# Stability analysis for wireless networked control system in unslotted IEEE 802.15.4 protocol

Bo Wu, Hai Lin, Michael Lemmon

Abstract-Wireless networked control systems (WNCS) with the control loops closed over a wireless network are prevailing these days. But it also produces new challenges for stability analysis when considering the nuance of the practical communication protocols. The IEEE 802.15.4 protocol has been very popular among communication protocols utilized in WNCS. However, usually its medium access control (MAC) is not tailored towards WNCS which may be greatly affected by the choice of MAC parameters. Most previous research either focuses on WNCS or the communication protocol but few of them considered the modelling and analysis on both. To this end, this paper serves to bridge these two interactive parts. We first propose a Markov chain model to analyze the communication network's impact on the delay from sensor to controller and apply an extended Bernoulli jump linear system framework to study how MAC parameters affect control system stability. It has been revealed that there are certain MAC parameters greatly affect system stability and they have to be carefully designed and tuned systematically.

#### I. INTRODUCTION

Wireless networked control systems (WNCS) are spatially distributed systems in which the communication between sensors, controllers and actuators occurs through a shared wireless communication network. There has been growing interest in WNCS because of its flexible architectures, reduced installation and maintenance costs [1]. However control loops being closed over shared communication networks also introduces new challenges [2]. It is no longer suitable to assume that the delays between sensor and actuator are negligible or constant. Indeed, WNCS will inevitably suffer from random delays or packet dropouts introduced by transmission, channel access, retransmission and routing which may affect the control system stability or even destabilize the system [3].

The stability of WNCS subject to data rate, time-varying transmission delay and communication constraints has been considered in [4], [5], [6] and references therein. However, it has been pointed out that in most control applications the packet size is relatively small [7], [8] and thus the end to end delay is actually dominated by the duration of random access [8] instead of data transmission. Therefore the effect of random access plays a significant role in the delay of WNCS, which is a function of data traffic, medium access control (MAC) protocol and topology [9]. But most of such studies either assume an oversimplified communication pro-

tocol model or assume a delay characteristic as given a priori without considering practical communication protocols. Hence this paper first gives a detailed analytical model of delay effects in the WNCS using a practical communication protocol, particularly the IEEE 802.15.4 standard [10].

Most of the theoretical analysis on the IEEE 802.15.4 protocol is based on the multi-state Markov model proposed in Bianchi's work [9]. The key approximation in [9] which is also adopted in most subsequent researches such as [11], [12] is the assumption of a constant and independent channel busy probability at each attempt for each node, regardless of the number of backoffs or retransmissions already suffered. This assumption has been proven to be quite accurate in saturated traffic scenario meaning that the transmission queue of each node is assumed to be always nonempty, and has also been widely adopted in the literature for unsaturated traffic [8], [13]. We will also adopt this assumption in our work.

The main concern of this paper is the stability analysis of WNCS with the IEEE 802.15.4 communication protocol. Unlike previous related works that either concern only stability or protocol, in this study we bridge them and study both to provide a control and communication system co-design framework. We first model and analyze the MAC protocol and end to end delay, show that they are functions of MAC and WNCS parameters. Then we apply the WNCS stability analysis based on the extended Bernoulli jump linear system theory. From the results we identify the most significant MAC parameters to stabilize the system with the potential to maximize the stability margin. In this way, when designing the protocol tailored to control applications, it is possible to assess the stability of the control system subject to the communication network beforehand and adjust the corresponding MAC or WNCS parameters. Also when there is variation in the size of WNCS or the channel conditions, this work can give guidance on the MAC parameter adaptation.

The paper is organized as follows. Section II formulates the problem. Section III gives an overview of the unslotted IEEE 802.15.4. In Section IV, we propose a Markov chain model for the standard unslotted IEEE 802.15.4 protocol followed by Section V where we study the end to end delay distribution using approximations. In Section VI, we apply an extended Bernoulli jump linear system framework to study the stability of WNCS.

## II. PROBLEM FORMULATION

Let's consider a WNCS with the star topology consisting of N identical linear time invariant (LTI) systems. The sensors are spatially distributed while the controllers and

Authors are with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, 46556 USA e-mail: (bwu3@nd.edu, hlin1@nd.edu, lemmon@nd.edu)

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the actuators are collocated in the center of the network. The communication conforms to the IEEE 802.15.4 unslotted protocol. Assume that each control system has the dynamic as the following:

$$\frac{dx}{dt} = Ax(t) + Bu(t) \tag{1}$$

where  $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, A, B$  are matrices of appropriate sizes and u(t) = -Kx(t) is the control input.

The control system is sampled with a random sampling period and then the sampled value is transmitted to the controller via a wireless channel. Once the controller receives the data, it immediately computes the corresponding control and applies it to the corresponding actuator. The control value is then used until next update comes. We assume that there is no delay in this stage.

In a typical contention based MAC protocol, there are three possible outcomes for each transmission attempt. First the sender successfully gets access to the wireless channel and transmits the data while no other senders was transmitting. So no collision occurs and the receiver gets the data without any distortion. Here we assume that the wireless channel is perfect so once the data is transmitted without collision, the receiver will get the data. Second, the sender gets access to the channel and sends the data, but it is dropped due to collision. In this case, the receiver will not get anything and no retransmission is assumed. The third case is that the data is dropped due to accessing failures within allowed number of attempts.

Let  $r_k \in \{0, 1, 2\}$  denote the above states of successful transmissions, packet drop due to collisions and packet drop due to access failures respectively at the  $k_{th}$  sampling period. Since we assumed that at each attempt, the channel busy probability is constant and independent and also each sensor tries to access the channel independently, it can be obtained that  $r_k$  is a Bernoulli process. Let  $h_0, h_1, h_2$  denote the random sampling period in each state, consequently they are i.i.d with certain distributions respectively.

When  $r_k = 0$ , the sampled system can be written as [14]

$$x_{k+1} = \phi_0(k)x_k + \Gamma_{0,0}u_k + \Gamma_{1,0}u_{k-1} \tag{2}$$

where

$$\phi_0(k) = e^{Ah_0}$$

$$u_k = -Kx_k$$

$$\Gamma_{0,0} = \int_0^{h_0 - D_k} e^{As} \, dsB$$

$$\Gamma_{1,0} = \int_{h_0 - D_k}^{h_0} e^{As} \, dsB$$
(3)

 $D_k$  denotes the delay experienced in the  $k_{th}$  transmission which is the sum of the backoff delay and the time spent on transmitting the data packet. Set  $z_k = [x_k, u_{k-1}]^T$ . The new augmented system is as follows:

$$z_{k+1} = \Phi_0 z_k \tag{4}$$

where

$$\Phi_0 = \begin{bmatrix} \phi_0(k) - \Gamma_{0,0}K & \Gamma_{1,0} \\ -K & 0 \end{bmatrix}$$
(5)

For  $r_k = i$  where i = 1, 2, the sample system can be written as

$$x_{k+1} = \phi_i(k)x_k + \Gamma_{0,i}u_{k-1} \tag{6}$$

where

$$\phi_i(k) = e^{Ah_i}$$
  

$$\Gamma_{0,i} = \int_0^{h_i} e^{As} \, dsB$$
(7)

The sampled data was not received by the system, the controller will use the previous control value. Therefore, in the augmented system for i = 1, 2 we have

$$\Phi_i = \begin{bmatrix} \phi_i(k) & \Gamma_{0,i} \\ 0 & 1 \end{bmatrix}$$
(8)

Consequently, the single control system can be modelled as an extended Bernoulli jump linear system in which the system dynamic in one of the three states is random with a certain distribution. It is different from the traditional Bernoulli jumper linear system in which the system dynamic in each state is deterministic [15]. In our case, the mean square stability condition for such a system is [16]:

$$R = \rho(\sum_{i=0}^{2} p_i E[\Phi_i \otimes \Phi_i]) < 1$$
(9)

where  $p_i$  denotes the probability in the  $i_{th}$  state,  $\rho(G)$  denotes the spectral radius of some matrix G and  $\otimes$  is the Kronecker product. Note that the stability results also apply if we consider additive Gaussian noises.

Observe that the stability of the system depends on  $p_i$ and  $h_i$  which are functions of MAC parameters, data length and N. Thus we need the protocol modelling and analysis which will be conducted in section IV. Before that, let's first introduce the IEEE 802.15.4 protocol.

## III. OVERVIEW OF THE UNSLOTTED IEEE 802.15.4

The IEEE 802.15.4 standard defines two channel access modalities: the beacon-enabled modality, which uses a slotted CSMA/CA and the optional GTS allocation mechanism, and a simpler unslotted CSMA/CA without beacons. This paper focuses on the latter modality.

According to the standard, there are three types of nodes: coordinator, routers and end devices. The coordinator establishes a network and can allow others to join. The routers are similar to the coordinators but do not start a network. The end devices only join the network. There must be and can only be one coordinator in a network but multiple routers and end devices. The nodes have to use the unslotted CSMA/CA protocol to access the channel. The algorithm is implemented using units of time called backoff periods  $T_b$  which contains 20 symbol time  $T_s$ . One symbol consists of 4 bits and the bit rate is 250 kbps. Therefore it can be obtained that  $T_s = 16\mu s$  and  $T_b = 320\mu s$ .

Basically at the beginning of every transmission attempt,



Fig. 1. The unslotted IEEE 802.15.4 CSMA/CA algorithm.

each node initializes two variables : NB and BE. NB is the number of times that the CSMA/CA algorithm was required to backoff because the channel is sensed to be busy. This value is initialized to 0 before each new transmission attempt and is upper bounded by  $NB_{max}$ , which is equal to 4 by default. BE is the backoff exponent related to the maximum number of backoff periods. It will be initialized to  $BE_{min}$  and cannot exceed  $BE_{max}$  whose default values are 3 and 5 respectively. There is no retransmission assumed so no ACK is transmitted.

As Fig. 1 illustrates, a node first initializes the two variables and then uniformly delays a discrete time duration between 0 to  $(2^{BE_{min}} - 1) * T_b$ . When the waiting time is up, it performs channel sensing which takes  $T_b$ . If the channel is busy, NB and NE will be increased by 1 and the node performs second round of backoff. If the channel is idle, the node will simply transmit the packet immediately. If at least two nodes happen to sense the channel idle at the same time slot and thus start transmission at the same time, the packets will collide and be lost. When  $NB \leq$  $NB_{max}$  and if channel is busy repeatedly, it will continue to backoff. When the number of backoff exceeds the maximum allowable value, the node will stop transmission attempt and the packet is dropped. Since  $T_b$  is 320  $\mu s$ , it can be calculated that the maximum backoff time that a node can have is 38.4 ms. We assume that the wireless channel is perfect, so in the unslotted CSMA/CA, the packets are lost due to two reasons: channel access failure and packet collision. The former happens when a packet fails to obtain an idle channel after channel sensing within m+1 backoff stages. The latter happens if the transmission collides with other packets. After each transmission, whether it is successful or not, we assume that the node will enter the idle state for a constant time period and prepare for the next data arrival.



Fig. 2. Markov chain model for unslotted IEEE 802.15.4 CSMA/CA.

#### IV. MARKOV CHAIN MODEL

In this section, we propose an analytical model of the unslotted CSMA/CA mechanism of IEEE 802.15.4 following the discussion in [17]. The key difference is that [17] discussed the slotted IEEE 802.15.4 and assumes that each node goes to the idle state with certain probability.

As discussed in problem formulation, a star topology network is considered. The coordinator is the collocated controllers and actuators for N control systems. The N end devices are the sensors contending to send data to the coordinator, which is the data sink. The behavior of each single node in the network is studied by the following Markov model.

Let s(t), c(t) be the stochastic processes representing the backoff stage and the state of the backoff counter at time t respectively, experienced by a node to transmit a packet. By assuming the independent probability that the nodes start sensing, the stationary probability  $\tau$  that a node attempts carrier sensing in a randomly chosen time slot is constant and independent of other nodes [17], and the tuple (s(t), c(t)) is a two dimensional Markov chain.

We define  $W_0 = 2^{BE_{min}}, m_0 = BE_{min}, m_b = BE_{max},$  $m = NB_{max}$ . The states (i, 1) to  $(i, W_m - 1), i \in [0, m]$ denote backoff states. For example, state (i, j) denotes that a node is in the backoff stage i and its counter is j. States  $Q_0, ..., Q_{L_0-1}$  denote the idle states, where  $L_0$  is the predefined idle duration. So whenever the state reaches  $Q_{L_0-1}$ , a new data packet will be ready for transmission. This is the main difference with the Markov chain model proposed in [18] which assumes that the generation of a new packet in each time unit is governed by a specified probability q. States  $(i,0), i \in [0,m]$  denote the counter decreases to 0. States (-1,k) and (-2,k) model the successful transmission and packet collision.  $P_c$  is the collision probability and  $P_b$  is the probability for channel being busy. L is the data length in integer number of  $T_h$ . If we denote h as the end to end delay for each transmission which is also the sampling period, it is obvious that h is a random variable. The model is illustrated in Fig. 2. The transition probabilities associated with the Markov chain are:

$$P(i,k|i,k+1) = 1, k \ge 0 \tag{10}$$

which represents the decrement of backoff counter that happens with probability 1.

$$P(i,k|i-1,0) = \frac{P_b}{W_i}, i \le m$$
(11)

which represents the probability of finding busy channel and then of selecting a state uniformly the in the next backoff stage.

$$P(-1, L-1|i, 0) = (1 - P_b)(1 - P_c)$$
(12)

which gives the probability of successful transmission without collision after finding an idle channel.

$$P(-2, L-1|i, 0) = (1 - P_b)P_c$$
(13)

which denotes the probability of unsuccessful transmission due to collision after finding channel to be idle.

$$P(Q_0|m,0) = P_b$$
(14)

which represents the probability of going back to the idle stage due to the channel access failure and backoff limits.

$$P(Q_0|-i,0) = 1 \tag{15}$$

(15) for i = 1, 2 are the probabilities of going back to the idle stage after a transmission with or without collision.

$$P(0,k|Q_{L_0}) = \frac{1}{W_0} \tag{16}$$

(16) models the probability of going back to the first backoff stage from the idle stage. Also, in idle stage,  $Q_i$  moves to  $Q_{i+1}$  with probability 1.

We then compute the stationary distribution of the Markov chain based on (10)-(16). Define  $b_{i,k} = \lim_{t\to\infty} P(s(t) = i, c(t) = k)$  to be the stationary distribution of the Markov chain. Then from the transition probabilities above, we can see that

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, \ i \ge 0$$
(17)

Where

$$W_i = \begin{cases} 2^i W_0 & i \in [0, m_b - m_0] \\ 2^{m_b - m_0} W_0 & i \in [m_b - m_0 + 1, m] \end{cases}$$

and

$$b_{i,0} = P_b^i b_{0,0} \tag{18}$$

The normalization condition is that

$$\sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k} + \sum_{k=0}^{L-1} b_{-1,k} + \sum_{k=0}^{L-1} b_{-2,k} + \sum_{l=0}^{L_0-1} Q_l = 1$$
(19)

From (17) and (18), we can take a closer look at the four terms in (19).

$$\sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k} = b_{0,0} \sum_{i=0}^{m} \frac{P_b^i}{W_i} \sum_{k=0}^{W_i-1} W_i - k$$
$$= \frac{b_{0,0} W_0}{2} \left( \frac{1 - P_b^{m+1}}{1 - P_b} + \frac{1 - (2P_b)^{m_b - m_0 + 1}}{1 - 2P_b} + 2^{m_b - m_0} \frac{P_b^{m_b - m_0 + 1} (1 - P_b^{m_0})}{1 - P_b} \right)$$
(20)

$$\sum_{k=0}^{L-1} b_{-1,k} = L(1-P_c)(1-P_b) \sum_{i=0}^{m} b_{i,0}$$

$$= L(1-P_c)(1-P_b^{m+1}b_{0,0})$$
(21)

Similarly,

$$\sum_{k=0}^{L-1} b_{-2,k} = LP_c(1 - P_b^{m+1})b_{0,0}$$
(22)  
$$\sum_{l=1}^{L_0-1} Q_l = L_0 Q_0$$

$$=L_0(P_b^{m+1}b_{0,0}+b_{0,0}(1-P_b^{m+1}))=L_0b_{0,0}$$
(23)

Plug (20)-(23) into (19), we can get an equation of  $P_b, b_{0,0}$ and  $P_c$ . On the other hand, the probability  $\tau$  that a node attempts to sense the channel at any given time is

$$\tau = \sum_{i=0}^{m} b_{i,0} = \frac{1 - P_b^{m+1}}{1 - P_b} b_{0,0} \tag{24}$$

Then the probability  $P_c$  that the package transmission collides is the probability that at least one of the N-1 remaining nodes transmits in the same time slot. It can be written as

$$P_c = 1 - (1 - \tau)^{N-1} \tag{25}$$

Similarly, the channel busy probability  $P_b$  can be expressed as

$$P_b = L(1 - (1 - \tau)^{N-1})(1 - P_b)$$
(26)

Combine these expressions it is possible to solve for  $b_{0,0}$ ,  $\tau$ ,  $P_c$ ,  $P_b$  from a set of nonlinear equations.

#### V. BACKOFF TIME ANALYSIS

Basically there are two cases for sampling period h at the  $k_{th}$  sampling time. First is when the data gets transmitted regardless of a collision. In this case,  $h = \tau_{b,k} + (L + L_0)T_b$  where  $\tau_{b,k}$  denotes the delay in the backoff stage,  $LT_b$  is the time to transmit the data and  $L_0T_b$  is the time spent in the idle state waiting for new data. In the second case the node fails to access the channel within the backoff time limit so  $h = \tau_{f,k}$  where  $\tau_{f,k}$  denotes the total time spent in m backoff stages without success. Since  $P_b$  is independent of time and the backoff selection is also an independent event, it can be obtained that  $\tau_{b,k}$  and  $\tau_{f,k}$  are i.i.d so they can be simply denoted as  $\tau_b$  and  $\tau_f$ .

For  $\tau_b$ , it is possible to get an exact distribution. For example in [11], a probability generation function approach is proposed to compute the discrete probability distribution of the delay. However, such an approach is computationally expensive and can only produce lengthy expressions without a closed form. A more general way is to obtain the approximated distribution by a moment matching approach in which the mean and variance of the actual delay is derived and then the probability distribution function of the delay is approximated by known distributions. Exponential distribution has been shown to match well the actual distribution in scenarios like our study [17]. Therefore, in this paper we use exponential distribution with derived actual average of the delay to approximate the backoff delay distribution. It can be seen that

$$\tau_b = \sum_{i=0}^{m} 1(B_i | B_s) D_i$$
 (27)

where 1(.) is the indicator function and  $B_s$  denotes the event of successful channel access (though still could collide with other packets).  $B_i$  denotes the event of successful transmission at  $i_{th}$  backoff stage,  $D_i$  denotes the corresponding total delay from backoff stage 0 to i. Then the expectation of  $\tau_b$  is

$$E[\tau_b] = E[\sum_{i=0}^m 1(B_i|B_s)D_i]$$

$$= \sum_{i=0}^m E[1(B_i|B_s)D_i] = \sum_{i=0}^m E[1(B_i|B_s)]E[D_i]$$
(28)

where  $E[D_i] = \sum_{j=0}^{i} E[d_j]$  and  $d_j$  denotes the random delay at the  $j_{th}$  backoff stage and is uniformly distributed so  $E[d_j] = \frac{W_j}{2}$ . Furthermore  $E[1(B_i|B_s)] = P(B_i|B_s)$  where  $P(B_i, B_s) = P_b^i(1 - P_b)$  and  $P(B_s) = \sum_{i=0}^{m} P_b^i(1 - P_b)$ . So from (28) we know that

$$E[\tau_b] = \frac{1}{1 - P_b^{m-1}} \sum_{i=0}^m P_b^i E[D_i]$$
(29)

which is a function of  $P_b$ . From the above equation we can see that  $P_b$  is the key parameter to bridge the control and communication system which is influenced by the total number of nodes N, the data length L and the idle time length  $L_0$ .

For  $\tau_f$ , we know that  $\tau_f = \sum_{i=0}^m d_i$ . Since  $d_i$ 's are independent of each other and each one of them is uniformly distributed,  $\tau_f$ 's distribution can be obtained.

## VI. STABILITY ANALYSIS

Now we are ready to analyze the stability of WNCS with the unslotted 802.15.4 MAC protocol. We assume that the WNCS starts at the time when the Markov chain of the communication network reaches its steady state. The control system is modelled as in Section II.

In state 0, from the protocol we know that  $h_0 = \tau_b + (L_0 + L)T_d$ . Since  $\tau_b$ 's distribution has been analyzed and approximated in section V,  $L_0$  and L are predefined constants, the distribution for  $h_0$  is also known. Likewise in state 1,  $h_1 = \tau_b + (L_0 + L)T_d$  and in state 2,  $h_2 = \tau_f + L_0Td$ . Then we can apply our analytical protocol model and delay analysis to examine the significance of different MAC parameters and shed some lights on the control and communication system co-design assuming a given system dynamic as (1) where A = 1, B = 1, K = 1.5.

# A. Number of nodes N

We evaluate the spectral radius R given in (9) as the function of N. Fig. 3(a) illustrates the effect of number of nodes N on the stability of our Bernoulli jump linear system. The protocol parameters are default ( $m_0 = 3$ ,  $m_b =$ 



Fig. 3. Spectral radius R and  $p_0, p_1, p_2$  as function of N.



Fig. 4. Spectral radius R and  $p_0, p_1, p_2$  as function of  $m_0$ .

5, m = 4) and we set L = 10,  $L_0 = 5$ . It can be observed that the system starts to become unstable when N > 17. The reasons are twofold. With more nodes, there is a higher busy channel probability which results in longer delays in successful transmissions. Furthermore, it is also more likely to have collisions or dropouts due to channel access failures as shown in Fig. 3(b). Observe that it is interesting to see Rfirst decreases and then increases indicating an optimal N.

# B. Initial backoff exponent $m_0$

Fig. 4(a) illustrates the R as a function of the initial backoff exponent  $m_0$  with N = 20,  $m_b = 8$ , m = 5, L =10,  $L_0 = 5$ . A smaller  $m_0$  gives less backoff delay if successfully transmitted but at the same time the channel being busy is more likely. Likewise, a larger  $m_0$  will decrease the channel busy probability and thus increase the chance to transmit the data but at the cost of longer delays occurring with higher probability. So increasing  $m_0$  to a certain point the longer delay will cancel the benefit of a higher success transmission probability which can be seen in Fig. 4(a) as Rfirst decreases and then increases. Obviously there is a trade off to choose  $m_0$  to minimize R.

Also from Fig. 4(b) it can be found that  $P_b$  decreases rapidly and the chance of successful transmission increases dramatically as  $m_0$  increases. So  $m_0$  is a key parameter that affects the channel busy probability and delay.



Fig. 5. Spectral radius R and  $p_0, p_1, p_2$  as function of  $m_b$ .



Fig. 6. Spectral radius R and  $p_0, p_1, p_2$  as functions of  $L_0$ .

# C. Maximum backoff exponent $m_b$

Fig. 5(a) and Fig. 5(b) are plotted with N = 10,  $m_0 = 3$ , m = 5, L = 10,  $L_0 = 5$ . With a higher  $m_b$ , it can be observed that the channel being busy is less likely because at the higher backoff stage there will be exponentially more time slots available to choose from. But at the same time the system may experience a longer delay for the same reason. It can be seen that R starts to level off as  $m_b$  increases because of these two competing factors. So increasing  $m_b$  initially can be beneficial but when  $m_b$  is already large it will not greatly improve the system performance.

## D. Idle time length $L_0$

With larger  $L_0$ , the average sampling period will be higher since the control system spends more time in the idle state. However, it is also less likely to sense the channel busy because of less traffic in the network. From Fig. 6(a) and Fig. 6(b) it can be observed that increasing  $L_0$  has very little impact on R which decreases by less than 0.004 while  $L_0$ changes from 5 to 30.

#### VII. CONCLUSION

This paper mainly addressed two problems. First we presented a two dimensional Markov chain model and delay distribution analysis in the unslotted IEEE 802.15.4 protocol. We analyzed the model, derived the channel busy probability and collision probability. Based on the derived analytic model, we studied the WNCS stability problem. It can be seen that before deploying a WNCS with IEEE 802.15.4

protocol, it is possible to assess the system stability. If the system tends to be unstable, the main parameters to be considered tuning is  $m_0$  and  $m_b$ . Increasing  $L_0$  may help very little and finally, if no MAC parameter tuning can stabilize the system, we may decrease the number of nodes in the network.

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