edges $e_1, e_2, \ldots, e_k$ have distinct end vertices,
(ii) for $i = 1, \ldots, k$, $S_i = \{i\}$ or there exist r distinct integers $i_1, i_2, \ldots, i_r$ such that $e_{j_1} \in \mathcal{I}_{i_1}, e_{j_2} \in \mathcal{I}_{i_2}, \ldots, e_{j_r} \in \mathcal{I}_{i_r}$, where $j_1, j_2, \ldots, j_r$ are all the elements of S_i.

Roughly speaking, k edges are *S-disjoint* if their end vertices are all distinct and if all the edges which have the same begin vertex can be associated with distinct indexes i. For this new graphic property, the following lemma can be given.

Lemma 10. *Considering switched linear system (1), there exist n S-disjoint edges in the associated colored union graph $\tilde{\mathcal{G}}$ if and only if $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ has g-rank $= n$.*

Proof. *Necessity:* If there exist n *S-disjoint* edges in $\tilde{\mathcal{G}}$, matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ contains at least n free parameters. Since the n *S-disjoint* edges have distinct end vertices, the corresponding n free parameters lie on n different rows. Besides, the n *S-disjoint* edges have distinct begin vertices or have same begin vertex that can be associated with distinct indexes i. This implies that these n free parameters lie on n different columns. Keep these n free parameters and set all the other free parameters to be zero. We can see that matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ has following form: $\begin{bmatrix} 0 & \lambda_1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & & . \\ \lambda_n & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}$, which has g-rank $= n$.

Sufficiency: From the Definition 12.3 and the following discussions of Reinschke (1988), for a structured matrix Q, g-rank $Q = s$-rank Q, where s-rank of Q is defined as the maximal number of free parameters that no two of which lie on the same row or column. If matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ has g-rank $= n$, it follows that there exists n free parameters from n different rows, which implies that the corresponding n edges have different end vertices, from n different columns, which implies that these n edges start from different vertices or start from the same vertices but can be associated with different indexes. Hence the condition that matrix has g-rank $= n$ is sufficient to ensure the existence of n *S-disjoint* edges.

With the above definition and lemma, another necessary and sufficient condition for structural controllability of system (1) can be proposed here.

Theorem 11. *The switched linear system (1) with graphic representations $\mathcal{G}_i$, $i \in \{1, \ldots, m\}$, is structurally controllable if and only if its colored union graph $\tilde{\mathcal{G}}$ satisfies the following two conditions:*

(i) *there is no nonaccessible vertex in the colored union graph $\tilde{\mathcal{G}}$,*
(ii) *there exist n S-disjoint edges in the colored union graph $\tilde{\mathcal{G}}$.*

Proof. Lemmas 6 and 10 show that there exist n *S-disjoint* edges in the colored union graph $\tilde{\mathcal{G}}$ if and only if there is no *S-dilation* in $\tilde{\mathcal{G}}$. Then this theorem follows immediately.

3.3. Computation complexity of the proposed criteria

Compared with the condition using '*S-dilation*', this condition using '*S-disjoint* edges' does not require to check all the vertex subsets, which is a more efficient criterion. The maximal number of '*S-disjoint* edges' can be calculated using bipartite graphs. For example, we can use the algorithm in Micali and Vazirani (1980), which allows us to compute the cardinality of maximum matching into a bipartite graph. A bipartite graph is a graph whose vertices can be divided into two disjoint sets $\mathcal{U}$ and $\mathcal{W}$ such that every edge connects a vertex in $\mathcal{U}$ to one in $\mathcal{W}$. To build a bipartite graph in directed subgraph $\mathcal{G}_i(\mathcal{V}_i, \mathcal{I}_i)$, what we need to do is adding some vertices and making $\mathcal{U}_i = \{v \in \mathcal{V}_i | \exists (v, v') \in \mathcal{I}_i\}$, which implies that cardinality $|\mathcal{U}_i|$ equals to the number of nonzero columns in matrix $[A_i, B_i]$. Besides, $\mathcal{W}_i = \mathcal{X}_i$, i.e., the state vertex set. Then it follows that the maximum matching in this bipartite graph is the same as the maximal *S-disjoint* edge set in $\mathcal{G}_i(\mathcal{V}_i, \mathcal{I}_i)$. According to the definition of *S-disjoint* edges, the beginning vertex from different subgraphs should be differentiated when building the bipartite graph for the colored union graph $\tilde{\mathcal{G}}$. Therefore for the bipartite graph of $\tilde{\mathcal{G}}$, $\mathcal{U} = \{v | \exists (v, v') \in \mathcal{I}_i, i = 1, 2, \ldots, m\}$, which implies that cardinality $|\mathcal{U}|$ equals to the number of nonzero columns in matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$. And $\mathcal{W} = \mathcal{X}$, i.e., the state vertex set. Similarly, the maximum matching in this bipartite graph is the same as the maximal *S-disjoint* edge set in the colored union graph. Therefore the complexity order of the algorithm using the method in Micali and Vazirani (1980) is $O(\sqrt{p+n} \cdot q)$, where q is the number of edges in the colored union graph, i.e., the number of free parameters in all system matrices, p is the number of nonzero columns in matrix $[A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m]$ and n is the number of state variables. Compared with condition (ii) of Theorem 11, condition (i) of Theorem 11 is easier to check. We have to look for paths which connect each state vertex with one of the input vertex. This is a standard task of algorithmic graph theory. For example, the depth-first search or breadth-first search algorithm for traversing a graph can be adopted and the complexity order is $O(|V| + |E|)$, where $|V|$ and $|E|$ are cardinalities of vertex set and edge set in the union graph.

4. Conclusions and future work

In this paper, structural controllability for switched linear systems has been investigated. Combining the knowledge in the literature of switched linear systems and graph theory, several graphic necessary and sufficient conditions for the structurally controllability of switched linear systems have been proposed. These graphic interpretations provide us a better understanding on how the graphic topologies of switched linear systems will influence or determine the structural controllability of switched linear systems. This shows us a new perspective that we can design the switching algorithm to make the switched linear system structurally controllable conveniently just having to make sure that some properties of the corresponding graph (union or colored union graph) are kept during the switching process. In this paper, the parameters in different subsystem models are assumed to be independent. A more general assumption is that some free parameters remain the same among different subsystems switching, i.e., dependence among subsystems. It turns out that our necessary and sufficient condition derived here would be a necessary condition under this dependence assumption. Besides, our result can be treated as basic starting point for exploring the structural controllability of switched nonlinear systems: using Lie algebra or transfer function methods to get full characterization for controllability of the switched non-linear system, then try to interpret each condition into graphic one and finally combine these conditions together to get graphic interpretations for structural controllability for the switched nonlinear system. To obtain a full characterization for the dependent case or switched nonlinear case needs further investigation.

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