Distributed Switched Supervisory Control to Achieve Almost Sure Safety for a Class of Interconnected Networked Systems

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***Abstract*— An interconnected wireless networked system consists of numerous coupled subsystems that need to exchange information over wireless communication channels. The use of these wireless networks induces a great deal of stochastic uncertainty that often results from *deep fades*, where a severe drop in the quality of communication link occurs. Such uncertainty negatively impacts the system's performance and causes unexpected safety issues. This paper proposes a distributed switched supervisory control scheme under which the local controller is reconfigured in response to the changes of channel state, to assure *almost sure safety* for the interconnected system. Here, *almost sure safety* means that the likelihood of the system state entering a safe region asymptotically goes to one as time goes to infinity. Sufficient conditions are provided for each local supervisor to determine when and which controller is placed in the feedback loop to assure *almost sure safety* in the presence of *deep fades*.**

I. Introduction

A distributed wireless networked systems(WNCS) consists of numerous coupled dynamical subsystems that coordinate their behaviors by exchanging information over wireless radio communication (RF) networks. It is well known that these RF networks are subject to deep fades, where the network's quality of service drops precipitously and remains low for an extended interval of time. These deep fades inject a great deal of stochastic uncertainty into the system, and negatively impact the system's performance and stability by interfering with the coordination between subsystems. The loss of coordination may cause serious safety issues in applications like smart transportation system [1], [2], unmanned aerial vehicles systems [3] and underwater autonomous vehicles [4]. These issues could be addressed by developing a distributed switched supervisory control system that detects such deep fades and adaptively reconfigures its controller to enforce a minimum safety requirement.

In real application, the safety issue is often examined in a stochastic setting by discussing the likelihood of a system state entering a forbidden or unsafe region. Traditionally, this has been done using mean square concepts in which the variance of some important system state, such as inter-vehicle distance, remains bounded. Such a concept is also analogous to the notion of *stochastic safety in probability* [5]. The common feature of above work is that they bound the likelihood of unsafe action occurring with a nonzero value, which still allows a finite probability for the system to be unsafe. This mean square safety or *stochastic safety in probability* criterion is simply not appropriate for many safety-critical systems such as smart transportation system where a small probability of danger can incur catastrophic failure. This paper suggests using a stronger notion of *almost sure safety* to assure the system state asymptotically goes to a safe equilibrium or a bounded safe set with probability one as time goes to infinity. In particular, *almost sure safety* in this paper refers to two strong notions of stochastic stability: *almost sure asymptotic stability* and *almost sure practical stability*.

The channel model that is used to attain *almost sure stability* must be carefully specified. Traditionally, this has been done by modeling channel fading as an *independent and identical distributed (i.i.d)* random process having either a Rayleigh or Rician distribution. This characterization might be reasonable for most stationary wireless networks, the use of i.i.d model is questionable in vehicular communication since the channel state is functionally dependent on the vehicle's physical state [6], [2]. A more realistic fading channel model was examined in [7], in which the channel is exponentially bursty and is dependent on the norm of the physical system's states. Such model is often referred to *exponentially bounded burstiness* (EBB) [8], and is more general in the sense that it can characterize the i.i.d channels as well as bursty channels that are often modeled as a two state Markov chain [9].

By using the EBB model that is functionally dependent on the physical state, one can develop a distributed switched supervisory control strategy to assure *almost sure safety* for a class of interconnected networked systems. The interconnected system consists of a collection of subsystems that are connected in a cascaded structure with upper systems driving the lower systems via their control inputs. This structure normally exists in vehicular systems, such as the chain of leader-follower formations discussed in [10]. Assuming an exponentially bursty channel model, this paper derives conditions that are sufficient for the entire system to have *almost sure asymptotic stability* and *almost sure practical stability* [11]. These sufficient conditions are used by the supervisor in each subsystem to decide when and how to select controller in the presence of deep fades.

The layout of the paper is as follows. Section II introduces mathematical notations. Section III provides a system description and problem setup. After that, we discuss the main results in Section IV. Finally, Section V concludes the paper.

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II. Mathematical Preliminaries

Let $\mathbb{Z}$ and $\mathbb{R}$ denote the set of integers and real numbers, respectively. Let $\mathbb{Z}_+$ and $\mathbb{R}_+$ denote the set of positive integers and non-negative real numbers, respectively. Let $\mathbb{R}^n$ denote the n-dimensional Euclidean vector space. The ∞-norm on the vector $x \in \mathbb{R}^n$ is $|x| = \max|x_i| : 1 \leq i \leq n$, and the corresponding induced matrix norm is $\|A\| = \max_{1\leq i\leq n}\sum_{j=1}^{n}|A_i^j|$. Given a vector $x \in \mathbb{R}^n$, we let $x_i \in \mathbb{R}$ for $i = 1,2,\ldots,n$ denote the ith element of vector x.

We let $f(\cdot) : \mathbb{R} \to \mathbb{R}^n$ denote a function mapping the real line onto vectors in $\mathbb{R}^n$. Let $f(t) \in \mathbb{R}^n$ denote the value that function f takes at time $t \in \mathbb{R}$. Given a time interval $\mathscr{I}$, let $\|f\|_{\mathscr{I}}$ denote the essential supremum of the function f defined over the time interval $\mathscr{I}$. We let $\|x\|_\infty$ denote the case when $\mathscr{I} = \mathbb{R}_+$. Let $\{\tau_k\}_{k=0}^{\infty}$ denote a strictly monotone increasing sequence with $\tau_k \in \mathbb{R}_+$ for all $k \in \mathbb{Z}_+$ and $\tau_k < \tau_{k+1}$. Then, $f(\tau_k)$ denotes the value of function f at time τ_k. For brevity, we let $f(k)$ denote $f(\tau_k)$ if its meaning is clear from the context. The left-hand limit at $\tau_k \in \mathbb{R}$ of a function $f(\cdot) : \mathbb{R} \to \mathbb{R}^n$ is denoted by $f(k^-)$. Similarly, the right-hand limit of the function $f(k)$ is denoted by $f(k^+)$. A function $\alpha(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathscr{K}$ function if it is continuous, strictly increasing with $\alpha(0) = 0$. It is said to be of class $\mathscr{K}_\infty$ if it is class $\mathscr{K}$ and $\alpha(s) \to \infty$ as $s \to \infty$. In addition, a function $\beta(\cdot,\cdot) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathscr{KL}$ if $\beta(\cdot,t)$ is class K for each fixed $t \geq 0$ and $\beta(s,t) \to 0$ for each fixed $s \geq 0$ as $t \to \infty$.

III. System Description and Problem Setup

The system under study is a collection of cascaded wireless networked subsystems shown in Figure 1. The subsystems are connected in a chain structure in the sense that the upper systems drive the lower systems through their control inputs. In the interconnected systems, the upper subsystem can observe the state of its immediately connected lower system. Each subsystem consists of three components: *Plant*, *Wireless Network* and *Controller*.

A. Plant

The plant of the cascaded system satisfies the following ODEs:

$$\dot{x}_1 = f_1(x_1,d) + g_1(x_1,u_1) \tag{1}$$

$$\dot{x}_i = f_i(x_i,u_{i-1}) + g_i(x_i,u_i),\ i = 2,\ldots,N \tag{2}$$

where $x_i \in \mathbb{R}^n$ is the state of subsystem i, $u_{i-1} \in \mathbb{R}^m$ and $u_i \in \mathbb{R}^m$ are respectively the control inputs of subsystem $i-1$ and i, $i = 2,3,\ldots,N$. $d(\cdot) : \mathbb{R}_+ \to \mathbb{R}^l$ is the external disturbance to the cascaded system. We assume internal control signals $u_i(\cdot), i = 1,2,\ldots,N$ are piecewise continuous functions and are Lebesgue measurable and locally bounded. The functions $f_i(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $g_i(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ are locally Lipschitz and satisfy the following assumption.

Assumption 3.1: Let $\mathscr{X}_i \subset \mathbb{R}^n$, $\mathscr{U}_{i-1} \subset \mathbb{R}^m$ and $\mathscr{U}_i \subset \mathbb{R}^m$ be compact sets. $\forall x_i \in \mathscr{X}_i \subset \mathbb{R}^n$, $u_{i-1} \in \mathscr{U}_{i-1} \subset \mathbb{R}^m$ and $u_i \in \mathscr{U}_i \subset \mathbb{R}^m$ compact sets, there exist Lipschitz constants L_{i_1}, L_{i_2} and L_{g_i} such that

$$\begin{aligned} |f_i(x_i,u_{i-1}) - f_i(\hat{x}_i,0)| &\leq L_{i_1}|x_i - \hat{x}_i| + L_{i_2}|u_{i-1}| \\ |g_i(x_i,u_i) - g_i(\hat{x}_i,u_i)| &\leq L_{g_i}|x_i - \hat{x}_i| \end{aligned}$$

B. Wireless Network

As shown in Figure 1, the system states of each subsystem must be transmitted over a wireless communication channel to its *Controller*. The information about the system's states are limited by the following two constraints,

- The state measurement $x_i(t)$ is only taken at a sequence of discrete time instants $\{\tau_k\}_{k=0}^{\infty}$, with $\tau_k < \tau_{k+1}$, $k = 0,1,\ldots,\infty$.
- The sampled data $x_i(\tau_k)$ is quantized with a finite number of blocks $\overline{R}_i$ by the *Encoder*. Each block contains n number of bits with each bit representing the information for each dimension of the states. $\overline{R}_i$ blocks of bits are transmitted over an unreliable wireless channel with only the first $R_i(\tau_k)$ blocks ($R_i(\tau_k) \leq \overline{R}_i$) being received at the *Decoder* side.

We assume a noiseless feedback channel, with each successfully received bit being acknowledged to the *Decoder*. This allows *Decoder* to use traditional dynamic quantization methods [12], [13] to construct an estimate of the sampled state. Let $x_i^q(\tau_k)$ denote the state estimate at time τ_k with $R_i(k)$ blocks of bits received. Let $\overline{U}_i(k)$ represent the length of a n dimensional hypercube $\mathscr{H}_i(k)$ with $x_i^q(\tau_k)$ as its center. The pair $\{x_i^q(\tau_k), \overline{U}_i(k)\}$ characterizes the information structure that is available to the control system, and is constructed such that the sampled state $x_i(\tau_k)$ is guaranteed to lie in the hypercube $\mathscr{H}_i(k)$ at each sampling time instant.

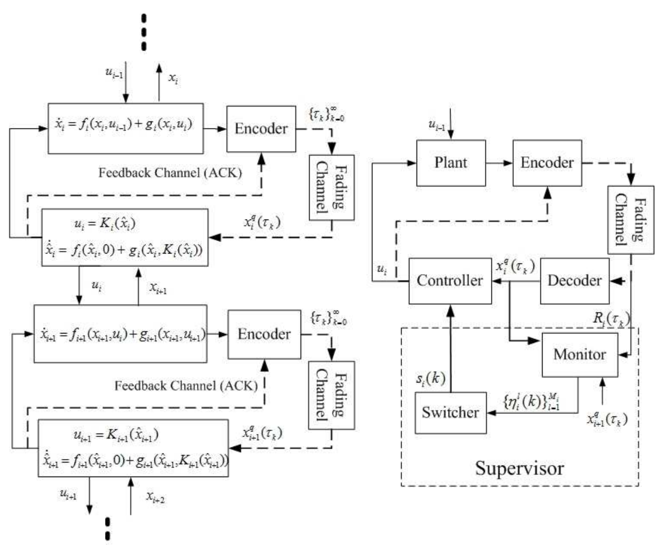

Fig. 1. Distributed supervisory switching control structure for serial connected wireless networked system

We adopt an exponential bounded burstiness (EBB) [8] model to characterize the stochastic changes of $R_i(k)$ in the fading channel. EBB characterizations can be used to describe a wide range of channel models including traditional i.i.d models as well as two-state Markov chain models. In particular, let $h(\cdot)$ and $\gamma(\cdot)$ denote continuous, positive and monotone increasing functions from $\mathbb{R}_+$ to $\mathbb{R}_+$. Assume the probability of successfully receiving $\{R_i(k)\}$ blocks of bits satisfies

$$\Pr\{R_i(k) \leq h(|x_i(\tau_k)|) - \sigma\} \leq e^{-\gamma(|x_i(\tau_k)|)\sigma} \quad (3)$$

for $\sigma \in [0, h(|x_i(\tau_k)|)]$. The function $h(|x_i(\tau_k)|)$ is a threshold for a low bit rate region that is varied as a function of the system state norm $|x_i(\tau_k)|$. The exponent associated with exponential decrease is represented by a function $\gamma(|x_i(\tau_k)|)$. The two functions play different roles in the EBB model. The function $h(|x_i(\tau_k)|)$ characterizes the fact that as the norm of the system state increases, the low bit rate threshold shrinks and moves toward the origin. On the other hand, the function $\gamma(|x_i(\tau_k)|)$ in the exponential bound models the fact that the likelihood of exhibiting lower bit rate than the threshold $h(|x_i(\tau_k)|)$ increases as the system state is away from the origin. Such relationship exists, for example, in Vehicle to Vehicle applications [2] where large inter-vehicle distance and velocity yield low bit rate, or in wireless mobile communication systems where the relative orientation changes on the transmitter and receiver may cause a deep fade.

C. Distributed Switched Supervisory Control System

Distributed switched supervisor system is a "high-level" decision system that uses available information including the estimate of local system state $x_i^q(\tau_k)$, local channel state $R_i(k)$ and the estimate of system state $x_{i+1}^q(\tau_k)$ from subsystem $i+1$, to orchestrate the switching among a family of candidate controllers. The switching supervisor decision decides when and which controller is selected and placed in the feedback loop.

Switching only occurs at each transmission time instant τ_k, and there is a family of candidate controllers $\mathscr{K}_i = \{K_i^\ell(\cdot), \ell \in \mathscr{C}_i\}$ that are selected ahead of time for each subsystem, where $\mathscr{C}_i$ is an index set taking value in $\{1, 2, \ldots, M_i\}$. For any $\ell \in \mathscr{C}_i$, the controller function $K_i^\ell(\cdot)$ is locally Lipschitz with $K_i^\ell(0) = 0$. With a selected control function $K_i^\ell(\cdot) \subset \mathscr{K}_i$, the control input u_i over time interval $[\tau_k, \tau_{k+1})$ is generated by

$$\begin{aligned} \dot{\hat{x}}_i &= f_i(\hat{x}_i, 0) + g_i(\hat{x}_i, K_i^l(\hat{x}_i)), \quad \hat{x}_i(\tau_k) = x_i^q(\tau_k) & (4) \\ u_i(t) &= K_i^l(\hat{x}_i), t \in [\tau_k, \tau_{k+1}) & (5) \end{aligned}$$

where $x_i^q(\tau_k)$ is the state estimate at time τ_k.

Let $e_i(t) := x_i(t) - \hat{x}_i(t)$ denote the estimation error. For each selected controller $K_i^\ell(\cdot) \subset \mathscr{K}_i$, we assume each closed-loop subsystem i generated by equations (1-2) and (4) is *input to state stable*(ISS) with respect to u_{i-1} and e_i. There exists a corresponding ISS triple $\{\beta_i^\ell(\cdot,\cdot), \chi_{i,1}^\ell(\cdot), \chi_{i,2}^\ell(\cdot)\}$ such that

$$|x_i(t)| \leq \beta_i^\ell(|x_i(\tau_0)|, t - \tau_0) + \chi_{i,1}^\ell(\|u_{i-1}\|_{[\tau_0,t]}) + \chi_{i,2}^\ell(\|e_i\|_{[\tau_0,t]}) \quad (6)$$

with $\ell = 1, 2, \ldots, M_i$ and $u_0 = d$, where $\beta_i^\ell(\cdot,\cdot)$ is class $\mathscr{KL}$ function, $\chi_{i,1}^\ell(\cdot)$ and $\chi_{i,2}^\ell(\cdot)$ are class $\mathscr{K}_\infty$ functions.

A supervisory in each subsystem consists of two components: *Monitor* and *Switcher*.

Monitor: A dynamic system whose inputs are local system estimate $x_i^q(\tau_k)$, local channel state $R_i(k)$ and system estimate $x_{i+1}^q(\tau_k)$ from subsystem $i+1$ at time instant τ_k, and whose output is a set $\{\eta_i^1, \eta_i^2, \ldots, \eta_i^\ell, \ldots, \eta_i^{M_i}\}$. Each element of the set $\eta_i^\ell : \mathscr{C}_i \to \mathbb{R}_+$ denotes a *monitoring signal* that characterizes performance level that can be achieved by a controller $K_i^l(\cdot)$ based on the available information. Such characterization will be clear in Section IV.

Switcher: A logic system whose inputs are the *monitoring signals* $\{\eta_i^\ell\}_{\ell=1}^{M_i}$, and output is a piecewise continuous switching signal $s_i(\cdot) : \mathbb{R}_+^{M_i} \to \mathscr{C}_i$ that determines the control law. The logic system could be simply a function that outputs a controller index with the minimum *monitoring signal*.

IV. Main Results

The main results of this paper provide sufficient conditions to assure *almost sure asymptotic stability* and *almost sure practical stability* for a cascaded wireless networked system in equations (1-2). Subsection IV-A gives a state dependent dwell time function that is used to determine the switching time. Under the dwell time result, subsections IV-B and IV-C present sufficient conditions on the selection of controller to assure *almost sure asymptotic stability* and *almost sure practical stability* respectively for each subsystem.

A. Dwell Time Function

Under the assumption that the switching only occurs at each sampling time, this subsection constructs a state dependent dwell time function in terms of the sampling time interval. The sampling time interval $T_k = \tau_{k+1} - \tau_k$ can be viewed as a minimum separation for two switches.

The following technical Lemma is used to characterize a class $\mathscr{KL}$ function in terms of the composition of two class $\mathscr{K}_\infty$ functions.

Lemma 4.1 ([14]): Assume $\beta(\cdot,\cdot)$ is a class $\mathscr{KL}$ function. Then, there exist two class $\mathscr{K}_\infty$ functions $\theta_1(\cdot)$ and $\theta_2(\cdot)$ such that

$$\beta(s,t) \leq \theta_1(e^{-t}\theta_2(s))$$

for all $(r,s) \in [0,a) \times [0,\infty)$ where $a \in \mathbb{R}_+$.

The following Lemma makes use of the technical Lemma 4.1 to construct a state-dependent dwell time function. The dwell time function provides a lower bound on the transmission time interval for the cascaded system in equations (1-2) and (4).

Lemma 4.2: Consider the interconnected system in equations (1-2), let $K_i^{\ell_k}(\cdot)$ denote the selected controller function for subsystem i at time instant τ_k and let $\beta_i^{\ell_k}(\cdot,\cdot)$ represent the corresponding class $\mathscr{KL}$ function in equation (6) with class $\mathscr{K}_\infty$ functions $\theta_{1,i}^{\ell_k}(\cdot)$ and $\theta_{2,i}^{\ell_k}(\cdot)$ defined in Lemma 4.1, suppose $\theta_{1,i}^{\ell_k}(\cdot)$ and $\theta_{2,i}^{\ell_k}(\cdot)$ satisfy $\lim_{r\to 0} \ln \frac{\theta_{2,i}^{\ell_k}(r)}{\theta_{1,i}^{\ell_k -1}(r)} < +\infty$

for $\lambda_i \in (0,1), i=1,2,\ldots,N$, if the minimal time interval between two consecutive switches satisfies

$$T_k = \tau_{k+1} - \tau_k \geq \max_{1\leq i\leq N} \left\{ \ln \frac{\theta_{2,i}^{\ell_k}(|x_i(\tau_k)|)}{\theta_{1,i}^{\ell_k^{-1}}(\lambda_i|x_i(\tau_k)|)}, 0 \right\} \quad (7)$$

then

$$\beta_i^{\ell_k}(|x_i(\tau_k)|, T_k) \leq \lambda_i|x_i(\tau_k)|, i=1,2,\ldots,N \quad (8)$$

Proof: By technical Lemma 4.1, we know that for a selected controller $K_i^{\ell_k}(\cdot)$ at time interval $[\tau_k, \tau_{k+1})$, its class $\mathscr{KL}$ function $\beta_i^{\ell_k}(|x_i(\tau_k)|, T_k)$ can be bounded by

$$\beta_i^{\ell_k}(|x_i(\tau_k)|, T_k) \leq \theta_{1,i}^{\ell_k}(e^{-T_k}\theta_{2,i}^{\ell_k}(|x_i(\tau_k)|))$$

By condition (7) and assumption $\lim_{r\to 0} \ln \frac{\theta_{2,i}^{\ell_k}(r)}{\theta_{1,i}^{\ell_k^{-1}}(r)} < +\infty$, it is clear that for all subsystem $i=1,2,\ldots,N$, there exists a dwell time function defined in (7) such that

$$\beta_i^{\ell_k}(|x_i(\tau_k)|, T_k) \leq \lambda_i|x_i(\tau_k)|, i=1,2,\ldots,N$$

holds. ■

B. Almost sure Asymptotic Stability

This subsection presents sufficient conditions to assure *almost sure asymptotic stability* in the following definition.

Definition 4.3: The cascaded system in (1-2) is said to be *almost sure asymptotically stable*, if for arbitrary $\varepsilon > 0$ there exists Δ_i such that if $|x_i(0)| \leq \Delta_i$,$i=1,2,\ldots,N$ then

$$\lim_{t\to+\infty} \Pr\left\{\sup_t |x_i(t)| \geq \varepsilon\right\} \to 0$$

for all $i=1,2,\ldots,N$.

The following assumption is necessary to assure *almost sure stability* for the system in (1-2), which requires that the external disturbance to system in equation (1) varnishes when the system state approaches the equilibrium.

Assumption 4.4: Consider the system in equation (1), for a given class $\mathscr{K}_\infty$ function $W_1(\cdot)$, the external disturbance $d(t)$ is upper bounded by

$$\|d(t)\|_{[\tau_k,\tau_{k+1})} \leq W_1(|x_1^q(\tau_k)|), k\in\mathbb{Z}_+$$

With Assumption 4.4, Lemma 4.5 provides a switching rule to restrain upper system's control input as a function of lower system's state.

Lemma 4.5: Consider the closed-loop subsystems formed by equations (2) and (4), suppose Assumption 4.4 holds, given a class $\mathscr{K}_\infty$ function $W_{i+1}(\cdot)$ and a family of controller functions $\{K_i^{\ell_k}(\cdot)\}_{\ell_k=1}^{M_i}$ for subsystem i, if there exists a controller function $K_i^{\ell_k}(\cdot) \in \{K_i^{\ell_k}(\cdot)\}_{\ell_k=1}^{M_i}$, and a corresponding class $\mathscr{K}_\infty$ function $\kappa_i^{\ell_k}(\cdot)$ with property that $|K_i^{\ell_k}(r)| \leq \kappa_i^{\ell_k}(|r|), \forall r$, such that

$$\kappa_i^{\ell_k}(\beta_i^{\ell_k}(|x_i^q(k)|, 0)) \leq W_{i+1}(|x_{i+1}^q(k)|) \quad (9)$$

holds for all $k\in\mathbb{Z}_+$, then

$$\sup_{\tau_k\leq t<\tau_{k+1}} |u_i(t)| \leq W_{i+1}(|x_{i+1}^q(k)|), k\in\mathbb{Z}_+, i=1,2,\ldots,N-1$$

where $\beta_i^{\ell_k}(\cdot,\cdot)$ is class $\mathscr{KL}$ function corresponding to the selected controller function $K_i^{\ell_k}(\cdot)$ for subsystem i.

Proof: Consider subsystem i with controller defined in equation (4), let function $K_i^{\tilde{\ell}_k}(\cdot)$ be the control law that is used to compute the control input $u_i(t)$ over time interval $[\tau_k, \tau_{k+1})$. The ISS characterization in inequality (6) implies the control system in equation (4) satisfy

$$|\hat{x}_i(t)| \leq \beta_i^{\tilde{\ell}_k}(|x_i^q(\tau_k)|, t-\tau_k), t\in[\tau_k, \tau_{k+1}) \quad (10)$$

Therefore, we have $|\hat{x}_i(t)| \leq \beta_i^{\tilde{\ell}_k}(|x_i^q(\tau_k)|, 0)$. Since $K_i^{\tilde{\ell}_k}(\cdot)$ is locally Lipschitz, there always exists a class $\mathscr{K}_\infty$ function $\kappa_i^{\tilde{\ell}_k}(\cdot)$ such that

$$|K_i^{\tilde{\ell}_k}(r)| \leq \kappa_i^{\tilde{\ell}_k}(|r|), \forall r \quad (11)$$

By equation (4), we know $\sup_{\tau_k\leq t<\tau_{k+1}} |u_i(t)| = \sup_{\tau_k\leq t<\tau_{k+1}} |K_i^{\tilde{\ell}_k}(\hat{x}_i(t))|$. By inequalities (10) and (11), we have

$$\sup_{\tau_k\leq t<\tau_{k+1}} |u_i(t)| \leq \sup_{\tau_k\leq t<\tau_{k+1}} \kappa_i^{\tilde{\ell}_k}(\hat{x}_i(t)) = \kappa_i^{\tilde{\ell}_k}(\beta_i^{\tilde{\ell}_k}(|x_i^q(\tau_k)|, 0))$$

Then, it is obvious that inequality (9) is a sufficient condition to assure the conclusion hold. ■

Remark 4.6: The inequality (9) can be viewed as a switching rule for the upper system i to react to the changes on the state's estimate of lower system $i+1$. The switching rule is applied over time interval $[\tau_k, \tau_{k+1})$ and is distributed and feasible because it only depends on information $x_i^q(\tau_k)$ and $x_{i+1}^q(\tau_k)$ available at time instant τ_k.

Recall that $\{x_i^q(k), \overline{U}_i(k)\}_{k=0}^\infty$ characterizes the information structure at each time instant τ_k. Under Assumptions 3.1 and 4.4, the following lemma gives a recursive construction for the sequence $\{\overline{U}_i(k)\}_{k=0}^\infty$ such that the quantization error remains bounded by $\overline{U}_i(k)$ for all $k\geq 0$. This predictable bound is used to switch controllers to assure almost sure performance. Note that the technique used to prove Lemma 4.7 is similar to traditional dynamic quantization method[12], [15].

Lemma 4.7: Consider the closed-loop system in equations (1-2) and (4), given the transmission time sequence $\{\tau_k\}_{k=0}^\infty$ and a family of controller functions $\{K_i^{\ell_k}(\cdot)\}_{k=0}^\infty$, $\ell_k \in \{1,2,\ldots,M_i\}$. Let $T_k = \tau_{k+1} - \tau_k$ and suppose the hypothesis of Lemma 4.5 holds, the initial information structure pair $\{x_i^q(0), \overline{U}_i(0)\}$ is known to both *Encoder* and *Decoder*, and the initial system state $x_i(0) \in [-\overline{U}_i(0), \overline{U}_i(0)]^n$. If the sequence $\{\overline{U}_i(k)\}_{k=0}^\infty$ is constructed by the following recursive equation

$$\overline{U}_i(k+1) = 2^{-R_i(k+1)}\left(\overline{U}_i(k)e^{L_iT_k} + \left(e^{L_iT_k} - 1\right)\frac{B_i(k)}{L_i}\right) \quad (12)$$

where

$$L_i = L_{i_1} + L_{g_i}$$
$$B_i(k) = L_{i_2}W_i\left(|x_i^q(k)|\right)$$

then the estimation error $e_i(k) := x_i(k) - x_i^q(k)$, $i = 1, 2, \ldots, N$ can be bounded as

$$|x_i(k) - x_i^q(k)| \le \overline{U}_i(k), k \in \mathbb{Z}_+ \quad (13)$$

where $R_i(k)$ is the number of blocks received at time instant τ_k.

Proof: Consider the propagation of the estimation error $e_i(t) = x_i(t) - \hat{x}_i(t)$ over time interval $[\tau_k, \tau_{k+1})$, we have

$$\begin{aligned}\dot{e}_i(t) &= \dot{x}_i(t) - \dot{\hat{x}}_i(t) \\ &= f_i(x_i, u_{i-1}) - f_i(\hat{x}_i, u_{i-1}) + g_i(x_i, u_i) - g_i(\hat{x}_i, u_i)\end{aligned}$$

Using inequality $\frac{d|e_i|}{dt} \le |\dot{e}_i|$ and Lipschitz Assumption 3.1, the dynamic changes of infinity norm of estimation error $|e_i(t)|$ can be bounded as

$$\begin{aligned}\frac{d|e_i|}{dt} &\le L_{i,1}|e_i| + L_{i,2}|u_{i-1}| + L_{g_i}|e_i| \\ &\le (L_{i,1} + L_{g_i})|e_i| + L_{i,2}W_i(|x_{i+1}^q(k)|)\end{aligned}$$

The last inequality holds because of Lemma 4.5. Then, using Gronwall-Bellman inequality over time interval $[\tau_k, \tau_{k+1})$ yields

$$|e_i(t)| \le e^{L_i(t-\tau_k)}|e_i(\tau_k)| + (e^{L_i(t-\tau_k)} - 1)\frac{B_i(k)}{L_i}$$

where $L_i = L_{i,1} + L_{g_i}$ and $B_i(k) = L_{i,2}W_i(|x_{i+1}^q(k)|)$. For $t \to \tau_{k+1}$, one can get $|e_i(k+1^-)| \le e^{L_iT_k}|e_i(\tau_k)| + (e^{L_iT_k} - 1)\frac{B_i(k)}{L_i}$. Assume that $|e_i(k)| \le \overline{U}_i(k)$, then $|e_i(k+1^-)| \le e^{L_iT_k}\overline{U}_i(k) + (e^{L_iT_k} - 1)\frac{B_i(k)}{L_i}$. Upon receiving $R_i(k+1)$ blocks of bits at time instant τ_{k+1}, We know that

$$\begin{aligned}|e_i(k+1)| &\le 2^{-R_i(k+1)}|e_i(k+1^-)| \\ &\le 2^{-R_i(k+1)}e^{L_iT_k}\overline{U}_i(k) + (e^{L_iT_k} - 1)\frac{B_i(k)}{L_i}\end{aligned}$$

holds by uniform quantization method. Then from recursive equation (12), the final conclusion holds with $|e_i(k+1)| \le \overline{U}_i(k+1)$. ■

With Lemma 4.7, the following lemma presents a sufficient condition to ensure that the changes of the estimation error satisfy a stochastic inequality. The inequality is used to prove almost sure asymptotic stability for system in equations (1-2).

Lemma 4.8: Suppose the wireless communication channel for each subsystem in equations (1) and (2) satisfies the EBB characterization in (3), given the sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}, i = 1, 2, \ldots, N$ that is constructed in Lemma 4.7, let $G(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ be a monotone increasing function taking the form,

$$G(y) = e^{-h(y)\gamma(y)}\left(1 + h(y)\gamma(y)\right), y \in \mathbb{R}_+$$

for any given $\eta_i > 0$, if

$$G(|x_i(k+1)|) \le \eta_i \cdot e^{-L_iT_k} \quad (14)$$

then,

$$\mathbb{E}\left(\overline{U}_i(k+1)|\overline{U}_i(k)\right) \le \eta_i\overline{U}_i(k) + \eta_i(1 - e^{-L_iT_k})\frac{B_i(k)}{L_i} \quad (15)$$

Proof: The proof follows the same line in Lemma 4.4 of [7], and is omitted here. ■

Remark 4.9: The function $G(\cdot)$ in condition (14) is directly related to the EBB model, and it generates a partition of the system state. Each partition associates with a threshold η_i that characterizes the rate of stochastic changes for the sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}$.

Remark 4.10: The inequality in (15) characterizes the stochastic changes of the sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}$. The term η_i represents the rate of change resulting from the variation of local channel state. The term $B_i(k)$ characterizes the impact of disturbance from the upper system, and it reflects the coupling strength between subsystems.

With the result of Lemma 4.8, Theorem 4.11 states that for each candidate controller, there exists a corresponding η_i such that the expectation inequality in (15) holds. This result is used in the *Monitor* of each local supervisor to generate *monitoring signals* $\{\eta_i^{\ell}\}_{\ell=1}^{M_i}$.

Theorem 4.11: Consider the interconnected system in equations (1) and (2), and suppose the wireless communication channel satisfies the EBB characterization in equation (3), the sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}$ is recursively constructed by equation (12). Given the state estimate $x_i^q(k)$ at time instant τ_k, for each candidate controller $K_i^{\ell_k}(\cdot) \in \mathscr{K}_i$ selected over time interval $[\tau_k, \tau_{k+1})$, there always exists a corresponding $\eta_i^{\ell_k} \ge 0$ with

$$\eta_i^{\ell_k} = G\left(\beta_i^{\ell_k}(|x_i^q(k)|, T_k) + U_i(k+1)\right) \cdot e^{L_iT_k} \quad (16)$$

where $U_i(k+1) = \overline{U}_i(k)e^{L_iT_k} + \left(e^{L_iT_k} - 1\right)\frac{B_i(k)}{L_i}$, such that

$$\mathbb{E}\left(\overline{U}_i(k+1)|\overline{U}_i(k)\right) \le \eta_i^{\ell_k}\overline{U}_i(k) + \eta_i^{\ell_k}(1 - e^{-L_iT_k})\frac{B_i(k)}{L_i} \quad (17)$$

Proof: Consider the time interval $[\tau_k, \tau_{k+1})$, the system state $x_i(k+1)$ at time τ_{k+1} can be bounded by considering

$$\begin{aligned}|e_i(k+1^-)| &= |x_i(k+1^-) - \hat{x}_i(k+1^-)| \\ &\le U_i(k+1) = \overline{U}_i(k)e^{L_iT_k} + \left(e^{L_iT_k} - 1\right)\frac{B_i(k)}{L_i}\end{aligned}$$

Since $x_i(k+1^-) = x_i(k+1)$, for each candidate controller $K_i^{\ell_k}(\cdot)$, we have

$$\begin{aligned}|x_i(k+1)| &\le |\hat{x}_i(k+1^-| + U_i(k+1) \\ &\le \beta_i^{\ell_k}(|x_i^q(k)|, T_k) + U_i(k+1) \quad (18)\end{aligned}$$

From Lemma 4.8, we know that $G(\cdot)$ is continuous, positive and monotone increasing function. By inequalities (18), we have if

$$\eta_i^{\ell_k} = G\left(\beta_i^{\ell_k}(|x_i^q(k)|, T_k) + U_i(k+1)\right) \cdot e^{L_iT_k}$$

then

$$G(|x_i(k+1)|) \le \eta_i^{\ell_k}e^{-L_iT_k}$$

holds. Then, the final conclusion holds. ■

Remark 4.12: The *monitoring signal* $\eta_i^{\ell_k}$ is constructed based on the local information $x_i^q(k)$ and $\overline{U}_i(k)$ at time instant τ_k, and the controller function $K_i^{\ell_k}(\cdot)$. In order to guarantee the almost convergence of sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}$, $\eta_i^{\ell_k}$ must be sufficiently small. This suggests that the controller must be reconfigured in response to the changes of information $x_i^q(k)$ and $\overline{U}_i(k)$.

With the state-dependent dwell time function and the *monitoring signals* generated in (16), the following theorem provides a sufficient condition on the selection of the controller to assure almost sure asymptotic stability for the system in equations (1-2).

Theorem 4.13: Consider the interconnected system in equations (1-2). Suppose wireless communication channel condition in each subsystem satisfies the EBB characterization in equation (3). Let the hypothesis of Theorem 4.11 and Lemma 4.2 hold, suppose the coupling between the subsystems is sufficiently weak, i.e. there exists a constant positive value ε_i such that $B_i(k) = L_{i_2} W_i\left(|x_i^q(k)|\right) \leq \varepsilon_i |x_i^q(k)|$, and if there exists a candidate controller $K_i^{\ell_k}(\cdot)$ with $\eta_i^{\ell_k}$ such that for a given $\delta_i \in (0,1)$, we have

$$\max\{\overline{r}_i, \overline{J}_i\} < \delta_i \tag{19}$$

where

$$\overline{r}_i = \eta_i^{\ell_k} + e^{L_i T_k}(1 - 2^{-\bar{R}_i})$$
$$\overline{J}_i = \lambda_i + (1 - 2^{-\bar{R}_i} + \eta_i^{\ell_k} e^{-L_i T_k})(e^{L_i T_k} - 1)\frac{\varepsilon_i}{L_i}$$

and $\bar{R}_i$ is the total number of blocks that are transmitted at each time instant τ_k. Then the interconnected system in equations (1-2) is almost sure asymptotically stable

Proof: Consider time interval $[\tau_k . \tau_{k+1})$, by Lemma 4.7, we know that

$$|x_i(k) - \hat{x}_i(k)| \leq \overline{U}_i(k)$$

then $|x_i(k)| \leq |\hat{x}_i(k)| + \overline{U}_i(k)$. Let $E_i(k+1) = \hat{x}_i(k+1) - \hat{x}_i(k+1^-)$, then

$$|\hat{x}_i(k+1)| \leq |\hat{x}_i(k+1^-)| + |E_i(k+1)|$$

Since $|E_i(k+1)| \leq (\overline{U}_i(k)e^{L_i T_k} + (e^{L_i T_k} - 1)\frac{B_i(k)}{L_i})(1 - 2^{-R_i(k+1)})$, then we have

$$\begin{aligned}
&|\hat{x}_i(k+1)| \\
&\leq \beta_i^{\ell_k}(|\hat{x}_i(k)|, T_k) \\
&+ \left(\overline{U}_i(k)e^{L_i T_k} + (e^{L_i T_k} - 1)\frac{B_i(k)}{L_i}\right)\left(1 - 2^{-R_i(k+1)}\right) \\
&\leq \lambda_i |\hat{x}_i(k)| + \left(\overline{U}_i(k)e^{L_i T_k} + (e^{L_i T_k} - 1)\frac{\varepsilon_i |\hat{x}_i(k)|}{L_i}\right)\left(1 - 2^{-\bar{R}_i}\right)
\end{aligned} \tag{20}$$

where $\bar{R}_i$ is the number of blocks(packets) that are transmitted at each time instant τ_k. The second inequality holds because of Lemma 4.2, weak coupling property and $R_i(k+1) \leq \bar{R}_i$ for any $k \in \mathbb{Z}_+$. Taking expectation on both sides of the inequality (20) and inequality (15), and adding both sides of the resulting expectation inequalities yields

$$\begin{aligned}
&\mathbb{E}\left[\overline{U}_i(k+1) + |\hat{x}_i(k+1)|\right] \\
&\leq \left(\lambda_i + (1 - 2^{-\bar{R}_i} + \eta_i e^{-L_i T_k})(e^{L_i T_k} - 1)\frac{\varepsilon_i}{L_i}\right)\mathbb{E}[|\hat{x}_i(k)|] \\
&+ \left(\eta_i + e^{L_i T_k}(1 - 2^{\bar{R}_i})\right)\mathbb{E}[\overline{U}_i(k)] \\
&\leq \delta_i \mathbb{E}\left[\overline{U}_i(k) + |\hat{x}_i(k)|\right]
\end{aligned}$$

The second inequality holds by condition (19). Then, we have

$$\lim_{k\to\infty} \mathbb{E}\left[\overline{U}_i(k+1) + |\hat{x}_i(k+1)|\right] \leq \lim_{k\to\infty} (\delta_i)^{k+1}\left(\overline{U}_i(0) + |\hat{x}_i(0)|\right)$$

Since $|x_i(k)| \leq |\hat{x}_i(k)| + \overline{U}_i(k)$, it is clear that $\lim_{k\to\infty} \mathbb{E}[|x_i(k)|] \to 0$. Because of the locally Lipschitz Assumption 3.1, we further know that the trajectory between each transmission time interval $[\tau_k, \tau_{k+1})$ is bounded, then the subsystem i is almost sure asymptotical stable. Since the index i is arbitrary, the whole interconnected system is guaranteed to be almost sure asymptotical stable. ■

Remark 4.14: The inequality in (19) can be used in the *Switcher* of local supervisor to determine which controller is selected and placed in the loop. The input of the *Switcher* will be the *monitoring signals* $\{\eta_i^{\ell_k}\}_{\ell_k=1}^{M_i}$ that are generated by Theorem 4.11, and the output will be the index of the controller that satisfies condition (19).

C. Almost sure practical stability

This subsection presents sufficient conditions that assure *almost sure practical stability* for the cascaded system in (1-2).

Definition 4.15: The interconnected system is said to be *almost sure practical stability*, for arbitrary $\varepsilon > 0$ if there exists (Δ_i, Δ_i^*) with $\Delta_i^* > \Delta_i > 0$ such that if $|x_i(0)| \leq \Delta_i$, then

$$\lim_{t\to+\infty} \Pr\left\{\sup_t |x_i(t)| - \Delta_i^* \geq \varepsilon\right\} \to 0$$

Theorem 4.16: Consider the interconnected system in equations (1-2), Suppose the Assumption 4.4 holds, let the hypothesis of the Lemma 4.2 and Theorem 4.11 hold, and given the sequence $\{\overline{U}_i(k)\}_{k=0}^{\infty}$ constructed by equation (12) and positive values Δ_i^*. If there exists a candidate controller $K_i^{\ell_k}(\cdot)$ with $\eta_i^{\ell_k}$ such that $r_i < 1$ for $i = 1, 2, \ldots, N$ and

$$B_i(k) = L_{i_2} W_i\left(|x_i^q(k)|\right) \leq \frac{1 - r_i}{J_i}\min\{\Delta_i^*, |x_i^q(k)| + \overline{U}(k)\} \tag{21}$$

where

$$r_i = \max\{\eta_i^{\ell_k} + e^{L_i T_k}(1 - 2^{-\overline{R}_i}), \lambda_i\}$$
$$J_i = (1 - 2^{-\bar{R}_i} + \eta_i^{\ell_k} e^{-L_i T_k})(e^{L_i T_k} - 1)\frac{1}{L_i}$$

then the system in equations (1-2) is almost sure practical stable with respect to a compact set defined by $\Omega_i = \{x_i(t) : |x_i(t)| \leq \Delta_i^*\}$.

Proof: By Theorem 4.11, we know that for a selected controller $K_i^{\ell_k}(\cdot)$, there exists a corresponding $\eta_i^{\ell_k}$ such that given available information $\mathscr{I}_i(k) = \{\overline{U}_i(k), x_i^q(k)\}$ at time instant τ_k

$$\mathbb{E}\left(\overline{U}_i(k+1)|\mathscr{I}_i(k)\right) \leq \eta_i^{\ell_k}\overline{U}_i(k) + \eta_i(1-e^{-L_iT_k})\frac{B_i(k)}{L_i} \quad (22)$$

Similar to the proof of Theorem 4.13, we have following conditional expectation with available information at time instant τ_k,

$$\begin{aligned}&\mathbb{E}[|\hat{x}_i(k+1)||\mathscr{I}_i(k)]\\ &\leq \lambda_i|\hat{x}_i(k)| + \left(\overline{U}_i(k)e^{L_iT_k} + (e^{L_iT_k}-1)\frac{B_i(k)}{L_i}\right)\left(1-2^{-\overline{R}_i}\right)\end{aligned} \quad (23)$$

then, by adding inequalities (22-23) yields

$$\begin{aligned}&\mathbb{E}\left(\overline{U}_i(k+1) + |\hat{x}_i(k+1)||\mathscr{I}_i(k)\right)\\ &\leq \underbrace{\max\{\eta_i^{\ell_k} + e^{L_iT_k}(1-2^{-\overline{R}_i}), \lambda_i\}}_{r_i}(\overline{U}_i(k) + |x_i^q(k)|)\\ &+ \underbrace{\left(\eta_i^{\ell_k}e^{-L_iT_k} + 1 - 2^{-\overline{R}_i}\right)(e^{L_iT_k}-1)\frac{1}{L_i}}_{J_i}B_i(k)\end{aligned} \quad (24)$$

Let $V_i(k) = \overline{U}_i(k) + |x_i^q(k)|$, and consider the function $V_i(k)$ as a candidate Lyapunov function. It is clear that $V_i(k) \geq 0$ for any $k \in \mathbb{Z}_+$. Then, we can rewrite conditional inequality (24) into

$$\begin{aligned}\mathbb{E}[V_i(k+1)|V_i(k)] &\leq \mathbb{E}[V_i(k+1)|\mathscr{I}_i(k)]\\ &\leq r_iV_i(k) + J_iB_i(k)\end{aligned}$$

Furthermore, if the controller function $K_i^{\ell_k}(\cdot)$ is selected to assure $r_i < 1$ and condition (21), then we have

$$\begin{aligned}&\mathbb{E}[V_i(k+1)|V_i(k)]\\ &\leq V_i(k) - [(1-r_i)V_i(k) - J_iB_i(k)]\\ &\leq V_i(k) + (1-r_i)\min\{\Delta_i^* - V_i(k), 0\}\\ &= V_i(k) - (1-r_i)\max\{V_i(k) - \Delta_i^*, 0\}\end{aligned} \quad (25)$$

It is clear from inequality (25) that it preserves the supermartingale property when $V_i(k)$ lies in the compact set $\Omega_i = \{V_i(k) : V_i(k) \leq \Delta_i^*\}$ which guarantees the invariance of set Ω_i. When $V_i(k)$ lies out of the set Ω_i, $V_i(k)$ will decease almost surely until the system's state reaches the set Ω_i. This condition can be viewed as a stochastic version of the LaSalle Theorem in discrete time system. It is worth noting that using condition (25) to prove almost sure convergence to a compact set is not new, and it is well studied in [16], [17]. With condition (25), we can prove that for arbitrary $\varepsilon > 0$, the following *almost sure convergence* property for $V_i(k)$ with respect to set Ω_i holds

$$\lim_{k\to+\infty} \Pr\left\{\sup_k V_i(k) - \Delta_i^* \geq \varepsilon\right\} \to 0$$

Since $|x_i(k)| \leq |x_i^q(k)| + \overline{U}_i(k) = V_i(k)$, the *almost sure convergence* property for $V_i(k)$ leads to *almost sure convergence* for $|x_i(k)|$ with respect to set Ω_i. By ISS assumption in (6), the state trajectories will remain bounded within each transmission time interval $[\tau_k, \tau_{k+1})$ for all $k \in \mathbb{Z}_+$. Therefore, for arbitrary $\varepsilon > 0$, we have

$$\lim_{t\to+\infty} \Pr\left\{\sup_t |x_i(t)| - \Delta_i^* \geq \varepsilon\right\} \to 0$$

The proof is complete. ■

Remark 4.17: As expected, the sufficient conditions in Theorem 4.16 to assure *almost sure practical stability* are weaker than the conditions in Theorem 4.13 for *almost sure asymptotic stability*. Such weakness lies in two aspects. First, the results in Theorem 4.16 do not require weak coupling assumption between subsystems. Instead, it relies on controller reconfiguration to ensure a weaker condition (21) holds. Secondly, the condition $r_i < 1$ in Theorem 4.16 is weaker than condition (19) in Theorem 4.13. In fact, it only requires $\eta_i^{\ell_k} + e^{L_iT_k}(1-2^{-\overline{R}_i}) < 1$ holds since $\lambda_i < 1$ always holds by Lemma 4.2.

V. Conclusion

This paper studies the *almost sure safety* property of a cascaded networked system in the presence of deep fades that exhibit exponentially bounded burstiness. The *almost sure safety* is assured by developing a distributed switched supervisory control strategy under which the local controller is reconfigured online to guarantee *almost sure asymptotic stability* and *almost sure practical stability*. Sufficient conditions are derived for the supervisor in each subsystem to decide when and which controller is selected and placed in the feedback loop, to assure *almost sure safety* for the entire networked system. The application of this paper's results to a leader follower formation control problem is discussed in our companion paper [10].

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