Distributed Switching Control to Achieve Almost Sure Safety for Leader-Follower Vehicular Networked Systems

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***Abstract*—Leader-follower formation control is a widely used distributed control strategy that requires systems to exchange their information over a wireless radio communication network to attain and maintain formations. These wireless networks are often subject to *deep fades*, where a severe drop in the quality of the communication link occurs. Such deep fades inevitably inject a great deal of stochastic uncertainties into the system, which significantly impact the system's performance and stability, and cause unexpected safety problems in applications like smart transportation systems. Assuming an exponentially bursty channel that varies as a function of the vehicular states, this paper proposes a distributed switching control scheme under which the local controller is reconfigured in response to the changes of channel state, to assure *almost sure safety* for a chain of leader-follower system. Here *almost sure safety* means that the likelihood of vehicular states entering a safe region asymptotically goes to one as time goes to infinity. Sufficient conditions are provided for each local vehicle to decide which controller is placed in the feedback loop to assure *almost sure safety* in the presence of *deep fades*. Simulation results of a chain of leader-follower formation are used to illustrate the findings.**

***Index Terms*—Cyber-physical systems (CPS), quality of service (QoS), vehicular network (VN).**

I. Introduction

VEHICULAR networks (VNs) are cyber-physical systems (CPS) consisting of numerous autonomous vehicles that coordinate with each other by sharing information over wireless networks. VNs have recently received considerable attention due to rapid advances in Vehicle to Vehicle (V2V) communication technology, which promises significant safety improvement for applications like intelligent transportation systems [10]. Building safe VNs, however, is extremely challenging in two aspects. First, the mobile nature of VNs requires design of control strategies that are distributed and scalable. Secondly, V2V wireless networks in VNs are highly time varying due to the motion of transmitters and receivers. As a result, the V2V channel is inherently bursty and subject to *deep fading*, which causes a severe drop in the network's quality of service (QoS). These deep fades induce a great amount of stochastic uncertainties into the system, thereby negatively impacting the system's performance and causing serious safety issues. The objective of this paper is to design a distributed control strategy that could assure a certain level of safety for VNs in the presence of *bursty deep fading* channels.

Leader-follower scheme naturally serves as a distributed strategy for VNs due to its simplicity, scalability and the fact that communication is essential for assuring safe platooning in automated highway system (AHS) [23]. This has been illustrated by work that is based on either experimental validation [1], [3] or theoretical analysis [15], [38]. In leader-follower platoon systems, the question of safety is often analyzed under the concept of string stability [35]. This concept has been proven to be effective in characterizing the propagation of disturbances from the leader to downstream vehicles [32]. Recent results [15] showed that string stability can be improved by increasing the leader's communication connectivity to its followers. Such improvement, however, is compromised by reduced network connectivity arising from the delayed or dropped packets [22], [29]. This impact of unreliable network links on formation control has motivated studies of robust networked controllers under communication constraints, such as time varying but bounded delays [13], [22], [28] and Bernoulli [37] or two-state Markov chain dropouts [33]. So communication issues are critical in the development of safe VNs.

The channel model that is used to characterize V2V fading network, however, must be carefully specified. Traditionally, communication channels are modeled as an *independent and identical distributed (i.i.d)* random process with either a Rayleigh or Rician distribution [37] or a two-state Markov chain [33]. These characterizations are inadequate for V2V channel due to two reasons. First, fading channels are time varying and possess memory that cannot be captured by i.i.d models. Second, conventional two-state Markov chain ignores the potential dependence of the channel state (e.g., bit error rate) on the vehicle's physical states (e.g., inter-vehicle distance and bearing angle) [3], [7]. Such dependency in V2V communication systems has been extensively explored in communication community, see [4], [7], [14], [31], [40]. However, obtaining a V2V channel model is practically challenging due to its significant dependency on the dynamics of the vehicles and the surrounding environments [14]. Thus, most existing V2V channel models are obtained for specific environments and have limited use for control systems.

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From control perspective, there are two fundamental properties in V2V communication channel that have essential impacts on the system's performance and stability. The first property is the channel burstiness, which is characterized by a long string of consecutive dropouts in the network. Recent work [21] showed that system's stability can be seriously compromised if such burstiness is allowed with a sufficiently large probability. The second property is the dependency of channel state on the vehicular states. The knowledge of the correlation between physical and communication systems is valuable in designing control strategies for the safety of mobile vehicles. The vehicles could be safe if they use the knowledge of channel state to adaptively adjust their future actions so that the likelihood of the V2V channel exhibiting deep fades becomes smaller and smaller as time progresses.

Motivated by the observation of burstiness and state dependency in V2V network, a more realistic channel model was proposed in [16] for a single leader-follower pair, in which the channel is exponentially bursty and is dependent on the norm of the physical system's states. Such channel model is built upon the framework of *exponentially bounded burstiness* (EBB) originally developed by [41] and explicitly incorporates the state dependency. The EBB characterization discussd in this paper is closely related to the notion of *Outage Probability* [19], [39], which is a well studied performance metric for fading channel. The advantages of the EBB model are that (1) it directly characterizes the probability bound on the channel burstiness which is proven to be essential for system's stability; (2) it captures the state dependency of V2V channel in a simple but effective way that could be useful for control purpose; and (3) it is general in the sense that it can model a wide range of communication channels including i.i.d and two-state Markov chains. This paper extends the prior work in [16] to a chain of leader-follower system and shows that for any mobile channels modeled by either i.i.d or two-state Markov chain, there always exists an EBB characterization.

In the presence of V2V communication networks, the safety issues for VNs must be examined in a stochastic setting by discussing the likelihood of a system state entering a forbidden or unsafe region. Traditionally, this has been done using mean square concepts in which the variance of some important system state, such as inter-vehicle distance, remains bounded. Such a concept is also analogous to the notion of *stochastic safety in probability* [30]. The common feature of the above work is that they bound the likelihood of unsafe action occurring with a nonzero value, which still allows a finite probability for the system to be unsafe. This mean square safety or *stochastic safety in probability* criterion is not appropriate for many safety-critical systems such as smart transportation system where a small probability of danger can incur catastrophic failure. This paper suggests using a stronger notion of *almost sure safety* to assure the system state asymptotically goes to a safe equilibrium or a bounded safe set with probability one as time goes to infinity. In particular, *almost sure safety* in this paper refers to two strong notions of stochastic stability: *almost sure asymptotic stability* and *almost sure practical stability* [18].

Because of the challenge in modeling V2V channels, to the best of our knowledge, there is relatively little work that discusses the almost sure safety for VNs in the presence of realistic V2V channels. The most related work that assures similar safety property for networked systems is [33]. In [33], a H_∞ controller was developed to assure second moment stability which is a stronger notion than almost sure stability, for a linear networked system with a two-state Markov chain channel model. The approach used to guarantee safety in [33] relies on the fact that the linear system with the channel model can be formulated as a Markovian jump linear system (MJLS). Other recent work using MJLS approach to prove mean square stability includes [26], [27]. However, this MJLS approach cannot be applied to the vehicular systems with V2V channels for two reasons. First, the result based on MJLS framework is limited to the system with a single centralized controller, which is impractical for vehicular systems. Second, the state dependency in V2V channels introduces extra nonlinearity into vehicular systems that cannot be addressed in the framework of MJLS.

By using the EBB model that is functionally dependent on the vehicular state, this paper develops a distributed switching control scheme to assure *almost sure safety* for a chain of leader-follower systems. The leader-follower chain consists of a collection of leader-follower pairs that require each follower to manipulate its linear and angular velocity to achieve and maintain a desired separation and relative bearing. The information of the leader's bearing angle is transmitted over an exponentially bursty channel, which is accessed by a directional antenna mounted on each leading vehicle in the chain [17].

The results in this paper add to the prior literature on bearing-only leader-follower systems. Recent work in leader-follower systems [11], [24], [34], [36] studied bearing-only systems under the assumption of perfect or bounded measurements. In [36], the safety of leader-follower formation systems were examined by analyzing the disturbance amplification within the formations under perfect information. The studies in [11], [24] showed that bearing angles were important in addressing the localization problem in multi-agent systems when perfect information is available. This was extended in [34] to deal with uncertain measurements in a known bounded set. The present paper, on the other hand, considered the scenario that the leader's bearing angle was not perfectly known and the uncertainty of the information stochastically changed over time. This stochastic uncertainty results from *deep fading* in V2V communication and is prone to destabilizing a leader-follower chain.

This stochastic uncertainty prevents each leader-follower subsystem from maintaining the formation safely. The cascaded structure of the leader-follower chain amplifies such uncertainty from upper system to the lower system, and therefore leads to catastrophic failure for the entire system. This paper proposes two switching rules to recover the safe-behavior of the leader-follower chain by adaptively selecting local controller in response to the changes of channel state, and by enforcing the upper systems to constrain their control actions as a function of the lower system's states. Sufficient conditions are provided for each vehicle to decide which controller is placed in the feedback loop to assure *almost sure asymptotic stability* and *almost sure practical stability* for the entire leader-follower chain.

The layout of this paper is as follows. Section II introduces mathematical notations. Section III provides a system description and problem formulation. After that, Section IV discusses the main results. Then, Section V presents the simulation results of a leader-follower chain with four vehicles. Finally, Section VI concludes the paper.

II. Mathematical Preliminaries

Let $\mathbb{Z}$ and $\mathbb{R}$ denote the set of integers and real numbers, respectively. Let $\mathbb{Z}_+$ and $\mathbb{R}_+$ denote the set of non-negative integers and real numbers, respectively. Let $\mathbb{R}^n$ denote the n-dimensional Euclidean vector space. The ∞-norm on the vector $x \in \mathbb{R}^n$ is $|x| = \max |x_i| : 1 \leq i \leq n$, and the corresponding induced matrix norm is $\|A\| = \max_{1\leq i\leq n} \sum_{j=1}^{n} |A_{ij}|$. Let $f(t) \in \mathbb{R}^n$ denote the value that function f takes at time $t \in \mathbb{R}$. Let $\{\tau_k\}_{k=0}^{\infty}$ denote a strictly monotonically increasing sequence with $\tau_k \in \mathbb{R}_+$ for all $k \in \mathbb{Z}_+$ and $\tau_k < \tau_{k+1}$. Then, $f(\tau_k)$ denotes the value of function f at time τ_k. For simplicity, we let $f(k)$ denote $f(\tau_k)$ if its meaning is clear in the context. The left-hand limit at $\tau_k \in \mathbb{R}_+$ of a function $f(\cdot) : \mathbb{R} \to \mathbb{R}^n$ is denoted by $f(k^-)$. Similarly, the right-hand limit of the function $f(k)$ is denoted by $f(k^+)$.

Consider a continuous-time random process $\{x(t) \in \mathbb{R}^n : t \in \mathbb{R}_+\}$ whose sample paths are right-continuous and satisfy the following differential equation:

$$\dot{x}(t) = f\left(x(t), u(t), w(t), d(t)\right) \tag{1}$$

where $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}^m$ is a control input, $d(t)$ is an external $\mathscr{L}_\infty$ disturbance with $|d(t)|_{\mathscr{L}_\infty} = D$ and $w(t)$ is a jump process

$$w(t) = \sum_{\ell=1}^{\infty} w_\ell \delta(t - \tau_\ell) \tag{2}$$

in which $\{w_\ell, \ell \in \mathbb{Z}_+\}$ is a Markov process describing the ℓth jump's size at jump instants $\{\tau_\ell\}_{\ell=1}^{\infty}$. The expectation of this stochastic process at time t will be denoted as $\mathbb{E}(x(t))$.

Let x^* be the equilibrium of system (1) with $f(x^*, 0, 0, 0) = 0$. The system in (1), (2) is said to be *almost-surely asymptotically stable* with respect to x^*, if

$$\lim_{t\to\infty} \Pr\left\{\sup_t |x(t)| \to x^*\right\} = 1$$

Given a constant positive $\Delta^* \in \mathbb{R}_+$, let $\Omega(\Delta^*)$ be a bounded set defined as $\Omega(\Delta^*) = \{x \in \mathbb{R}^n | |x - x^*| \leq \Delta^*\}$. The system in (1), (2) is said to be *almost-surely practical stable* with respect to $\Omega(\Delta^*)$, if there exists $\Delta > 0$ with $\Delta^* > \Delta$ such that if $|x(0) - x^*| \leq \Delta$, then

$$\lim_{t\to\infty} \Pr\left\{\sup_t |x(t)| \in \Omega(\Delta^*)\right\} = 1$$

The system in (1), (2) is *almost sure safe* if it is *almost surely asymptotically stable* with respect to equilibrium x^* or *almost surely practical stable* with respect to set $\Omega(\Delta^*)$. x^* is called safe equilibrium, and the states in set $\Omega(\Delta^*)$ are safe states.

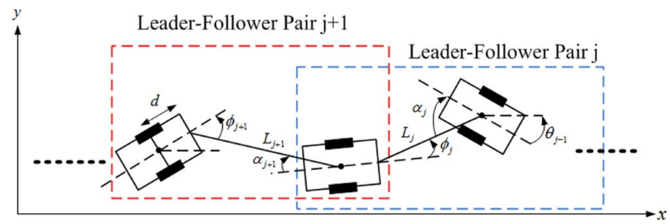

Fig. 1. A cascaded formation of nonholonomic vehicular system.

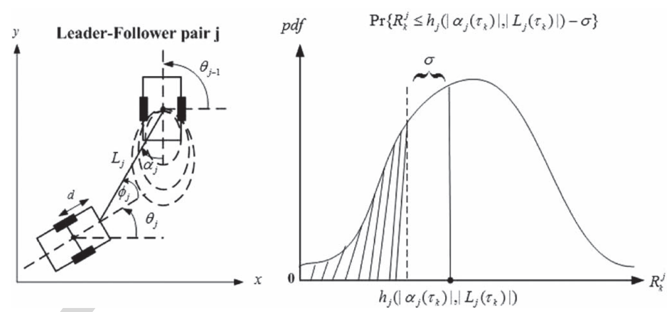

Fig. 2. Exponential Bounded Burstiness (EBB) Model for directional wireless channel.

III. System Description

A. System Model

Fig. 1 shows a string formation of N mobile robots. For each mobile robot, we consider the following kinematic model:

$$\dot{x}_i = v_i \cos(\theta_i),\ \dot{y}_i = v_i \sin(\theta_i),\ \dot{\theta}_i = \omega_i,\ i = 0, 1, \ldots, N-1 \tag{3}$$

where $(x_i(t), y_i(t))$ denotes the vehicle i's position at time $t \in \mathbb{R}_+$, $\theta_i(t)$ is the orientation of the vehicle relative to the x axis at time t. v_i and ω_i are the vehicle's speed and angular velocity that represent the control input.

As shown in Fig. 1, the cascaded formation with N mobile robots consists of $N-1$ leader-follower pairs. In each leader-follower pair j, we assume that the leader can directly measure its relative bearing angle α_j to the follower. Similarly, the follower can measure its bearing angle ϕ_j to the leader. Both of the vehicles are able to measure the relative distance L_j. What is not directly known to the follower is the relative bearing angle α_j. In this paper, we consider the case when information about leader's bearing angle α_j is transmitted over a wireless channel. The channel is accessed through a directional antenna whose radiation pattern is shown in Fig. 2.

The control objective of the cascaded formation is to have the follower in each leader-follower pair regulate its speed and angular velocity to achieve and maintain a desired distance and bearing angle. Let L_{d_j} and α_{d_j} denote the desired inter-vehicle distance and relative bearing angle, respectively, in the jth leader-follower pair. By using the similar technique in [12], the time rate of change of the relative distance L_j and leader's relative bearing angle α_j can be obtained as follows:

$$\begin{aligned}\dot{L}_j &= v_{j-1}\cos\alpha_j - v_j\cos\phi_j - d\omega_j\sin\phi_j\\ \dot{\alpha}_j &= \frac{1}{L_j}(-v_{j-1}\sin\alpha_j - v_j\sin\phi_j + d\omega_j\cos\phi_j) + \omega_{j-1}\end{aligned} \tag{4}$$

where d is the distance from the vehicle's center to its front.

Remark III.1: Note that $d > 0$ is a parameter for the vehicle and the relative distance measurement always satisfies $L_j \geq d$. When $d = 0$ (or if L_j is a distance from leader's center to follower's center), system (4) has a singular point at $L_j = 0$ and the state feedback linearization method in (8) cannot be applied due to [5].

B. Information Structure

As discussed in the previous section, the leader's bearing angle α_j in each leader-follower pair must be transmitted to the follower over a wireless channel. In this regard, the information about α_j that is available to the follower is limited by the following two constraints,

- The state measurement $\alpha_j(t)$ is only taken at a sequence of time instants $\{\tau_k\}_{k=0}^{\infty}$ that satisfy $\tau_k < \tau_{k+1}, \ k = 1, 2, \ldots, \infty$.
- The sampled data $\alpha_j(\tau_k)$ is quantized with a finite number of bits $\overline{R}_j$, and is transmitted over an unreliable wireless channel with only first $R_j(k)$ bits ($R_j(k) \leq \bar{R}_j$) received at the follower.

At the kth sampling time instant, the triple $\{\hat{\alpha}_j(k^-), U_j(k), c_j(k)\}$ characterizes the information structure of the leader's bearing angle $\alpha_j(\tau_k)$ at the leader side. Assume that the measurement $\alpha_j(\tau_k)$ lies in an interval $[-U_j(k) + \hat{\alpha}_j(k^-), U_j(k) + \hat{\alpha}_j(k^-)]$ with $\hat{\alpha}_j(k^-)$ representing the "center" of the interval and $U_j(k)$ representing the length of the interval. The codeword $c_j(k) = \{b_{jl}(k)\}_{l=1}^{\bar{R}_j}$ consists of bits $b_{jl}(k) \in \{-1, 1\}$, and is constructed by truncating the first $\overline{R}_j$ bits of the following infinite length of bits:

$$\left\{ \{b_{jl}(k)\}_{l=1}^{\infty} \in \{-1,1\}^{\infty} | \alpha_j(\tau_k) = \hat{\alpha}_j(k^-) + U_j(k) \sum_{l=1}^{\infty} \frac{1}{2^j} b_{jl}(k) \right\}.$$

This corresponds to a uniform quantization of the sampled state within the interval $[-U_j(k) + \hat{\alpha}_j(k^-), U_j(k) + \hat{\alpha}_j(k^-)]$ with $\overline{R}_j$ number of bits.

We assume that the follower only successfully receives the first $R_j(k)$ bits in the codeword $c_j(k)$. The information structure at the follower side is another triple $\{\hat{\alpha}_j(k), U_j(k), \hat{c}_j(k)\}$ with $\hat{c}_j(k) = \{b_{jl}\}_{l=1}^{R_j(k)}$ and $\hat{\alpha}_j(k)$ being constructed as follows:

$$\hat{\alpha}_j(k) = \hat{\alpha}_j(k^-) + U_j(k) \sum_{l=1}^{R_j(k)} \frac{1}{2^j} b_{jl}(k) \tag{5}$$

$\hat{\alpha}_j(k)$ is an estimate of the leader's bearing angle $\alpha_j(k)$ at time instant τ_k.

In order to reconstruct the estimate $\hat{\alpha}_j(k)$, it is necessary to synchronize the leader and follower in the sense that they have the same information structure. We assume a noiseless feedback channel, with each successfully received bit being acknowledged to the leader. This allows one to ensure that the information structures are synchronized between the leader and follower. The follower then uses the estimated bearing angle $\hat{\alpha}_j(k)$, and the measured inter-vehicle distance L_j, to select its speed, v_j, and angular velocity ω_j to achieve the control objective.

C. Wireless Channel

As shown in Fig. 2, the leading vehicle in each pair uses a directional antenna to access the V2V wireless channel. We assume the V2V channels are free of interference from other leader-follower pairs, but the channel does exhibit deep fading. Deep fades occur when the channel gain drops below a threshold and stays below that threshold level for a random interval of time. In mobile communication networks, the wireless channel exhibits deep fades and has memory of its past channel states. Such time varying channel will increase the likelihood of a burst of packet dropouts. This fact has been recently found to be a fundamental factor that destabilizes networked control systems [21].

Outage probability is a well studied performance metric for wireless fading channel [19], [39]. In wireless networks, outage probability is commonly defined as the probability of the signal to noise ratio of a received signal being less than the threshold for reliable reception. This metric is closely related to the concept of EBB which was introduced in [41]. In [41], the EBB model was used to bound the likelihood of a string of consecutive dropouts for a single communication link. This model has been further explored in [9], [20] to study the burstiness of a communication network with multiple nodes. Thus, this paper also adopts the EBB model to characterize channel outages. The definition of the EBB model is

Definition III.2: Let $R_j(k)$ denote a random variable that characterizes the possible number of successfully decoded bits over time interval $[\tau_k, \tau_{k+1})$, then random process $\{R_j(k)\}$ is EBB with continuous, positive and monotonically decreasing functions $(h(\cdot,\cdot), \gamma(\cdot,\cdot))$, if the probability of successfully decoding $R_j(k)$ bits at each sampling time τ_k satisfies

$$\Pr\left\{R_j(k) \leq h\left(|\alpha_j(\tau_k)|, |L_j(\tau_k)|\right) - \sigma\right\} \leq e^{-\gamma(|\alpha_j(\tau_k)|, |L_j(\tau_k)|)\sigma} \tag{6}$$

for $|\alpha_j(\tau_k)| \leq \pi/2$ and $\sigma \in [0, h(|\alpha_j(\tau_k)|, |L_j(\tau_k))|]$ with

$$\Pr\left\{R_j(k) = 0\right\} = 1 \tag{7}$$

for $|\alpha_j(\tau_k)| > \pi/2, \ \forall\, k \in \mathbb{Z}_+$.

The (6) and (7) characterize the fact that if the follower vehicle is out of the antenna's radiation scope, i.e., $|\alpha_j(\tau_k)| > \pi/2$, then the communication link between the vehicles is broken. If the vehicle is within the scope, i.e., $|\alpha_j(\tau_k)| \leq \pi/2$, the probability of having a string of dropouts is exponentially bounded.

Fig. 2 shows a distribution of channel state $R_j(k)$ at time τ_k. The function $h(|\alpha_j|, |L_j|)$ in the EBB model is a threshold that partitions channel state space (horizontal axis in the figure) into high bit rate region (right from $h(|\alpha_j|, |L_j|)$) and low bit rate region (left to $h(|\alpha_j|, |L_j|)$). This threshold is a decreasing function of the absolute value of the current formation states $L_j(\tau_k)$ and $\alpha_j(\tau_k)$. It models the impact of the path loss on the data rate [39]. In the low bit rate region, $\sigma \in [0, h(|\alpha_j(\tau_k)|, |L_j(\tau_k))|]$ characterizes the *dropout burst length*. The function $\gamma(|\alpha_j|, |L_j|)$ is the exponent in the exponential bound and characterizes how fast the probability of a bursty dropout decays as a function of *dropout burst length* within the low bit rate region. This decay rate is also a decreasing function of $L_j(\tau_k)$ and $\alpha_j(\tau_k)$. It models that fact that the likelihood of having a bursty dropout in V2V channel increases with inter-vehicle distance and relative bearing angle. Thus, the EBB characterization explicitly models state dependency and bursty dropouts for a realistic V2V channel.

The following lemma shows that the EBB characterization in (6), (7) can be used to describe a wide range of channel models that include traditional i.i.d models [39] as well as two-state Markov chain models [42].

Lemma III.3: Consider a bit stream $\{b_{jl}(k)\}_{l=1}^{\overline{R}_j}$ that is sequentially transmitted over the fading channel, define a corresponding random process $\{X_{jl}(k)\}_{l=1}^{\overline{R}_j}$ with random variable $X_{jl}(k) \in \{0, 1\}$ taking value 1 when the corresponding bit successfully decoded and 0 otherwise. For fading channels that are modeled by either a i.i.d. process [39] or a two-state Markov process [42], i.e., $\{X_{jl}(k)\}_{l=1}^{\overline{R}_j}$ is a i.i.d. process or a two-state Markov process, there always exists a EBB characterization in (6), (7) with $R_j(k) = \sum_{l=1}^{\overline{R}_j} X_{jl}(k)$.

Proof: The proof is provided in the Appendix. ■

What should be apparent from the EBB model is that we are explicitly accounting for the relationship between bursty channel state $(R_j(k))$ and formation configuration. A major goal of this paper is to exploit that relationship in deciding how to switch between different controllers to assure almost sure performance.

D. Distributed Switching Control

In this paper, the control objective is to steer the cascaded vehicular system shown in Fig. 1 to a sequence of desired distances $\{L_{d_j}\}_{j=1}^{N-1}$ and bearing angles $\{\alpha_{d_j}\}_{j=1}^{N-1}$ in a distributed fashion, and then maintain around those set-points.

At each time instant $\{\tau_k\}_{k=0}^{\infty}$, the follower of each leader-follower pair switches among a group of controller gains to regulate its velocity and angular velocity to achieve the control objective. Let $K(k) := \{K_{\alpha_j}(k), K_{L_j}(k)\}$ denote the controller gain pair used for leader-follower pair j at time instant τ_k. These controller gains are selected from one pair of a collection of values $\mathscr{K}_j = \{K_{j_1}, K_{j_2}, \dots, K_{j_M}\}$. Recall that the dynamic of formation configuration is equation (4), we use standard input to state feedback linearization to generate the control input

$$\begin{bmatrix} v_j \\ \omega_j \end{bmatrix} = \begin{bmatrix} -\cos\phi_j & -L_j \sin\phi_j \\ -\frac{\sin\phi_j}{d} & \frac{L_j}{d}\cos\phi_j \end{bmatrix} \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha_j}\right) \end{bmatrix} \tag{8}$$

over the time interval $[\tau_k, \tau_{k+1})$. The variable $\hat{\alpha}_j(t)$ is a continuous function over $[\tau_k, \tau_{k+1})$, and satisfies the following initial value problem:

$$\dot{\hat{\alpha}}_j = K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha}_j\right),\ \hat{\alpha}_j(\tau_k) = \hat{\alpha}_j(k) \tag{9}$$

where the estimate $\hat{\alpha}_j(k)$ is obtained from (5). With this control, the inter-vehicle distance L_j and bearing angle α_j satisfy the following differential equations over $[\tau_k, \tau_{k+1})$:

$$\begin{bmatrix} \dot{L}_j \\ \dot{\alpha}_j \end{bmatrix} = \begin{bmatrix} \cos\alpha_j & 0 \\ \frac{-\sin\alpha_j}{L_j} & 1 \end{bmatrix} \begin{bmatrix} v_{j-1} \\ \omega_{j-1} \end{bmatrix} + \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha}_j\right) \end{bmatrix} \tag{10}$$

for all $k = 1, 2, \dots, \infty$.

The (9), (10) represent the closed-loop system for the leader-follower pair j and can be viewed as an example of a jump nonlinear system given in (1), (2). The $\mathscr{L}_\infty$ disturbance in the jth leader-follower system is $[v_{j-1}, \omega_{j-1}]$. The estimate of the bearing angle $\hat{\alpha}_j$ forms a jump process with jumps occurring at discrete time instants $\{\tau_k\}_{k=1}^{\infty}$. As shown in (5), the magnitude of the jump at each time instant is stochastically governed by the length of the uncertainty interval $U_j(k)$ and the number of received bits $R_j(k)$. Such jump process significantly impacts the formation performance of the cascaded system by pushing the formation state away from the equilibrium, which in turn leads to deep fades with a high probability. In the next section, we will show how to reconfigure the local controller gain in response to the changes of $U_j(k)$ and $R_j(k)$ such that almost sure performance is assured.

It is apparent from Fig. 1 that vehicle j for $j = 1, 2, \dots, N-2$ plays a leader in leader-follower pair $j+1$ as well as a follower in leader-follower pair j. In this regard, vehicle j could observe the full state α_{j+1} of the leader-follower subsystem $j+1$ because it serves the leadership in that system. By observing the behavior of the following vehicle, vehicle j for $j = 1, 2, \dots, N-1$ can adjust its controller gain to overcome large overshoots in the following system. Such cooperative control strategy lessens the amplification on the disturbance from the upper leader-follower systems to the lower systems.

IV. Main Results

This paper's main results consist of two parts regarding the *safe* behavior of inter-vehicle distance L_j and bearing angle α_j for each leader-follower pair. Specifically, "safe" means that the vehicle does not collide with each other and the bearing angle is regulated to stay in a specified bounded set almost surely. The first part of the results provides a sufficient condition under which the inter-vehicle distance L_j for $j = 1, 2, \dots, N-1$ is almost surely convergent to a compact invariant set regardless of the changes on channel state. Furthermore, we show that the inter-vehicle distance is almost surely convergent to the desired separation $L_{d_j},\ j = 1, 2, \dots, N-1$ if the bearing angle $\alpha_j,\ j = 1, 2, \dots, N-1$ is almost surely convergent. The second part of the results derive sufficient conditions for the almost sure asymptotic stability and practical stability for the bearing angle $\alpha_j,\ j = 1, 2, \dots, N-1$.

In the main results, we use the fact that the leader's action in each leader-follower pair can be constrained as a function of the following system's state to assure the stability for the whole leader-follower system. Proposition IV.1 provides an explicit characterization of the bound on the leader's action, as well as a distributed way to achieve that bound. Using the results from Proposition IV.1, one can easily prove the first main result in this paper (Lemma IV.5), i.e., the convergence of inter-vehicle distance since the distance is measurable to both leader and follower. The more challenging and interesting part of the results is to guarantee almost sure stability for the bearing angle α_j, which is presented in Section IV-B.

The following Proposition is provided to assure the control input from upper leader-follower subsystem is bounded as a function of state estimates of the bottom system. The proof is provided in the Appendix.

Proposition IV.1: Consider the closed-loop system in (9), (10), let $d \geq 1$, if there exists a sequence of controller gains $\{K_j(k)\}_{k=0}^{\infty}$, $K_j(k) = \{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$ such that for given monotonically increasing functions $W_j(\cdot): \mathbb{R}_+ \to \mathbb{R}_+$, $j = 1, 2, \ldots, N-1$, the following inequality holds for all $k = 0, 1, \ldots, \infty$:

$$\max\left\{\tilde{L}_{j,\max}, K_{\alpha_j}(k)\left|\tilde{\alpha}_j(k)\right|\right\} \leq \frac{W_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right)}{\left(1+M_{L_j}(k)\right)} \tag{11}$$

where

$$\begin{aligned}
\tilde{L}_{j,\max} &= K_{L_j}(k)\left|\tilde{L}_j(k)\right| e^{K_{L_j}(k)T_k} \\
&\quad + W_{j-1}\left(\left|\tilde{\alpha}_j(k)\right|\right)\left(e^{K_{L_j}(k)T_k} - 1\right) \\
M_{L_j}(k) &= \max\left\{\overline{L}_j(\tau_k), \overline{L}_j(\tau_{k+1})\right\} \\
\overline{L}_j(t) &= \left(L_{d_j} + \frac{W_{j-1}\left(\left|\tilde{\alpha}_j(k)\right|\right)}{K_{L_j}(k)}\right)\left(1 - e^{-K_{L_j}(k)(t-\tau_k)}\right) \\
&\quad + L_j(k)e^{-K_{L_j}(k)(t-\tau_k)} \\
\tilde{\alpha}_j(k) &= \alpha_{d_j} - \hat{\alpha}_j(k), \quad \tilde{L}_j(k) = L_{d_j} - L_j(k)
\end{aligned}$$

then

$$\sup_t \left|\left[\begin{matrix} v_j(t) \\ \omega_j(t) \end{matrix}\right]\right| \leq W_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right), t \in [\tau_k, \tau_{k+1}). \tag{12}$$

Because of inequality (12), each leader-follower subsystem j in (10) can bound the external disturbance $[v_{j-1}, \omega_{j-1}]$ by observing its local state estimate $\tilde{\alpha}_j$ at each time instant τ_k. Meanwhile, the subsystem $j-1$ can select its controller gain so that the control input $[v_{j-1}, \omega_{j-1}]$ satisfies the bound in inequality (12) because the estimate of bearing angle $\tilde{\alpha}_j$ is always available to subsystem $j-1$. Such property provides a basis for designing a distributed and cooperative switching law to assure the stability for the whole formation system.

Remark IV.2: Functions $W_j(\cdot)$ are upper bounds on the control inputs of upper leader-follower system and the values of $W_j(\cdot)$ at each time instant τ_k can also be seen as feedback signals from the bottom system. Such feedback signals directly constrain the magnitude of control input from upper system, so that the disturbances are not amplified from upper system to bottom system.

Remark IV.3: The inequality (11) could be viewed as a switching rule for the leader-follower pair j to react to the changes on system $j+1$'s bearing angle. The switching rule applied over each time interval $[\tau_k, \tau_{k+1})$ is feasible because it is only based on the information that is available at time τ_k.

With the validity of Proposition IV.1, the following corollary characterizes the propagated bound on the external inputs of the leader-follower chain as a function of the bearing angle's estimate in each leader-follower pair.

Corollary IV.4: Suppose the hypothesis of Proposition IV.1 holds then

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0 \circ \tilde{W}_1 \circ \cdots \circ \tilde{W}_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right) \tag{13}$$

where $v_0(k)$ and $\omega_0(k)$ are the speed and angular velocity of the first vehicle in the chain, and

$$\tilde{W}_j(\cdot) := \frac{1}{\left(1+M_{L_j}(k)\right)K_{\alpha_j}(k)} W_j(\cdot),\ j = 1, \ldots, N-2.$$

Proof: Consider the first leader-follower pair, the Proposition IV.1 implies

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0\left(\left|\tilde{\alpha}_1(k)\right|\right).$$

Since

$$\left(1+M_{L_1}(k)\right)K_{\alpha_1}(k)\left|\tilde{\alpha}_1(k)\right| \leq W_1\left(\left|\tilde{\alpha}_2(k)\right|\right)$$

holds due to inequality (11), then

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0 \circ \tilde{W}_1\left(\left|\tilde{\alpha}_2(k)\right|\right).$$

Repeating above procedure leads to the final conclusion (13). ■

A. *Almost Sure Convergence of Inter-Vehicle Distance L_j*

In this section, we present the first main result of this paper involving the almost sure convergence of inter-vehicle distance. First, the following lemma provides a sufficient condition on the controller gain K_{L_j}, under which one can show $L_j(t)$ converges at an exponential rate to an invariant set $\Omega_{\mathrm{inv,j}}$ centered at the desired inter-vehicle distance L_{d_j}, for $j = 1, 2, \ldots, N-1$ regardless of the change on channel state.

Lemma IV.5: Let the hypothesis of Proposition IV.1 hold, consider the system in (9), (10) with the selected controller gain $\{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$. If $K_{L_j}(k) > (W_j(|\tilde{\alpha}_j(k)|)/\rho(L_{d_j} - d))$ and $L_j(0) > d$, then for any sample path, $L_j(t) \geq d$ for all $t \in \mathbb{R}_+$ and there exists a finite time $\overline{T} > 0$ such that $L_j(t)$ enters and remains in the set

$$\Omega_{\mathrm{inv,j}} \equiv \left\{L_j \in \mathbb{R}_+ \left| \left|L_j - L_{d_j}\right| \leq \frac{W_j\left(\left|\tilde{\alpha}_j(k)\right|\right)}{\rho K_{L_j}(k)}\right.\right\}$$

for all $t \geq \overline{T}$ and any $\rho \in (0, 1]$.

Proof: Consider the function $V(L_j) = (1/2)(L_j - L_{d_j})^2$ and closed-loop state (10). Taking the directional derivative of V over time interval $[\tau_k, \tau_{k+1})$ one obtains

$$\begin{aligned}
\dot{V}(L_j) &= -K_{L_j}\left(L_j - L_{d_j}\right)^2 + \left(L_j - L_{d_j}\right) \cdot v_{j-1}\cos\alpha_j \\
&\leq -K_{L_j}(1-\rho)\left(L_j - L_{d_j}\right)^2 - \rho \cdot K_{L_j}\left(L_j - L_{d_j}\right)^2 \\
&\quad + \left|L_j - L_{d_j}\right| W_j\left(\left|\tilde{\alpha}_j(k)\right|\right)
\end{aligned}$$

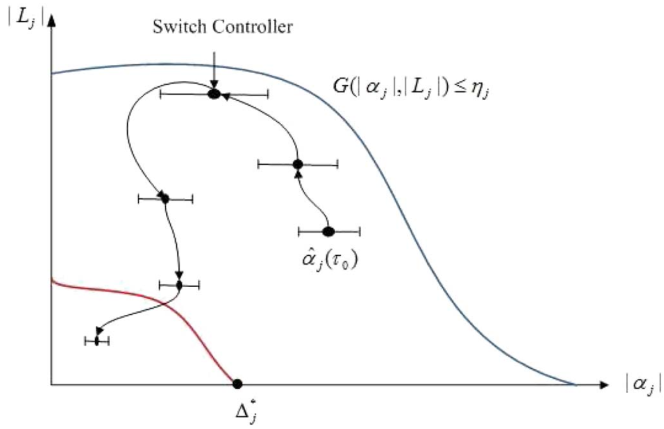

Fig. 3. Partition of formation state space.

for any $\rho \in (0,1]$. The last inequality holds because of Proposition IV.1. When $|L_j - L_{d_j}| \geq W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}$, the following dissipative inequality holds:

$$\begin{aligned}\dot{V}(L_j) \leq & -K_{L_j}(1-\rho)\left(L_j - L_{d_j}\right)^2 \\ = & -2K_{L_j}(1-\rho)V(L_j). \end{aligned} \quad (14)$$

This implies that $V(L_j(t))$ is exponentially decreasing when the state $L_j(t)$ is outside the set $\Omega_{\text{inv, j}}$. Since $L_j(0) > d$, the feasible region outside the invariant set is $L_j(t) \geq (W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)) + L_{d_j} > d$. By inequality (14), it is clear that $L_j(t)$ converges to the set $\Omega_{\text{inv, j}}$ in finite time and $L_j(t) > d$ for all time since for any $L_j \in \Omega_{\text{inv, j}}$, it satisfies

$$L_j \geq -\frac{W_j\left(|\tilde{\alpha}_j(k)|\right)}{\rho K_{L_j}} + L_{d_j} > d.$$

Since the time interval $[\tau_k, \tau_{k+1})$ is selected arbitrarily, the conclusion holds for any $k \in \mathbb{Z}_+$. ■

Remark IV.6: Note that d is the distance from the center of the vehicle to the front of the vehicle. As shown in Fig. 1, $L_j(t) > d$ means that the two vehicles do not collide with each other.

Corollary IV.7: Consider closed-loop system in (9), (10), let the hypotheses of Proposition IV.1 and Lemma IV.5 hold. If the bearing angle α_j is almost surely convergent to α_{d_j} with $W_{j-1}(0) = 0,\ j = 1, 2, \ldots, N-1$, then the separation distance L_j almost surely converges to L_{d_j}.

Proof: From Lemma IV.5, one knows that the inter-vehicle separation converges to a invariant set with size of $W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)$. With $W_{j-1}(0) = 0, \quad j = 1, 2, \ldots, N-1$, and $\lim_{k\to\infty} \Pr\{\alpha_j(k) \to \alpha_{d_j}\} = 1$, it is easy to show that the event $\lim_{k\to\infty}(W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)) = 0$ occurs with probability one as time goes infinity, i.e., the separation $L_j(t)$ almost surely converges to L_{d_j}. ■

B. Almost Sure Asymptotic Stability and Practical Stability for Bearing Angle α_j

This section provides the second main result of this paper that assures almost sure asymptotic stability and almost sure practical stability for the bearing angle α_j. Fig. 3 shows the basic idea and results. Two types of sets are depicted in Fig. 3 with one enclosed by the blue solid curve, and the other one enclosed by the red dashed curve. The set enclosed by the blue solid curve represents the partition generated by inequality $G(|\alpha_j|, |L_j|) \leq \eta_j$ with associated threshold $\eta_j \in (0,1)$, which is shown in Lemma IV.9. The area enclosed by the red dashed curve characterizes the target set where the system trajectory will converge to almost surely. The size of the target set is characterized by Δ_j^*. The almost sure asymptotic stability result is interpreted as a special case when the target set contains only origin.

The main result states that the bearing angle α_j will almost surely converge to the target set if the system trajectory enters and remains in the set enclosed by the blue solid curve. To assure the invariance of the set enclosed by the blue solid curve, we adopt a switching control strategy to reconfigure the control gain for each leader-follower pair. Fig. 3 shows one possible evolution of the system trajectory α_j and L_j with the switching strategy. We use black dots to represent the estimates of the bearing angle $\hat{\alpha}_j(\tau_k)$ at each sampling time τ_k. A bar is used to characterize the uncertainty interval with the estimate $\hat{\alpha}_j(\tau_k)$ as its center. The length of bar can be viewed as an upper bound of the quantization error $|\alpha_j(\tau_k) - \hat{\alpha}_j(\tau_k)|$, and increases as the channel condition decreases. Therefore, the basic idea for switching is that when the system trajectory approaches the blue set's boundary with an increasing uncertainty length, an appropriate controller is re-selected to assure that the stochastic variation on the uncertainty length satisfies a super-martingale inequality, which guarantees the convergence of system states to the target set with probability one.

To be more specific about the main result, first, a dynamic quantization method is used to show that the quantization error $|\alpha_j(\tau_k) - \hat{\alpha}_j(\tau_k)|$ can be bounded by a recursively constructed sequence (Lemma IV.8). Then, a sufficient condition is presented to select controllers, under which the sequence (Lemma IV.9) and bearing angle estimate (Lemma IV.11) satisfy super-martingale like inequalities. Finally, the super-martingale inequality condition leads to the proof of almost sure asymptotic stability (Theorem IV.12) and practical stability (Theorem IV.14) for the bearing angle α_j.

Recall that $\{\alpha_j(k^-), U_j(k)\}_{k=0}^{\infty}$ characterizes the quantizer's state at each time instance τ_k. The following lemma gives a recursive construction for this sequence such that the quantization error remains bounded by some function of $U_j(k)$ for all $k \geq 0$. This bound is used to switch controllers to assure almost sure performance. Note that the technique used to prove the following lemma follows the pattern in dynamic quantization [6].

Lemma IV.8: Consider the closed-loop system in (9), (10), given the transmission time sequence $\{\tau_k\}_{k=0}^{\infty}$, and controller pairs $\{K_{L_j}(k), K_{\alpha_j}(k)\}_{k=0}^{\infty}$. Let $T_k = \tau_{k+1} - \tau_k$, let the hypothesis of Proposition IV.1 and Lemma IV.5 hold, the quantizer's initial state $\{\hat{\alpha}_j(0), U_j(0)\}$ is known to both leader and follower, and the initial state $\alpha_j(0) \in [-U_j(0), U_j(0)]$, $U_j(0) \leq \pi/2$. If the sequence $\{\alpha_j(k^-), U_j(k)\}_{k=0}^{\infty}$ is constructed by the following recursive equation:

$$U_j(k+1) = B_j(k)T_k + 2^{-R_j(k)}U_j(k) \quad (15)$$

$$\hat{\alpha}_j(k+1^-) = \left(\hat{\alpha}_j(k^+) - \alpha_{d_j}\right)e^{-K_{\alpha_j}(k)T_k} + \alpha_{d_j} \quad (16)$$

where

$$B_j(k) = \max\left\{\frac{1}{\min\left\{L_{j\min}, L_j(k)\right\}}, 1\right\} W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)$$
$$L_{j\min} = \left[-\tilde{L}_j(k) + \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)}\right] e^{-K_{L_j}(k)T_k} + L_{d_j} - \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)}$$
$$\tilde{L}_j(k) = L_{d_j} - L_j(k)$$

then the bearing angle $\alpha_j(k)$ for all $j = 1, 2, \ldots, N-1$ generated by system (9), (10) can be bounded as

$$\left|\alpha_j(k) - \hat{\alpha}_j(k^+)\right| \leq \overline{U}_j(k) \tag{17}$$

where $\overline{U}_j(k) = 2^{-R_j(k)}U_j(k)$ and $R_j(k)$ is the number of bits received over the time interval $[\tau_k, \tau_{k+1})$.

Proof: Let $e_j(t) = \alpha_j(t) - \hat{\alpha}_j(t)$ denote the estimation error. By inequality $d|e_j|/dt \leq |de_j/dt|$, the dynamic of $e_j(t)$ over time interval $[\tau_k, \tau_{k+1})$ is bounded by

$$\begin{aligned} \frac{d|e_j|}{dt} &\leq \left|\begin{bmatrix} -\frac{\sin\alpha_j}{L_j} & 1 \end{bmatrix}\begin{bmatrix} v_{j-1} \\ \omega_{j-1}\end{bmatrix}\right| \\ &\leq \left(\frac{1}{|L_j|} + 1\right)\left|\begin{bmatrix} v_{j-1} \\ \omega_{j-1}\end{bmatrix}\right| \\ &\leq \left(\frac{1}{|L_j|} + 1\right) W_{j-1}\left(|\tilde{\alpha}_j(k)|\right). \end{aligned} \tag{18}$$

The last inequality holds because of Proposition IV.1. Since $\dot{L}_j \geq K_{L_j}(k)(L_{d_j} - L_j) - |v_{j-1}| \geq K_{L_j}(k)(L_{d_j} - L_j) - W_{j-1}(|\tilde{\alpha}_j(k)|)$, using Gronwall-Bellman inequality over $[\tau_k, \tau_{k+1})$ yields

$$L_j(t) \geq \left[L_j(\tau_k) - \left(L_{d_j} - \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)}\right)\right] e^{-K_{L_j}(k)(t-\tau_k)} + L_{d_j} - \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)}\Bigg).$$

Since $L_{d_j} \geq (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k))$ and $L_j(t) > d$ from Lemma IV.5, we know $\inf_{\tau_k \leq t < \tau_{k+1}} L_j(t)$ is obtained at either $t = \tau_k$ or $t = \tau_{k+1}$

$$L_j(t) \geq \inf_{\tau_k \leq t < \tau_{k+1}} L_j(t) = \min\left\{L_{j\min}, L_j(\tau_k)\right\} \tag{19}$$

where $L_{j\min} = [-\tilde{L}_j(k) + (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k))]e^{-K_{L_j}(k)T_k} + (L_{d_j} - (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k)))$. By inequality (19), (18) is rewritten as

$$\frac{d|e_j|}{dt} \leq \left(\frac{1}{\min\left\{L_{j\min}, L_j(\tau_k)\right\}} + 1\right) W_{j-1}\left(|\tilde{\alpha}_j(k)|\right).$$

Solving above differential inequality, we have

$$|e_j(t)| \leq \underbrace{\left(\frac{1}{\min\left\{L_{j\min}, L_j(\tau_k)\right\}} + 1\right) W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}_{B_j(k)} \times (t - \tau_k) + |e_j(\tau_k)|.$$

For $t \to \tau_{k+1}$, one can get $|e(k+1^-)| \leq B_j(k)T_k + |e_j(k)|$. And assume that $|e_j(k)| \leq \overline{U}_j(k)$, then $|e(k+1^-)| \leq B_j(k)T_k + \overline{U}_j(k)$. We know that

$$\begin{aligned} \left|e(k+1^+)\right| &\leq 2^{-R_j(k+1)}\left|e(k+1^-)\right| \\ &\leq 2^{-R_j(k+1)}\left(B_j(k)T_k + \overline{U}_j(k)\right). \end{aligned}$$

From (15) and $\overline{U}_j(k+1) = 2^{-R_j(k+1)}U_j(k+1)$, we have $|e(k+1^+)| \leq \overline{U}_j(k+1)$. The (16) holds by simply considering the solution to the ODE $\dot{\tilde{\alpha}}_j = -K_{\alpha_j}\tilde{\alpha}_j$ with initial value $\tilde{\alpha}_j = \alpha_{d_j} - \hat{\alpha}_j(k^+)$. ■

With Lemma IV.8, the following lemma provides a sufficient condition on the selection of controller gains that assures a super-martingale like property for the sequence $\{U_j(k)\}_{k=0}^{+\infty}$, $j = 1, 2, \ldots, N-1$.

Lemma IV.9: Consider the closed-loop system in (9), (10). Let

$$G\left(|\alpha_j|, |L_j|\right) = e^{-h(|\alpha_j|,|L_j|)\gamma(|\alpha_j|,|L_j|)}\left(1 + h\left(|\alpha_j|, |L_j|\right)\gamma\left(|\alpha_j|, |L_j|\right)\right)$$

be a non-negative, monotonically increasing function with respect to $|\alpha_j|$ and $|L_j|$ respectively. If there exists a sequence of controller gains $\{K_{L_j}(k), K_{\alpha_j}(k)\}_{k=0}^{\infty}$ with $K_j(k) = \{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$ for all $k \in \mathbb{Z}$ such that the Proposition IV.1 and following inequality hold for any $\eta_j \in (0, 1)$

$$\begin{aligned} &G\left(\overline{\alpha}_j(k+1), \overline{L}_j(k+1)\right) \leq \eta_j \\ &\overline{\alpha}_j(k+1) = \left|-\tilde{\alpha}_j(k)e^{-K_{\alpha_j}(k)T_k} + \alpha_{d_j}\right| + B_j(k)T_k + \overline{U}_j(k) \\ &\overline{L}_j(k+1) = L_{d_j} + \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)} - \left[\tilde{L}_j(k) + \frac{W_{j-1}\left(|\tilde{\alpha}_j(k)|\right)}{K_{L_j}(k)}\right] e^{-K_{L_j}(k)T_k} \end{aligned} \tag{20}$$

then

$$\mathbb{E}\left[\overline{U}_j(k+1)|\overline{U}_j(k)\right] \leq \eta_j\overline{U}_j(k) + \eta_j B_j(k)T_k, \qquad \forall\, k \in \mathbb{Z}_+. \tag{21}$$

Proof: Consider the sequence $\{\overline{U}_j(k)\}_{k=0}^{\infty}$ that satisfies (15) in Lemma IV.8, using the argument in [16], one has $\mathbb{E}[\overline{U}_j(k+1)|\overline{U}_j(k)] \leq G(|\alpha_j(k+1)|, |L_j(k+1)|)(B_j(k)T_k + \overline{U}_j(k))$. Let $G(|\alpha_j(k+1)|, |L_j(k+1)|) \leq \eta_j$, we have final conclusion (21) hold. In order to select the controller gain $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ for the time interval $[\tau_k, \tau_{k+1})$, the selection decision is made based only on the information at time instant τ_k. Thus, we further bound the state $|\alpha_j(k+1)|$ and $|L_j(k+1)|$ by considering $|e_j(k+1^-)| = |\alpha_j(k+1^-) - \hat{\alpha}_j(k+1^-)| \leq B_j(k)T_k + \overline{U}_j(k)$. Since $\alpha_j(k+1) = \alpha_j(k+1^-)$, we have

$$\begin{aligned} |\alpha_j(k+1)| &\leq \left|\hat{\alpha}_j(k+1^-)\right| + B_j(k)T_k + \overline{U}_j(k) \\ &\leq \left|\alpha_{d_j} - \left(\alpha_{d_j} - \hat{\alpha}_j(k)\right)e^{-K_{L_j}(k)T_k}\right| \\ &\quad + B_j(k)T_j + \overline{U}_j(k) \\ &\triangleq \overline{\alpha}_j(k+1). \end{aligned}$$

Similarly, one also has $|L_j(k+1)| \leq \overline{L}_j(k+1) = (L_{d_j} + (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k)))(1 - e^{-K_{L_j}(k)T_k}) + L_j(k)e^{-K_{L_j}(k)T_k}$

that is shown in Proposition IV.1. Since the function $G(|\alpha_j(k+1)|, |L_j(k+1)|)$ is a monotonically increasing function w.r.t $|\alpha_j(k+1)|$ and $|L_j(k+1)|$, and then if $G(\overline{\alpha}_j(k+1), \overline{L}_j(k+1)) \leq \eta_j$, we have $G(|\alpha_j(k+1)|, |L_j(k+1)|) \leq \eta_j$, then the final conclusion holds. ∎

Remark IV.10: Function $G(\alpha_j, L_j)$ in condition (20) is directly related to the EBB model, and it generates a partition of the formation state space as shown in Fig. 3. Each partition associates with a threshold η_j that characterizes the convergent rate for the uncertainty set. The aim of switching control strategy is to guarantee that the condition (20) holds with a selected η_j.

Similar to Lemma IV.9, the following lemma shows that the sequence of the estimate of bearing angle $\{\tilde{\alpha}_j(k)\}_{k=0}^{\infty}$ for $j = 1, 2, \ldots, N-1$ satisfies a super-martingale like property as sequence $\{\overline{U}_j(k)\}_{k=0}^{\infty}$ does.

Lemma IV.11: Consider the system in (9), (10), given a sequence of controller pair $\{K_{L_i}(k), K_{\alpha_i}(k)\}_{k=0}^{\infty}$ with each $\{K_{L_i}(k), K_{\alpha_i}(k)\}$ selected at time instants $\{\tau_k\}_{k=0}^{\infty}$ and $\{K_{L_i}(k), K_{\alpha_i}(k)\} \in \mathscr{K}_i$. Let $K_{\alpha_i}^* = \min\{K_{\alpha_i} | K_{\alpha_i} \in \mathscr{K}_i\}$ and let $\mathscr{I}_k$ denote the information available at time instant τ_k, then we have

$$\mathbb{E}\left[|\tilde{\alpha}_i(k+1)| \,|\mathscr{I}_k\right] \leq e^{-K_{\alpha_i}^* T_k} |\tilde{\alpha}_i(k)| + \left(B_i(k)T_k + \overline{U}_i(k)\right)(1 - 2^{-\bar{R}_i}).$$

Proof: Consider the time interval $[\tau_k, \tau_{k+1})$, by (9), we know that $\dot{\hat{\alpha}}_j = K_{\alpha_j}(k)(\alpha_{d_j} - \hat{\alpha}_j(t))$ with initial value $\hat{\alpha}_j(\tau_k)$. Therefore, let $\tilde{\alpha}_j(k) = \alpha_{d_j} - \hat{\alpha}_j(k)$, we have $\tilde{\alpha}_j(k+1^-) = e^{-K_{\alpha_j}(k)T_k}\tilde{\alpha}_j(k)$. Let $E_j(k+1) = \tilde{\alpha}_j(k+1) - \tilde{\alpha}_j(k+1^-)$, then $\tilde{\alpha}_j(k+1) = e^{-K_{\alpha_j}(k)T_k}\tilde{\alpha}_j(k) + E_j(k+1)$. Let $K_{\alpha_j}^* = \min\{K_{\alpha_j} | K_{\alpha_j} \in \mathscr{K}_j\}$, then

$$|\tilde{\alpha}_j(k+1)| \leq e^{-K_{\alpha_j}^* T_k} |\tilde{\alpha}_j(k)| + |E_j(k+1)| \quad (22)$$

The term $|E_j(k+1)|$ can be bounded as $|E_j(k+1)| \leq (B_j(k)T_k + \overline{U}_j(k))(1 - 2^{-R_j(k+1)})$. Taking the conditional expectation on both sides of inequality (22) with respect to the information $\mathscr{I}_k$ available at time instant τ_k and using above bound on $|E(k+1)|$ yield $\mathbb{E}[|\tilde{\alpha}_j(k+1)||\mathscr{I}_k] \leq e^{-K_{\alpha_j}^* T_k}|\tilde{\alpha}_j(k)| + (B_j(k)T_k + \overline{U}_j(k))(1 - 2^{-R_j(k+1)})$. Since $R_j(k) \leq \overline{R}_j$ for all $k \in \mathbb{Z}_+$, the final conclusion holds. ∎

With Lemma IV.9 and IV.11, we proceed to state the main theorem of almost sure asymptotic stability as follows,

Theorem IV.12: Consider closed-loop system in (9), (10). Let the hypothesis of Lemma IV.9 hold, suppose there exists a positive constant value ε_j such that

$$B_j(k) = \max\left\{\frac{1}{\min\{L_{j\min}, L_j(k)\}}, 1\right\} W_{j-1}(|\tilde{\alpha}_j(k)|) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$$

for all $k \in \mathbb{Z}_+$, if

$$\max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, (\eta_j + 1 - 2^{-\bar{R}_j})\varepsilon_j T_k + e^{-K_{\alpha_j}^* T_k}\right\} \leq \delta \quad (23)$$

where $\delta \in (0,1)$. Then the system state of bearing angle α_j almost surely asymptotically converges to α_{d_j} for $j = 1, 2, \ldots, N-1$.

Proof: We prove the almost sure convergence of α_i by proving $\lim_{k\to\infty} \mathbb{E}[\overline{U}_i(k) + \tilde{\alpha}_i(k)] \to 0$. Since $\alpha_i = \hat{\alpha}_i + e_i$, then $|\alpha_i(k) - \alpha_{d_i}(k)| \leq |\hat{\alpha}_i(k) - \alpha_{d_i}| + \overline{U}_i(k)$. By Lemmas IV.9 and IV.11, one has $\mathbb{E}[\overline{U}_i(k+1) + \tilde{\alpha}_i(k+1)] \leq \delta_i \mathbb{E}[\overline{U}_i(k) + \tilde{\alpha}_i(k)]$ with $\delta_i \in (0,1)$, if inequality (23) holds. Then, it is clear that $\lim_{k\to\infty} \mathbb{E}[|\alpha_i(k) - \alpha_{d_i}(k)|] \to 0$. Using Markov inequality, we have $|\alpha_i(k) - \alpha_{d_i}(k)| \to 0$ almost surely, i.e., the bearing angle sequence $\{\alpha_i(k)\}$ almost surely converges to α_{d_i}. Because the state trajectory has no finite escape within each time interval $[\tau_k, \tau_{k+1}),\ \forall k \in \mathbb{Z}_+$. Then, the system state of bearing angle $\alpha_i(t)$ is almost surely convergent to α_{d_i}. ∎

Remark IV.13: The condition $B_j(k) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$ is equivalent to $W_{j-1}(\tilde{\alpha}_i(k)) \leq \varepsilon_i |\tilde{\alpha}_j(k)|$ since $L_j(t) > d > 1$ for $t \in \mathbb{R}_+$.

Almost sure practical stability is a weaker safety notion than almost sure asymptotic stability, and it allows the bearing angles to fluctuate within a reasonable safe set. Theorem IV.14 provides a sufficient condition to assure almost sure practical stability for bearing angle $\alpha_j(t),\ j = 1, 2, \ldots, N-1$.

Theorem IV.14: Consider closed-loop system in (9), (10). Let the hypothesis of Lemma IV.9 hold, for given positive values Δ_j^*, $j = 1, 2, \ldots, N-1$, if there exists a controller pair $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ with $\eta_j(k)$ such that

$$B_j(k) \leq \frac{1 - r_j}{J_j} \min\left\{\Delta_j^*, |\tilde{\alpha}_j(k)| + \overline{U}(k)\right\},\ j = 1, 2, \ldots, N-1 \quad (24)$$

with $r_j < 1$ where

$$r_j = \max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, e^{-K_{\alpha_j}^* T_k}\right\} \quad (25)$$

$$J_j = (\eta_j + 1 - 2^{-\bar{R}_j})T_k. \quad (26)$$

Then the bearing angle α_j of leader-follower pair i almost surely converges to a compact set defined by $\Omega_j = \{\alpha_j(t) : |\alpha_j(t) - \alpha_{d_j}| \leq \Delta_j^*\}$.

Proof: By Lemmas IV.9 and IV.11, one has

$$\mathbb{E}\left[|\tilde{\alpha}_j(k+1)| + \overline{U}_j(k+1)|\mathcal{I}_k\right] \leq \max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, e^{-K_{\alpha_j}^* T_k}\right\}\left(|\tilde{\alpha}_j(k)| + \overline{U}_j(k)\right) + (\eta_j + 1 - 2^{-\bar{R}_j})T_k B_j(k). \quad (27)$$

Let $V_j(k) = |\tilde{\alpha}_j(k)| + \overline{U}_j(k)$, and consider function $V_j(k)$ as a candidate Lyapunov function. It is clear that $V_j(k) \geq 0$ for any $k \in \mathbb{Z}_+$. Then, we can rewrite inequality (27) into $\mathbb{E}[V_j(k+1)|V_j(k)] \leq \mathbb{E}[V_j(k+1)|\mathscr{I}_j(k)] \leq r_j V_j(k) + J_j B_j(k)$. Furthermore, if the controller gains $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ are selected to assure $r_j < 1$, we have $\mathbb{E}[V_j(k+1)|V_j(k)] \leq V_j(k) - [(1 - r_j)V_j(k) - J_j B_j(k)]$. By condition (24), one can obtain

$$\mathbb{E}\left[V_j(k+1)|V_j(k)\right] \leq V_j(k) + (1 - r_j)\min\left\{\Delta_j^* - V_j(k), 0\right\} = V_j(k) - (1 - r_j)\max\left\{V_j(k) - \Delta_j^*, 0\right\}. \quad (28)$$

From inequality (28), one can prove the bounded set $\hat{\Omega}_j = \{V_j(k) : V_j(k) \leq \Delta_j^*\}$ is invariant with respect to system in (9) and (10) almost surely by considering

1) when $V_j(k) \leq \Delta_j^*$, inequality (28) is reduced to $\mathbb{E}[V_j(k+1)|V_j(k)] \leq V_j(k)$, which implies that sequence $\{V_j(k)\}$

is a super-martingale and remains in the set $\hat{\Omega}_j$ almost surely.

2) when $V_j(k) > \Delta_j^*$, $\exists \varepsilon > 0$ such that $\mathbb{E}[V_j(k+1)|V_j(k)] \leq V_j(k) - \varepsilon$. Clearly, the trajectory of $V_j(k)$ will asymptotically decrease until reaching the set $\hat{\Omega}_j$ almost surely.

This condition can be viewed as a stochastic version of the LaSalle Theorem in discrete time system. With condition (28), one can easily attain the almost sure convergence property for $V_j(k)$ with respect to set $\hat{\Omega}_j$, i.e., $\lim_{k\to+\infty} \Pr\{\sup_k V_j(k) \leq \Delta_j^*\} \to 1$. Since $|\alpha_j(k) - \alpha_{d_j}| \leq |\tilde{\alpha}_j(k)| + \overline{U}_j(k) = V_j(k)$, $|\alpha_j(k) - \alpha_{d_j}|$ converges to set Ω_j almost surely. Since the state trajectories remains bounded within each transmission time interval $[\tau_k, \tau_{k+1})$ for all $k \in \mathbb{Z}_+$. Therefore, we have $\lim_{t\to+\infty} \Pr\{\sup_t |\alpha_j(t) - \alpha_{d_j}| \leq \Delta_j^*\} \to 1$. ■

Remark IV.15: Inequality (24) characterizes an upper bound on the propagated disturbance $B_j(k)$ under which the leader-follower pair j is almost sure practically stable. This upper bound is a increasing function of the size of target set Δ_j^*, the worst-case of bearing angle $|\tilde{\alpha}_j(k)| + \overline{U}(k)$, and a decreasing function of the ratio η_j.

Remark IV.16: Inequality (24) can be viewed as a distributed rule to select $\eta_j(k)$ to assure almost sure practical stability for each leader-follower pair. The selected $\eta_j(k)$ is used in Lemma IV.9 to switch controller.

The following corollary shows an explicit bound on the bearing angle under which it is almost surely convergent to a "safe" set $\Omega_j(\Delta_j^*)$. Such bound is a function of η_j and Δ_j^*.

Corollary IV.17: In Theorem IV.14, suppose $W_j(\tilde{\alpha}_j(k)) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$ holds with $g_j(\eta_j) := (1 - r_j)/\varepsilon_j J_j \geq 1$ and $r_j < 1$ where r_j and J_j are defined in (25). If

$$|\tilde{\alpha}_j(k)| + \overline{U}_j(k) \leq g_j(\eta_j)\Delta_j^* \tag{29}$$

then the bearing angle α_j almost surely converges to a bounded set $\Omega_j = \{\alpha_j(t) : |\alpha_j(t) - \alpha_{d_j}| \leq \Delta_j^*\}$.

Proof: From Theorem IV.14, we know that the sufficient condition to assure almost sure practical stability with set Ω_j is $B_j(k) \leq (1 - r_j/J_j) \min\{\Delta_j^*, |\tilde{\alpha}_j(k)| + \overline{U}(k)\}$. By condition $W_j(\tilde{\alpha}_j(k)) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$, the above sufficient condition holds, if

$$\begin{aligned}|\tilde{\alpha}_j(k)| + \overline{U}_j(k) &\leq \frac{1 - r_j}{\varepsilon_j J_j} \min\left\{\Delta_j^*, |\tilde{\alpha}_j(k)| + \overline{U}(k)\right\}\\ &= g_j(\eta_j)\Delta_j^*\end{aligned}$$

holds. The equality holds because $g_j(\eta_j) := (1-r_j)/\varepsilon_j J_j \geq 1$. Therefore, the conclusion holds. ■

Remark IV.18: $g_j(\eta_j)$ is a monotonically decreasing function with respect to η_j, and it characterizes the size of the region from which the state almost surely converges to the set Ω_j with size Δ_j^*. The inequality (29) may be viewed as a partition of the physical state in the sense that small η_j gives rise to large contraction set.

V. Simulation Experiments

This section presents simulation experiments examining the resilience of our proposed switched controller to deep fades, and also demonstrates the benefits of using almost sure practical stability as a safety measurement over the traditional mean square stability.

A. Simulation Setup

In the simulation, we consider $N = 4$ vehicles that is cascaded in a string as shown in Fig. 1. Each leader-follower pair uses a two-state Markov chain model to simulate the fading channel between the leader and follower. The two-state Markov chain has two states with one representing the good channel condition and the other one representing the bad channel condition. Here, the "good channel state" simply means the transmitted bit is successfully received, while the "bad channel state" means the failure of receiving the bit.

Following the characterization of Makov chain model in [42], one can find that the conditional probability for good channel state is a monotonically decreasing function of $L_j(t)/\cos\alpha_j(t)$, while the conditional probability for bad channel state is a monotonically decreasing function of $\cos\alpha_j(t)/L_j(t)$. The explicit function form depends on the distribution of the channel gain. In this simulation, we use $p_{11} = e^{-3\times10^{-3}(L_j(t)/\cos\alpha_j(t))^2}$ to denote the conditional probability for the good channel state and $p_{22} = e^{-6\times10^{2}(\cos\alpha_j(t)/L_j(t))^2}$ to represent the conditional probability for the bad channel condition. The corresponding transition probabilities between these states are $1 - p_{11}$ and $1 - p_{22}$. Then, we use the EBB model in (6) to characterize the low bit region generated by the two-state Markov chain model. The corresponding functions in EBB model (6) are $h(\alpha_j, L_j) = \bar{R}_j e^{-3\times10^{-4}(L_j(t)/\cos\alpha_j(t))^2}$, $\gamma(\alpha_j, L_j) = e^{-4.5\times10^{-3}(L_j(t)/\cos\alpha_j(t))^2}$ with $\bar{R}_j = 2$ representing two bits that are transmitted at each sampling period.

The 100 ms sampling time that is consistent with the transmission frequency in V2V communication technology [10] is widely used in mobile robot system, is selected for each leader-follower pair ($j = 1, 2, 3$), i,e, $T_k = 0.1$ sec for all $k \in \mathbb{Z}_+$. The functions $W_{j-1}(\cdot)$ in Proposition IV.1 are selected to be linear functions $W_{j-1}(|\tilde{\alpha}_j(t)|) = a_j|\tilde{\alpha}_j(t)| + b_j$ with parameters being $a_1 = 0.1$, $b_1 = 0.01$; $a_2 = 0.8$, $b_2 = 2$; $a_3 = 1$, $b_3 = 4$. The value of the parameter sets are chosen to be increasing with respect to j to guarantee the feasibility of the controller selection for each leader-follower system.

In this simulation, we consider an interesting and realistic scenario that the fourth vehicle from far distance intends to join the other three closed-spaced vehicles. Hence, the initial states for three leader-follower pairs ($j = 1, 2, 3$) are $\alpha_1(\tau_0) = \pi/3$, $\alpha_2(\tau_0) = \pi/4$, $\alpha_3(\tau_0) = \pi/6$ and $L_1(\tau_0) = 7.1$ m, $L_2(\tau_0) = 7.1$ m, $L_3(\tau_0) = 99$ m. The initial uncertainties are $U_j(\tau_0) = \pi/6$. By switching controller pairs from sets $\mathscr{K}_j = \{(K_{L_j}, K_{\alpha_j}) : 0 < K_{L_j} \leq 100,\ 0 < K_{\alpha_j} \leq 100\}$, each leader-follower pair is required to achieve and maintain desired set-points $\alpha_{d_j} = 0$, $L_{d_j} = 2$ m, $j = 1, 2, 3$.

B. Simulation Results

A Monte Carlo method was used to verify that the system has almost surely practical stability when Proposition IV.1 and Theorem IV.14 hold. Each simulation example is run 100 times over a time interval from 0 to 10 seconds.

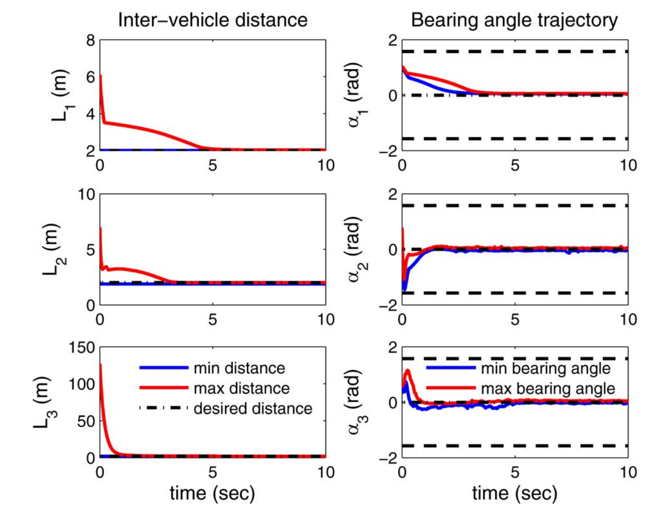

Fig. 4. The maximum and minimum value of separation L_j (m) and bearing angle α_j (rad) for leader-follower pair, $j = 1, 2, 3$.

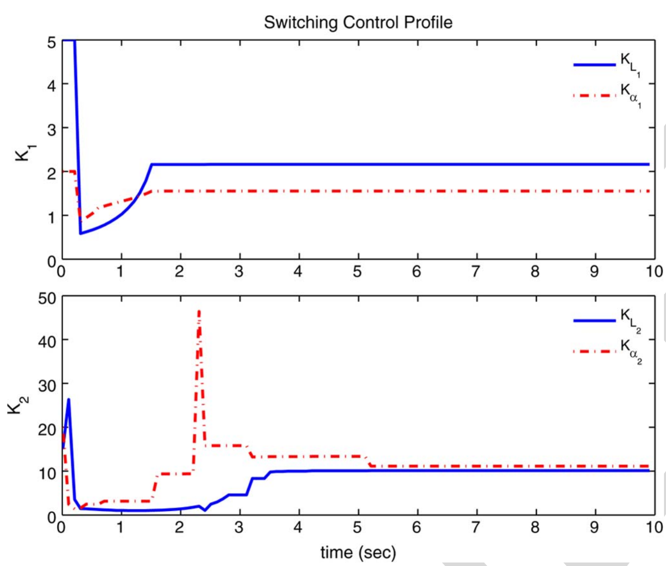

Fig. 5. One sample of switching controller profile for leader-follower pair 1 (Top) and 2 (Bottom): K_{L_j} and K_{α_j} are controller gains for the distance and bearing angle of leader-follower j, $j = 1, 2$.

In the first simulation, we select the controllers for each leader-follower pair from $\mathscr{K}_j$, $j = 1, 2, 3$ so that Proposition IV.1 and Theorem IV.14 hold at each time instant τ_k. Fig. 4 shows the maximum and minimum values of the system states L_j and α_j, $j = 1, 2, 3$ evaluated over all the 100 runs. The maximum value is marked by red lines and the minimum value is marked by blue lines. The two dashed lines in Fig. 4 represent the upper and lower bound for the relative bearing α, i.e., $|\alpha_j| \leq \pi/2$, which characterizes the safety region. We can see from Fig. 4 that the maximum and minimum values of the system states asymptotically converge to a bounded set containing the desired set-points $\alpha_{d_j} = 0$ and $L_{d_j} = 2$ m. This is precisely the behavior that one would expect if the system is almost sure practically stable. These results, therefore, seem to confirm our statement in Theorem IV.14.

Figs. 5 and 6 show one sample of switching controller profile and channel state for each leader-follower pair. The top plot

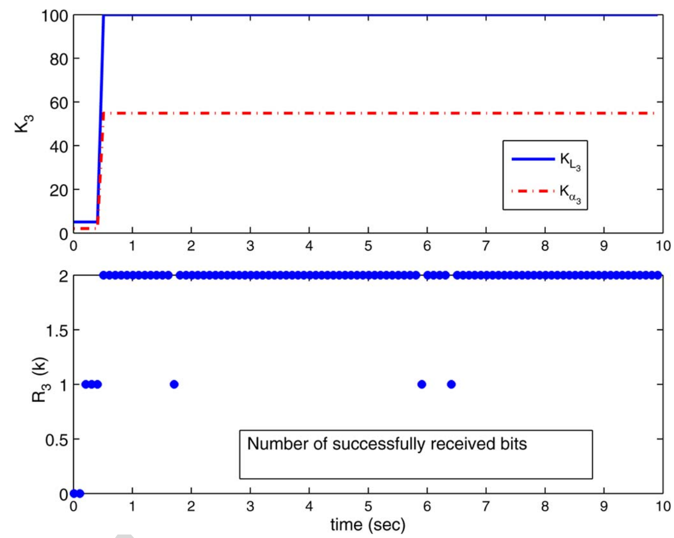

Fig. 6. One sample of switching profile (Top) and channel state (Bottom) for leader-follower pair 3: K_{L_3} and K_{α_3} are controller gains and $R_3(k)$ is the number of successfully received bits at each time interval $[\tau_k, \tau_{k+1})$.

in Fig. 5 shows the switching controller profile for the leader-follower pair 1 with red line marked as controller gain K_{α_1} and blue line as controller gain K_{L_1}. The bottom one is the switching controller profile for leader-follower pair 2 with the same marking rule. These plots show that the controller gains stay low at the first two seconds to avoid large disturbance to the bottom system, and then switch from low to high when the systems approach the equilibrium and are confident that the channel state will always stay good. The top plot in Fig. 6 is the switching controller profile for the leader-follower system 3 with same marking rule, and the bottom plot is the channel state $R_3(k)$ that characterizes the number of successfully received bits at each time interval. We can clearly see from the plots that the controller for system 3 starts with low gains to compensate the effect caused by a short string of zero bits at the beginning, and then switches from low gain to high gain when channel condition stays good. These results demonstrate that channel state indeed is used as a feedback signal to switch the controller.

In the second simulation, we studied the benefits of almost sure practical stability as a safety measurement over the traditional mean square stability. Traditional mean square stability requires the second moment of the system state converges to a positive constant value, but it does not put any constraint on the sample path which might potentially cause safety issues. For a fair comparison, the same simulation setup and parameters are applied in this simulation with the only difference being on the controllers. One type of controller used in this simulation is a mean square stabilizing controller, which is selected to guarantee mean square stability for each leader-follower pair. The other type of controller is the switching controller proposed in this paper to guarantee almost sure practical stability for each leader-follower pair. The switching control strategy uses the mean square stabilizing controller as its initial controller.

Fig. 7 shows a comparison of the maximum and minimum values of the bearing angle α_3 for leader-follower pair 3 with the switching controller case in the top plot and the mean square controller $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$

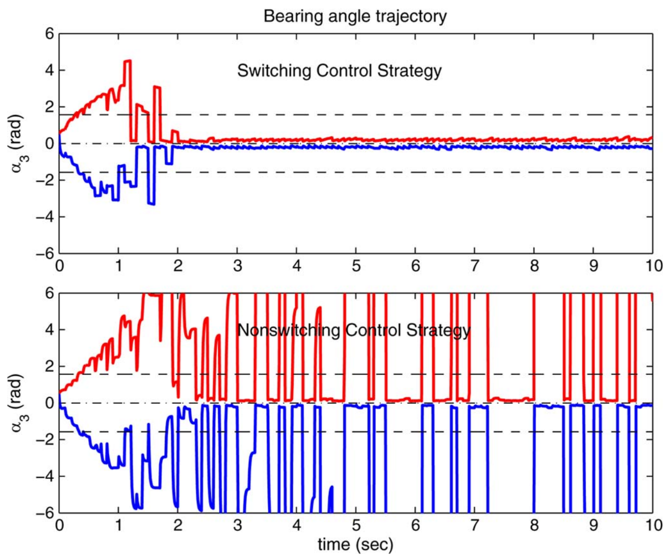

Fig. 7. The maximum and minimum system trajectory for leader-follower pair 3 with switching controller (Top) and non-switching controller pair (Bottom) $K_{L_3} = 2$ and $K_{\alpha_3} = 50$.

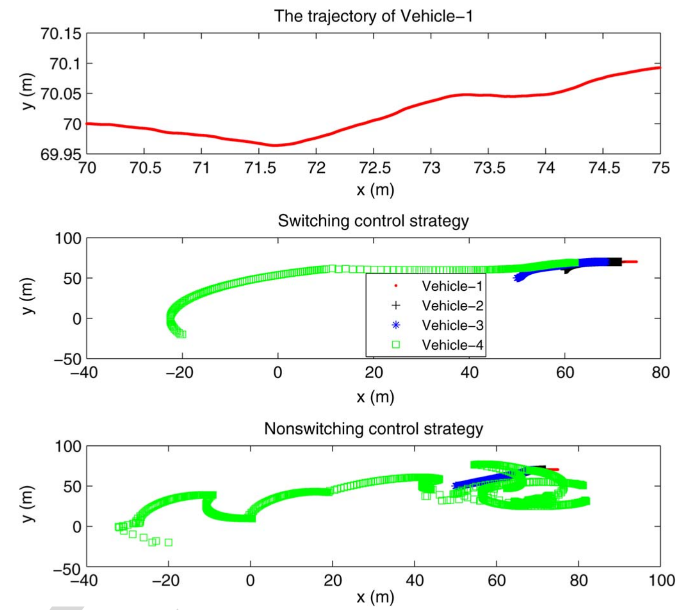

Fig. 8. The comparison of one sample run (10 seconds) of vehicles' trajectories generated by a switching control strategy (Middle) and a non-switching strategy (Bottom) with $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$. The top plot is the trajectory of the leading vehicle (Vehicle-1).

in the bottom plot. It is worth noting that (K_1, K_2, K_3) is just one of the many selections in our simulation. Because of the space limitations, we only use (K_1, K_2, K_3) as an example to demonstrate the results. It is clear from Fig. 7 that the system's sample path goes unbounded as time increases by using a mean square stabilizing controller, but it converges asymptotically to a bounded set by using a switching controller. These results suggest that the composition of mean square stable systems does not guarantee mean square stability for the whole system, while the composition of almost sure stable systems may still guarantee almost sure stability for the whole system.

Fig. 8 shows the comparison of one sample run of vehicles' trajectories in Euclidean space that are generated by the switching control strategy proposed in this paper and the non-switching strategy with controller gain $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$. The top plot of the figure is the leading vehicle's trajectory generated by a velocity profile $(v_1(t), \omega_1(t))$ which satisfies the condition in Corollary IV.4. The middle plot shows the trajectories of four vehicles that adopt the switching strategy where the red, black, blue and green dots represent the trajectories of leading vehicle (Vehicle-1), Vehicle-2, Vehicle-3 and Vehicle-4 respectively. It is clear from the plot that the leader-follower system almost surely converges to the specified formation. The bottom plot shows the result for non-switching control strategy using the mean square controller $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$ which exhibits significantly unsafe oscillatory behavior in Vehicle-4.

VI. Conclusion

This paper studies the almost sure safety property for a chain of leader-follower vehicular networked system in the presence of a V2V channel that exhibits exponentially bounded burstiness and varies as a function of vehicular state. The concept of almost sure safety is examined in terms of almost sure asymptotic stability and practical stability. Switching strategy is adopted to assure almost sure safety by adaptively reconfiguring local controller gains to the changes of channel state. Sufficient conditions are provided to decide which controller is placed in the feedback loop at each transmission time. As a result of the correlation between channel state and physical vehicular state, the sufficient conditions partition the vehicular state space into a set of regions in which controllers are designed to achieve almost sure safety. The simulation results of a four-vehicle leader-follower formation control are provided to support our theoretical analysis and illustrate the benefit of using almost sure practical stability as a safety measurement over traditional mean square stability.

It is important to note that this paper studies the effect of a V2V communication channel on the safety of leader-follower systems under the assumption that no measurement noise is present in the system. This assumption turns out to be a necessary and sufficient condition to assure almost sure stability due to negative results in [25]. One can only hope for a weaker notion of stochastic stability if state-independent noise is present in the system. Addressing this issue is beyond the scope of this paper and will be explored in our future work.

Appendix

Proof of Proposition IV.1: Consider the infinite norm of the control input given in (8)

$$\begin{aligned}
&\left\| \begin{bmatrix} v_j(t) \\ \omega_j(t) \end{bmatrix} \right\| \\
&\leq \left\| \begin{bmatrix} -\cos\phi_j & -L_j \sin\phi_j \\ -\frac{\sin\phi_j}{d} & \frac{L_j}{d}\cos\phi_j \end{bmatrix} \right\| \left\| \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha_j}\right) \end{bmatrix} \right\| \\
&\leq (1 + |L_j(t)|) \max\left\{ K_{L_j}(k) \left| \tilde{L}_j(t) \right|, K_{\alpha_j}(k) \left| \tilde{\alpha}_j(t) \right| \right\}
\end{aligned} \quad (30)$$

with $\tilde{L}_j(t) = L_{d_j} - L_j(t)$. The supreme of $|L_j(t)|$ over time interval $[\tau_k, \tau_{k+1})$ can be obtained by considering $\dot{L}_j(t) \leq K_{L_j}(k)(L_{d_j} - L_j)(t) + W_{j-1}(|\tilde{\alpha}_j(k)|)$. Using Gronwall Bellman theorem to solve above inequality and yield

$$\begin{aligned} L_j(t) &\leq L_j(k)e^{-K_{L_j}(k)(t-\tau_k)} \\ &\quad + \left(L_{d_j} + \frac{W_{j-1}\left(|\alpha_j(k)|\right)}{K_{L_j}(k)}\right)\left(1 - e^{-K_{L_j}(k)(t-\tau_k)}\right) \\ &\triangleq \overline{L}_j(t). \end{aligned}$$

Assume $L_j(t) > 0$ (In Lemma IV.8, we prove that if controller gain $K_{L_j}(k)$ is selected sufficiently large, $L_j(t) > d > 0$ holds for all $t \geq 0$), and because $d\overline{L}_j/dt \geq 0$ or $d\overline{L}_j/dt < 0$ over interval $[\tau_k, \tau_{k+1})$. In other words, $\overline{L}_j(t)$ is a monotonically function over $[\tau_k, \tau_{k+1})$. Thus $\sup_{\tau_k \leq t < \tau_{k+1}} L_j(t)$ is obtained when $t = \tau_k$ or $t \to \tau_{k+1}$, i.e.,

$$L_j(t) = \max\left\{\overline{L}_j(\tau_k), \overline{L}_j(\tau_{k+1})\right\} \triangleq M_{L_j}(k). \tag{31}$$

Note that over time interval $[\tau_k, \tau_{k+1})$, one has $d|\tilde{L}_j(t)|/dt \leq K_{L_j}(k)|\tilde{L}_j(t)| + W_{j-1}(\tilde{\alpha}_j(k))$ thus

$$\begin{aligned} \sup_{\tau_k \leq t \leq \tau_{k+1}} K_{L_j}(k)\left|\tilde{L}_j(t)\right| &= K_{L_j}(k)\left|\tilde{L}_j(k)\right| e^{K_{L_j}(k)T_k} \\ &\quad + W_{j-1}(|\tilde{\alpha}_j(k)|)\left(e^{K_{L_j}(k)T_k} - 1\right) \\ &\triangleq \tilde{L}_{j,\max}(k). \end{aligned} \tag{32}$$

By inequalities (31), (32), (30) can be further bounded

$$\left\|\begin{bmatrix} v_j(t) \\ \omega_j(t) \end{bmatrix}\right\|_\infty \leq \left(1 + M_{L_j}(k)\right)\max\left\{\tilde{L}_{j,\max}(k), K_{\alpha_j}(k)\left|\tilde{\alpha}_j(t)\right|\right\} \tag{33}$$

with $\tilde{\alpha}_j(t) = \alpha_{d_j} - \hat{\alpha}_j(t)$ satisfying $\dot{\tilde{\alpha}}_j = -K_{\alpha_j}(k)\tilde{\alpha}_j, t \in [\tau_k, \tau_{k+1})$ with initial value $\tilde{\alpha}_j(\tau_k)$. From the solution of the above ODE, it is obvious that $|\tilde{\alpha}_j(t)| < |\tilde{\alpha}_j(\tau_k)|$, then it is straightforward to show that if the condition (11) is satisfied, the inequality (12) holds. ■

Proof of Lemma III.3: Consider the case that the collection of random variables $\{X_{jl}(k)\}$ is i.i.d. and the probability of successfully decoding a packet is equal to the probability that the signal to noise ratio (SNR_{jl}) exceeds some fixed threshold γ_0 [39], i.e.,

$$\Pr\{X_{jl}(k) = 1\} = \Pr\{\mathrm{SNR}_{jl} \geq \gamma_0\}.$$

The selection of the threshold γ_0 is often directly related to the communication system (e.g. modulation scheme). We assume a pre-selected γ_0 for a fixed communication system. For Raleigh fading, one can explicitly compute the successfully decoding probability as

$$\Pr\{\mathrm{SNR}_{jl} \geq \gamma_0\} = e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \triangleq p(L_j, \alpha_j)$$

with $\overline{\gamma}(L_j, |\alpha_j|) = \mathbb{E}(P_{rec}(L_j, |\alpha_j|))/N_0$ where $P_{rec}(L_j, |\alpha_j|)$ and N_0 are powers of the receiving signal and noise respectively. According to directional antenna gain theory [2], one knows that $P_{rec}(L_j, |\alpha_j|)$ is a monotonically decreasing function with respect to $L_j \in (0, +\infty)$ and $|\alpha_j| \in [0, \pi/2]$ and so does $p(L_j, |\alpha_j|)$. Since $\{X_{jl}(k)\}$ is i.i.d., one has that $R_j(k) = \sum_{l=1}^{\overline{R}_j} X_{jl}(k)$ follows a binomial distribution with mean value $\mathbb{E}(R_j(k)) = \overline{R}_j p(L_j, |\alpha_j|)$. Using Chernoff inequality, one has

$$\begin{aligned} &\Pr\left\{R_j(k) \leq (1-\delta)\overline{R}_j p\left(L_j, |\alpha_j|\right)\right\} \\ &\qquad \leq e^{-\frac{\delta^2}{2}\overline{R}_j p(L_j, |\alpha_j|)}, \delta \in (0, 1). \end{aligned}$$

Let $h(L_j, |\alpha_j|) = \delta_d \overline{R}_j p(L_j, |\alpha_j|)$ for some $\delta_d \in (0, 1]$, then

$$\begin{aligned} &\Pr\{R_j(k) \leq (1-\delta)h(L_j, |\alpha_j|)\} \\ &\quad = \Pr\left\{R_j(k) \leq (1-\delta)\delta_d \overline{R}_j p\left(L_j, |\alpha_j|\right)\right\} \\ &\quad = \Pr\left\{R_j(k) \leq \left(1 - (1 - \delta_d + \delta_d\delta)\right)\overline{R}_j p\left(L_j, |\alpha_j|\right)\right\} \\ &\quad \leq e^{-\frac{(1-\delta_d+\delta_d\delta)^2}{2\delta_d\delta}\delta h(L_j, |\alpha_j|)} \end{aligned} \tag{34}$$

Let $\sigma = \delta h(L_j, |\alpha_j|)$, $\hat{\gamma}(\delta) = (1 - \delta_d + \delta_d\delta)^2/2\delta_d\delta$, then wen have

$$\Pr\left\{R_j(k) \leq h\left(|L_j|, |\alpha_j|\right) - \sigma\right\} \leq e^{-\hat{\gamma}(\delta)\sigma}$$

where $\sigma \in [0, h(|L_j|, \|\alpha_j\|))$. The last inequality holds due to Chernoff inequality. Taking the first derivative of $\hat{\gamma}(\delta)$ w.r.t δ, one has

$$\frac{d\hat{\gamma}}{d\delta} = \frac{\overbrace{(1 - \delta_d + \delta_d\delta)}^{>0}(\delta_d\delta - 1 + \delta_d)}{2\delta_d\delta^2}.$$

Clearly, given $0 < \delta_d < 1$, $\hat{\gamma}$ has the minimum value at $\delta^* = (1/\delta_d) - 1$, and $\hat{\gamma}(\delta^*) = 2(1 - \delta_d)$. One has a EBB characterization as follows:

$$\Pr\left\{R_j(k) \leq h\left(|L_j|, |\alpha_j|\right) - \sigma\right\} \leq e^{-2(1-\delta_d)\sigma}.$$

Consider the case that the collection of random variables $\{X_{jl}(k)\}$ is a two-state Markov process and for Rayleigh fading channels, the transition probability matrix $M(k)$ for a two-state Markov chain can be obtained by using the technique in [42], as seen in (35), as shown at the top of the next page, for $l = 2, 3, \ldots, N$ and c is the system parameter for a selected V2V wireless system and is sufficiently small to assure the transition probability is valid, i.e., within [0, 1]. Note that the function forms in (35) are particular for Rayleigh fading channels. One may not have explicit function form for other type of fading channel, but the fundamental relationship between physical state L_j, α_j and the fading function should remain the same. Given the transition probability in (35), the stationary distribution π_1 and π_0 can be obtained as

$$\begin{pmatrix} \pi_1 \\ \pi_0 \end{pmatrix} = \begin{pmatrix} e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \\ 1 - e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \end{pmatrix}.$$

Let $\lambda_2(M)$ denote the second largest eigenvalue of transition matrix M, and it is easy to obtain $\lambda_2(M)$ as follows:

$$\lambda_2(M) = 1 - c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}\left(L_j, |\alpha_j|\right)}} - c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} - 1}$$

$$M(k) = \begin{pmatrix} \Pr\left\{X_{jl}(k)=1|X_{j(l-1)}(k)=1\right\} & \Pr\left\{X_{jl}(k)=1|X_{j(l-1)}(k)=0\right\} \\ \Pr\left\{X_{jl}(k)=0|X_{j(l-1)}(k)=1\right\} & \Pr\left\{X_{jl}(k)=0|X_{j(l-1)}(k)=0\right\} \end{pmatrix}$$
$$= \begin{pmatrix} 1 - c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}} & c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}-1} \\ c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}} & 1 - c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}-1} \end{pmatrix} \tag{35}$$

With results in [8], we know that there also exists Chernoff type bound for finite Markov Chains. In particular, if the two-state Markov chain starts with its stationary distribution π_0 and π_1, then for $0 < \delta < 1$, we have

$$\Pr\left\{R_j(k) \leq (1-\delta)\pi_1\overline{R}_j\right\} \leq e^{-(1-\lambda_2(M))\delta^2\pi_1\overline{R}_j}. \tag{36}$$

The transformation used in inequality (34) can be applied to probability inequality (36). Let $h(L_j,\alpha_j) = \delta_d\overline{R}_j\pi_1(L_j,\alpha_j)$ for some selected $\delta_d \in (0,1]$, then

$$\begin{aligned} &\Pr\left\{R_j(k) \leq (1-\delta)h(L_j,\alpha_j)\right\} \\ &\leq e^{-(1-\lambda_2(M))\frac{(1-\delta_d+\delta_d\delta)^2}{\delta_d\delta}\delta h(L_j,\alpha_j)} \\ &\leq e^{-f(L_j,|\alpha_j|)4(1-\delta_d)\delta h(L_j,\alpha_j)} \end{aligned}$$

where $f(L_j,|\alpha_j|) = c(\sqrt{2\pi(\gamma_0/\overline{\gamma}(L_j,|\alpha_j|))}/(1-e^{-(\gamma_0/\overline{\gamma}(L_j,|\alpha_j|))}))$ with $c > 0$. It is easy to check that function $f(L_j,|\alpha_j|)$ is monotonically decreasing with respect to L_j and $|\alpha_j|$. Hence, one can always find corresponding EBB characterizations for both i.i.d and two-state Markov processes with monotonically decreasing function pairs $\{\delta_d\overline{R}_j p(L_j,\alpha_j), 2(1-\delta_d)\}$ and $\{\delta_d\overline{R}_j\pi_1(L_j,\alpha_j), 4(1-\delta_d)f(L_j,|\alpha_j|)\}$, respectively. ■

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Distributed Switching Control to Achieve Almost Sure Safety for Leader-Follower Vehicular Networked Systems

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***Abstract*—Leader-follower formation control is a widely used distributed control strategy that requires systems to exchange their information over a wireless radio communication network to attain and maintain formations. These wireless networks are often subject to *deep fades*, where a severe drop in the quality of the communication link occurs. Such deep fades inevitably inject a great deal of stochastic uncertainties into the system, which significantly impact the system's performance and stability, and cause unexpected safety problems in applications like smart transportation systems. Assuming an exponentially bursty channel that varies as a function of the vehicular states, this paper proposes a distributed switching control scheme under which the local controller is reconfigured in response to the changes of channel state, to assure *almost sure safety* for a chain of leader-follower system. Here *almost sure safety* means that the likelihood of vehicular states entering a safe region asymptotically goes to one as time goes to infinity. Sufficient conditions are provided for each local vehicle to decide which controller is placed in the feedback loop to assure *almost sure safety* in the presence of *deep fades*. Simulation results of a chain of leader-follower formation are used to illustrate the findings.**

***Index Terms*—Cyber-physical systems (CPS), quality of service (QoS), vehicular network (VN).**

I. Introduction

VEHICULAR networks (VNs) are cyber-physical systems (CPS) consisting of numerous autonomous vehicles that coordinate with each other by sharing information over wireless networks. VNs have recently received considerable attention due to rapid advances in Vehicle to Vehicle (V2V) communication technology, which promises significant safety improvement for applications like intelligent transportation systems [10]. Building safe VNs, however, is extremely challenging in two aspects. First, the mobile nature of VNs requires design of control strategies that are distributed and scalable. Secondly, V2V wireless networks in VNs are highly time varying due to the motion of transmitters and receivers. As a result, the V2V channel is inherently bursty and subject to *deep fading*, which causes a severe drop in the network's quality of service (QoS). These deep fades induce a great amount of stochastic uncertainties into the system, thereby negatively impacting the system's performance and causing serious safety issues. The objective of this paper is to design a distributed control strategy that could assure a certain level of safety for VNs in the presence of *bursty deep fading* channels.

Leader-follower scheme naturally serves as a distributed strategy for VNs due to its simplicity, scalability and the fact that communication is essential for assuring safe platooning in automated highway system (AHS) [23]. This has been illustrated by work that is based on either experimental validation [1], [3] or theoretical analysis [15], [38]. In leader-follower platoon systems, the question of safety is often analyzed under the concept of string stability [35]. This concept has been proven to be effective in characterizing the propagation of disturbances from the leader to downstream vehicles [32]. Recent results [15] showed that string stability can be improved by increasing the leader's communication connectivity to its followers. Such improvement, however, is compromised by reduced network connectivity arising from the delayed or dropped packets [22], [29]. This impact of unreliable network links on formation control has motivated studies of robust networked controllers under communication constraints, such as time varying but bounded delays [13], [22], [28] and Bernoulli [37] or two-state Markov chain dropouts [33]. So communication issues are critical in the development of safe VNs.

The channel model that is used to characterize V2V fading network, however, must be carefully specified. Traditionally, communication channels are modeled as an *independent and identical distributed (i.i.d)* random process with either a Rayleigh or Rician distribution [37] or a two-state Markov chain [33]. These characterizations are inadequate for V2V channel due to two reasons. First, fading channels are time varying and possess memory that cannot be captured by i.i.d models. Second, conventional two-state Markov chain ignores the potential dependence of the channel state (e.g., bit error rate) on the vehicle's physical states (e.g., inter-vehicle distance and bearing angle) [3], [7]. Such dependency in V2V communication systems has been extensively explored in communication community, see [4], [7], [14], [31], [40]. However, obtaining a V2V channel model is practically challenging due to its significant dependency on the dynamics of the vehicles and the surrounding environments [14]. Thus, most existing V2V channel models are obtained for specific environments and have limited use for control systems.

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From control perspective, there are two fundamental properties in V2V communication channel that have essential impacts on the system's performance and stability. The first property is the channel burstiness, which is characterized by a long string of consecutive dropouts in the network. Recent work [21] showed that system's stability can be seriously compromised if such burstiness is allowed with a sufficiently large probability. The second property is the dependency of channel state on the vehicular states. The knowledge of the correlation between physical and communication systems is valuable in designing control strategies for the safety of mobile vehicles. The vehicles could be safe if they use the knowledge of channel state to adaptively adjust their future actions so that the likelihood of the V2V channel exhibiting deep fades becomes smaller and smaller as time progresses.

Motivated by the observation of burstiness and state dependency in V2V network, a more realistic channel model was proposed in [16] for a single leader-follower pair, in which the channel is exponentially bursty and is dependent on the norm of the physical system's states. Such channel model is built upon the framework of *exponentially bounded burstiness* (EBB) originally developed by [41] and explicitly incorporates the state dependency. The EBB characterization discussd in this paper is closely related to the notion of *Outage Probability* [19], [39], which is a well studied performance metric for fading channel. The advantages of the EBB model are that (1) it directly characterizes the probability bound on the channel burstiness which is proven to be essential for system's stability; (2) it captures the state dependency of V2V channel in a simple but effective way that could be useful for control purpose; and (3) it is general in the sense that it can model a wide range of communication channels including i.i.d and two-state Markov chains. This paper extends the prior work in [16] to a chain of leader-follower system and shows that for any mobile channels modeled by either i.i.d or two-state Markov chain, there always exists an EBB characterization.

In the presence of V2V communication networks, the safety issues for VNs must be examined in a stochastic setting by discussing the likelihood of a system state entering a forbidden or unsafe region. Traditionally, this has been done using mean square concepts in which the variance of some important system state, such as inter-vehicle distance, remains bounded. Such a concept is also analogous to the notion of *stochastic safety in probability* [30]. The common feature of the above work is that they bound the likelihood of unsafe action occurring with a nonzero value, which still allows a finite probability for the system to be unsafe. This mean square safety or *stochastic safety in probability* criterion is not appropriate for many safety-critical systems such as smart transportation system where a small probability of danger can incur catastrophic failure. This paper suggests using a stronger notion of *almost sure safety* to assure the system state asymptotically goes to a safe equilibrium or a bounded safe set with probability one as time goes to infinity. In particular, *almost sure safety* in this paper refers to two strong notions of stochastic stability: *almost sure asymptotic stability* and *almost sure practical stability* [18].

Because of the challenge in modeling V2V channels, to the best of our knowledge, there is relatively little work that discusses the almost sure safety for VNs in the presence of realistic V2V channels. The most related work that assures similar safety property for networked systems is [33]. In [33], a H_∞ controller was developed to assure second moment stability which is a stronger notion than almost sure stability, for a linear networked system with a two-state Markov chain channel model. The approach used to guarantee safety in [33] relies on the fact that the linear system with the channel model can be formulated as a Markovian jump linear system (MJLS). Other recent work using MJLS approach to prove mean square stability includes [26], [27]. However, this MJLS approach cannot be applied to the vehicular systems with V2V channels for two reasons. First, the result based on MJLS framework is limited to the system with a single centralized controller, which is impractical for vehicular systems. Second, the state dependency in V2V channels introduces extra nonlinearity into vehicular systems that cannot be addressed in the framework of MJLS.

By using the EBB model that is functionally dependent on the vehicular state, this paper develops a distributed switching control scheme to assure *almost sure safety* for a chain of leader-follower systems. The leader-follower chain consists of a collection of leader-follower pairs that require each follower to manipulate its linear and angular velocity to achieve and maintain a desired separation and relative bearing. The information of the leader's bearing angle is transmitted over an exponentially bursty channel, which is accessed by a directional antenna mounted on each leading vehicle in the chain [17].

The results in this paper add to the prior literature on bearing-only leader-follower systems. Recent work in leader-follower systems [11], [24], [34], [36] studied bearing-only systems under the assumption of perfect or bounded measurements. In [36], the safety of leader-follower formation systems were examined by analyzing the disturbance amplification within the formations under perfect information. The studies in [11], [24] showed that bearing angles were important in addressing the localization problem in multi-agent systems when perfect information is available. This was extended in [34] to deal with uncertain measurements in a known bounded set. The present paper, on the other hand, considered the scenario that the leader's bearing angle was not perfectly known and the uncertainty of the information stochastically changed over time. This stochastic uncertainty results from *deep fading* in V2V communication and is prone to destabilizing a leader-follower chain.

This stochastic uncertainty prevents each leader-follower subsystem from maintaining the formation safely. The cascaded structure of the leader-follower chain amplifies such uncertainty from upper system to the lower system, and therefore leads to catastrophic failure for the entire system. This paper proposes two switching rules to recover the safe-behavior of the leader-follower chain by adaptively selecting local controller in response to the changes of channel state, and by enforcing the upper systems to constrain their control actions as a function of the lower system's states. Sufficient conditions are provided for each vehicle to decide which controller is placed in the feedback loop to assure *almost sure asymptotic stability* and *almost sure practical stability* for the entire leader-follower chain.

The layout of this paper is as follows. Section II introduces mathematical notations. Section III provides a system description and problem formulation. After that, Section IV discusses the main results. Then, Section V presents the simulation results of a leader-follower chain with four vehicles. Finally, Section VI concludes the paper.

II. Mathematical Preliminaries

Let $\mathbb{Z}$ and $\mathbb{R}$ denote the set of integers and real numbers, respectively. Let $\mathbb{Z}_+$ and $\mathbb{R}_+$ denote the set of non-negative integers and real numbers, respectively. Let $\mathbb{R}^n$ denote the n-dimensional Euclidean vector space. The ∞-norm on the vector $x \in \mathbb{R}^n$ is $|x| = \max|x_i| : 1 \leq i \leq n$, and the corresponding induced matrix norm is $\|A\| = \max_{1 \leq i \leq n} \sum_{j=1}^{n} |A_{ij}|$. Let $f(t) \in \mathbb{R}^n$ denote the value that function f takes at time $t \in \mathbb{R}$. Let $\{\tau_k\}_{k=0}^{\infty}$ denote a strictly monotonically increasing sequence with $\tau_k \in \mathbb{R}_+$ for all $k \in \mathbb{Z}_+$ and $\tau_k < \tau_{k+1}$. Then, $f(\tau_k)$ denotes the value of function f at time τ_k. For simplicity, we let $f(k)$ denote $f(\tau_k)$ if its meaning is clear in the context. The left-hand limit at $\tau_k \in \mathbb{R}_+$ of a function $f(\cdot) : \mathbb{R} \to \mathbb{R}^n$ is denoted by $f(k^-)$. Similarly, the right-hand limit of the function $f(k)$ is denoted by $f(k^+)$.

Consider a continuous-time random process $\{x(t) \in \mathbb{R}^n : t \in \mathbb{R}_+\}$ whose sample paths are right-continuous and satisfy the following differential equation:

$$\dot{x}(t) = f\left(x(t), u(t), w(t), d(t)\right) \tag{1}$$

where $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}^m$ is a control input, $d(t)$ is an external $\mathscr{L}_\infty$ disturbance with $|d(t)|_{\mathscr{L}_\infty} = D$ and $w(t)$ is a jump process

$$w(t) = \sum_{\ell=1}^{\infty} w_\ell \delta(t - \tau_\ell) \tag{2}$$

in which $\{w_\ell, \ell \in \mathbb{Z}_+\}$ is a Markov process describing the ℓth jump's size at jump instants $\{\tau_\ell\}_{\ell=1}^{\infty}$. The expectation of this stochastic process at time t will be denoted as $\mathbb{E}(x(t))$.

Let x^* be the equilibrium of system (1) with $f(x^*, 0, 0, 0) = 0$. The system in (1), (2) is said to be *almost-surely asymptotically stable* with respect to x^*, if

$$\lim_{t \to \infty} \Pr\left\{\sup_t |x(t)| \to x^*\right\} = 1$$

Given a constant positive $\Delta^* \in \mathbb{R}_+$, let $\Omega(\Delta^*)$ be a bounded set defined as $\Omega(\Delta^*) = \{x \in \mathbb{R}^n ||x - x^*| \leq \Delta^*\}$. The system in (1), (2) is said to be *almost-surely practical stable* with respect to $\Omega(\Delta^*)$, if there exists $\Delta > 0$ with $\Delta^* > \Delta$ such that if $|x(0) - x^*| \leq \Delta$, then

$$\lim_{t \to \infty} \Pr\left\{\sup_t |x(t)| \in \Omega(\Delta^*)\right\} = 1$$

The system in (1), (2) is *almost sure safe* if it is *almost surely asymptotically stable* with respect to equilibrium x^* or *almost surely practical stable* with respect to set $\Omega(\Delta^*)$. x^* is called safe equilibrium, and the states in set $\Omega(\Delta^*)$ are safe states.

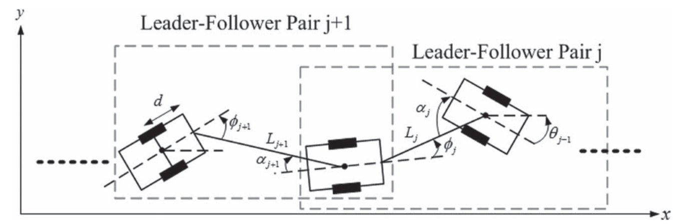

Fig. 1. A cascaded formation of nonholonomic vehicular system.

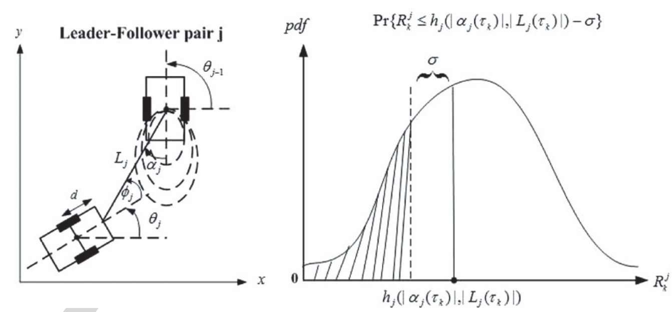

Fig. 2. Exponential Bounded Burstiness (EBB) Model for directional wireless channel.

III. System Description

A. System Model

Fig. 1 shows a string formation of N mobile robots. For each mobile robot, we consider the following kinematic model:

$$\dot{x}_i = v_i \cos(\theta_i),\ \dot{y}_i = v_i \sin(\theta_i),\ \dot{\theta}_i = \omega_i,\ i = 0, 1, \ldots, N-1 \tag{3}$$

where $(x_i(t), y_i(t))$ denotes the vehicle i's position at time $t \in \mathbb{R}_+$, $\theta_i(t)$ is the orientation of the vehicle relative to the x axis at time t. v_i and ω_i are the vehicle's speed and angular velocity that represent the control input.

As shown in Fig. 1, the cascaded formation with N mobile robots consists of $N-1$ leader-follower pairs. In each leader-follower pair j, we assume that the leader can directly measure its relative bearing angle α_j to the follower. Similarly, the follower can measure its bearing angle ϕ_j to the leader. Both of the vehicles are able to measure the relative distance L_j. What is not directly known to the follower is the relative bearing angle α_j. In this paper, we consider the case when information about leader's bearing angle α_j is transmitted over a wireless channel. The channel is accessed through a directional antenna whose radiation pattern is shown in Fig. 2.

The control objective of the cascaded formation is to have the follower in each leader-follower pair regulate its speed and angular velocity to achieve and maintain a desired distance and bearing angle. Let L_{d_j} and α_{d_j} denote the desired inter-vehicle distance and relative bearing angle, respectively, in the jth leader-follower pair. By using the similar technique in [12], the time rate of change of the relative distance L_j and leader's relative bearing angle α_j can be obtained as follows:

$$\begin{aligned} \dot{L}_j &= v_{j-1} \cos\alpha_j - v_j \cos\phi_j - d\omega_j \sin\phi_j \\ \dot{\alpha}_j &= \frac{1}{L_j}(-v_{j-1} \sin\alpha_j - v_j \sin\phi_j + d\omega_j \cos\phi_j) + \omega_{j-1} \end{aligned} \tag{4}$$

where d is the distance from the vehicle's center to its front.

Remark III.1: Note that $d > 0$ is a parameter for the vehicle and the relative distance measurement always satisfies $L_j \geq d$. When $d = 0$ (or if L_j is a distance from leader's center to follower's center), system (4) has a singular point at $L_j = 0$ and the state feedback linearization method in (8) cannot be applied due to [5].

B. Information Structure

As discussed in the previous section, the leader's bearing angle α_j in each leader-follower pair must be transmitted to the follower over a wireless channel. In this regard, the information about α_j that is available to the follower is limited by the following two constraints,

- The state measurement $\alpha_j(t)$ is only taken at a sequence of time instants $\{\tau_k\}_{k=0}^{\infty}$ that satisfy $\tau_k < \tau_{k+1},\ k = 1, 2, \ldots, \infty$.
- The sampled data $\alpha_j(\tau_k)$ is quantized with a finite number of bits $\overline{R}_j$, and is transmitted over an unreliable wireless channel with only first $R_j(k)$ bits ($R_j(k) \leq \bar{R}_j$) received at the follower.

At the kth sampling time instant, the triple $\{\hat{\alpha}_j(k^-), U_j(k), c_j(k)\}$ characterizes the information structure of the leader's bearing angle $\alpha_j(\tau_k)$ at the leader side. Assume that the measurement $\alpha_j(\tau_k)$ lies in an interval $[-U_j(k) + \hat{\alpha}_j(k^-), U_j(k) + \hat{\alpha}_j(k^-)]$ with $\hat{\alpha}_j(k^-)$ representing the "center" of the interval and $U_j(k)$ representing the length of the interval. The codeword $c_j(k) = \{b_{jl}(k)\}_{l=1}^{\bar{R}_j}$ consists of bits $b_{jl}(k) \in \{-1, 1\}$, and is constructed by truncating the first $\overline{R}_j$ bits of the following infinite length of bits:

$$\left\{ \{b_{jl}(k)\}_{l=1}^{\infty} \in \{-1,1\}^{\infty} | \alpha_j(\tau_k) = \hat{\alpha}_j(k^-) + U_j(k) \sum_{l=1}^{\infty} \frac{1}{2^j} b_{jl}(k) \right\}.$$

This corresponds to a uniform quantization of the sampled state within the interval $[-U_j(k) + \hat{\alpha}_j(k^-), U_j(k) + \hat{\alpha}_j(k^-)]$ with $\overline{R}_j$ number of bits.

We assume that the follower only successfully receives the first $R_j(k)$ bits in the codeword $c_j(k)$. The information structure at the follower side is another triple $\{\hat{\alpha}_j(k), U_j(k), \hat{c}_j(k)\}$ with $\hat{c}_j(k) = \{b_{jl}\}_{l=1}^{R_j(k)}$ and $\hat{\alpha}_j(k)$ being constructed as follows:

$$\hat{\alpha}_j(k) = \hat{\alpha}_j(k^-) + U_j(k) \sum_{l=1}^{R_j(k)} \frac{1}{2^j} b_{jl}(k) \tag{5}$$

$\hat{\alpha}_j(k)$ is an estimate of the leader's bearing angle $\alpha_j(k)$ at time instant τ_k.

In order to reconstruct the estimate $\hat{\alpha}_j(k)$, it is necessary to synchronize the leader and follower in the sense that they have the same information structure. We assume a noiseless feedback channel, with each successfully received bit being acknowledged to the leader. This allows one to ensure that the information structures are synchronized between the leader and follower. The follower then uses the estimated bearing angle $\hat{\alpha}_j(k)$, and the measured inter-vehicle distance L_j, to select its speed, v_j, and angular velocity ω_j to achieve the control objective.

C. Wireless Channel

As shown in Fig. 2, the leading vehicle in each pair uses a directional antenna to access the V2V wireless channel. We assume the V2V channels are free of interference from other leader-follower pairs, but the channel does exhibit deep fading. Deep fades occur when the channel gain drops below a threshold and stays below that threshold level for a random interval of time. In mobile communication networks, the wireless channel exhibits deep fades and has memory of its past channel states. Such time varying channel will increase the likelihood of a burst of packet dropouts. This fact has been recently found to be a fundamental factor that destabilizes networked control systems [21].

Outage probability is a well studied performance metric for wireless fading channel [19], [39]. In wireless networks, outage probability is commonly defined as the probability of the signal to noise ratio of a received signal being less than the threshold for reliable reception. This metric is closely related to the concept of EBB which was introduced in [41]. In [41], the EBB model was used to bound the likelihood of a string of consecutive dropouts for a single communication link. This model has been further explored in [9], [20] to study the burstiness of a communication network with multiple nodes. Thus, this paper also adopts the EBB model to characterize channel outages. The definition of the EBB model is

Definition III.2: Let $R_j(k)$ denote a random variable that characterizes the possible number of successfully decoded bits over time interval $[\tau_k, \tau_{k+1})$, then random process $\{R_j(k)\}$ is EBB with continuous, positive and monotonically decreasing functions $(h(\cdot,\cdot), \gamma(\cdot,\cdot))$, if the probability of successfully decoding $R_j(k)$ bits at each sampling time τ_k satisfies

$$\Pr\left\{R_j(k) \leq h\left(|\alpha_j(\tau_k)|, |L_j(\tau_k)|\right) - \sigma\right\} \leq e^{-\gamma(|\alpha_j(\tau_k)|, |L_j(\tau_k)|)\sigma} \tag{6}$$

for $|\alpha_j(\tau_k)| \leq \pi/2$ and $\sigma \in [0, h(|\alpha_j(\tau_k)|, |L_j(\tau_k))|]$ with

$$\Pr\left\{R_j(k) = 0\right\} = 1 \tag{7}$$

for $|\alpha_j(\tau_k)| > \pi/2,\ \forall\, k \in \mathbb{Z}_+$.

The (6) and (7) characterize the fact that if the follower vehicle is out of the antenna's radiation scope, i.e., $|\alpha_j(\tau_k)| > \pi/2$, then the communication link between the vehicles is broken. If the vehicle is within the scope, i.e., $|\alpha_j(\tau_k)| \leq \pi/2$, the probability of having a string of dropouts is exponentially bounded.

Fig. 2 shows a distribution of channel state $R_j(k)$ at time τ_k. The function $h(|\alpha_j|, |L_j|)$ in the EBB model is a threshold that partitions channel state space (horizontal axis in the figure) into high bit rate region (right from $h(|\alpha_j|, |L_j|)$) and low bit rate region (left to $h(|\alpha_j|, |L_j|)$). This threshold is a decreasing function of the absolute value of the current formation states $L_j(\tau_k)$ and $\alpha_j(\tau_k)$. It models the impact of the path loss on the data rate [39]. In the low bit rate region, $\sigma \in [0, h(|\alpha_j(\tau_k)|, |L_j(\tau_k))|]$ characterizes the *dropout burst length*. The function $\gamma(|\alpha_j|, |L_j|)$ is the exponent in the exponential bound and characterizes how fast the probability of a bursty dropout decays as a function of *dropout burst length* within the low bit rate region. This decay rate is also a decreasing function of $L_j(\tau_k)$ and $\alpha_j(\tau_k)$. It models that fact that the likelihood of having a bursty dropout in V2V channel increases with inter-vehicle distance and relative bearing angle. Thus, the EBB characterization explicitly models state dependency and bursty dropouts for a realistic V2V channel.

The following lemma shows that the EBB characterization in (6), (7) can be used to describe a wide range of channel models that include traditional i.i.d models [39] as well as two-state Markov chain models [42].

Lemma III.3: Consider a bit stream $\{b_{jl}(k)\}_{l=1}^{\overline{R}_j}$ that is sequentially transmitted over the fading channel, define a corresponding random process $\{X_{jl}(k)\}_{l=1}^{\overline{R}_j}$ with random variable $X_{jl}(k) \in \{0, 1\}$ taking value 1 when the corresponding bit successfully decoded and 0 otherwise. For fading channels that are modeled by either a i.i.d. process [39] or a two-state Markov process [42], i.e., $\{X_{jl}(k)\}_{l=1}^{\overline{R}_j}$ is a i.i.d. process or a two-state Markov process, there always exists a EBB characterization in (6), (7) with $R_j(k) = \sum_{l=1}^{\overline{R}_j} X_{jl}(k)$.

Proof: The proof is provided in the Appendix. ■

What should be apparent from the EBB model is that we are explicitly accounting for the relationship between bursty channel state $(R_j(k))$ and formation configuration. A major goal of this paper is to exploit that relationship in deciding how to switch between different controllers to assure almost sure performance.

D. Distributed Switching Control

In this paper, the control objective is to steer the cascaded vehicular system shown in Fig. 1 to a sequence of desired distances $\{L_{d_j}\}_{j=1}^{N-1}$ and bearing angles $\{\alpha_{d_j}\}_{j=1}^{N-1}$ in a distributed fashion, and then maintain around those set-points.

At each time instant $\{\tau_k\}_{k=0}^{\infty}$, the follower of each leader-follower pair switches among a group of controller gains to regulate its velocity and angular velocity to achieve the control objective. Let $K(k) := \{K_{\alpha_j}(k), K_{L_j}(k)\}$ denote the controller gain pair used for leader-follower pair j at time instant τ_k. These controller gains are selected from one pair of a collection of values $\mathscr{K}_j = \{K_{j_1}, K_{j_2}, \ldots, K_{j_M}\}$. Recall that the dynamic of formation configuration is equation (4), we use standard input to state feedback linearization to generate the control input

$$\begin{bmatrix} v_j \\ \omega_j \end{bmatrix} = \begin{bmatrix} -\cos\phi_j & -L_j \sin\phi_j \\ -\frac{\sin\phi_j}{d} & \frac{L_j}{d}\cos\phi_j \end{bmatrix} \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha_j}\right) \end{bmatrix} \tag{8}$$

over the time interval $[\tau_k, \tau_{k+1})$. The variable $\hat{\alpha}_j(t)$ is a continuous function over $[\tau_k, \tau_{k+1})$, and satisfies the following initial value problem:

$$\dot{\hat{\alpha}}_j = K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha}_j\right),\ \hat{\alpha}_j(\tau_k) = \hat{\alpha}_j(k) \tag{9}$$

where the estimate $\hat{\alpha}_j(k)$ is obtained from (5). With this control, the inter-vehicle distance L_j and bearing angle α_j satisfy the following differential equations over $[\tau_k, \tau_{k+1})$:

$$\begin{bmatrix} \dot{L}_j \\ \dot{\alpha}_j \end{bmatrix} = \begin{bmatrix} \cos\alpha_j & 0 \\ \frac{-\sin\alpha_j}{L_j} & 1 \end{bmatrix} \begin{bmatrix} v_{j-1} \\ \omega_{j-1} \end{bmatrix} + \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha}_j\right) \end{bmatrix} \tag{10}$$

for all $k = 1, 2, \ldots, \infty$.

The (9), (10) represent the closed-loop system for the leader-follower pair j and can be viewed as an example of a jump nonlinear system given in (1), (2). The $\mathscr{L}_\infty$ disturbance in the jth leader-follower system is $[v_{j-1}, \omega_{j-1}]$. The estimate of the bearing angle $\hat{\alpha}_j$ forms a jump process with jumps occurring at discrete time instants $\{\tau_k\}_{k=1}^{\infty}$. As shown in (5), the magnitude of the jump at each time instant is stochastically governed by the length of the uncertainty interval $U_j(k)$ and the number of received bits $R_j(k)$. Such jump process significantly impacts the formation performance of the cascaded system by pushing the formation state away from the equilibrium, which in turn leads to deep fades with a high probability. In the next section, we will show how to reconfigure the local controller gain in response to the changes of $U_j(k)$ and $R_j(k)$ such that almost sure performance is assured.

It is apparent from Fig. 1 that vehicle j for $j = 1, 2, \ldots, N-2$ plays a leader in leader-follower pair $j+1$ as well as a follower in leader-follower pair j. In this regard, vehicle j could observe the full state α_{j+1} of the leader-follower subsystem $j+1$ because it serves the leadership in that system. By observing the behavior of the following vehicle, vehicle j for $j = 1, 2, \ldots, N-1$ can adjust its controller gain to overcome large overshoots in the following system. Such cooperative control strategy lessens the amplification on the disturbance from the upper leader-follower systems to the lower systems.

IV. Main Results

This paper's main results consist of two parts regarding the *safe* behavior of inter-vehicle distance L_j and bearing angle α_j for each leader-follower pair. Specifically, "safe" means that the vehicle does not collide with each other and the bearing angle is regulated to stay in a specified bounded set almost surely. The first part of the results provides a sufficient condition under which the inter-vehicle distance L_j for $j = 1, 2, \ldots, N-1$ is almost surely convergent to a compact invariant set regardless of the changes on channel state. Furthermore, we show that the inter-vehicle distance is almost surely convergent to the desired separation $L_{d_j},\ j = 1, 2, \ldots, N-1$ if the bearing angle $\alpha_j,\ j = 1, 2, \ldots, N-1$ is almost surely convergent. The second part of the results derive sufficient conditions for the almost sure asymptotic stability and practical stability for the bearing angle $\alpha_j,\ j = 1, 2, \ldots, N-1$.

In the main results, we use the fact that the leader's action in each leader-follower pair can be constrained as a function of the following system's state to assure the stability for the whole leader-follower system. Proposition IV.1 provides an explicit characterization of the bound on the leader's action, as well as a distributed way to achieve that bound. Using the results from Proposition IV.1, one can easily prove the first main result in this paper (Lemma IV.5), i.e., the convergence of inter-vehicle distance since the distance is measurable to both leader and follower. The more challenging and interesting part of the results is to guarantee almost sure stability for the bearing angle α_j, which is presented in Section IV-B.

The following Proposition is provided to assure the control input from upper leader-follower subsystem is bounded as a function of state estimates of the bottom system. The proof is provided in the Appendix.

Proposition IV.1: Consider the closed-loop system in (9), (10), let $d \geq 1$, if there exists a sequence of controller gains $\{K_j(k)\}_{k=0}^{\infty}$, $K_j(k) = \{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$ such that for given monotonically increasing functions $W_j(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+,\ j = 1, 2, \ldots, N-1$, the following inequality holds for all $k = 0, 1, \ldots, \infty$:

$$\max\left\{\tilde{L}_{j,\max}, K_{\alpha_j}(k)\left|\tilde{\alpha}_j(k)\right|\right\} \leq \frac{W_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right)}{\left(1 + M_{L_j}(k)\right)} \qquad (11)$$

where

$$\begin{aligned}
\tilde{L}_{j,\max} &= K_{L_j}(k)\left|\tilde{L}_j(k)\right| e^{K_{L_j}(k)T_k} \\
&\quad + W_{j-1}\left(\left|\tilde{\alpha}_j(k)\right|\right)\left(e^{K_{L_j}(k)T_k} - 1\right) \\
M_{L_j}(k) &= \max\left\{\overline{L}_j(\tau_k), \overline{L}_j(\tau_{k+1})\right\} \\
\overline{L}_j(t) &= \left(L_{d_j} + \frac{W_{j-1}\left(\left|\tilde{\alpha}_j(k)\right|\right)}{K_{L_j}(k)}\right)\left(1 - e^{-K_{L_j}(k)(t-\tau_k)}\right) \\
&\quad + L_j(k)e^{-K_{L_j}(k)(t-\tau_k)} \\
\tilde{\alpha}_j(k) &= \alpha_{d_j} - \hat{\alpha}_j(k), \quad \tilde{L}_j(k) = L_{d_j} - L_j(k)
\end{aligned}$$

then

$$\sup_t \left|\left[\begin{array}{c} v_j(t) \\ \omega_j(t) \end{array}\right]\right| \leq W_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right), t \in [\tau_k, \tau_{k+1}). \qquad (12)$$

Because of inequality (12), each leader-follower subsystem j in (10) can bound the external disturbance $[v_{j-1}, \omega_{j-1}]$ by observing its local state estimate $\tilde{\alpha}_j$ at each time instant τ_k. Meanwhile, the subsystem $j-1$ can select its controller gain so that the control input $[v_{j-1}, \omega_{j-1}]$ satisfies the bound in inequality (12) because the estimate of bearing angle $\tilde{\alpha}_j$ is always available to subsystem $j-1$. Such property provides a basis for designing a distributed and cooperative switching law to assure the stability for the whole formation system.

Remark IV.2: Functions $W_j(\cdot)$ are upper bounds on the control inputs of upper leader-follower system and the values of $W_j(\cdot)$ at each time instant τ_k can also be seen as feedback signals from the bottom system. Such feedback signals directly constrain the magnitude of control input from upper system, so that the disturbances are not amplified from upper system to bottom system.

Remark IV.3: The inequality (11) could be viewed as a switching rule for the leader-follower pair j to react to the changes on system $j+1$'s bearing angle. The switching rule applied over each time interval $[\tau_k, \tau_{k+1})$ is feasible because it is only based on the information that is available at time τ_k.

With the validity of Proposition IV.1, the following corollary characterizes the propagated bound on the external inputs of the leader-follower chain as a function of the bearing angle's estimate in each leader-follower pair.

Corollary IV.4: Suppose the hypothesis of Proposition IV.1 holds then

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0 \circ \tilde{W}_1 \circ \cdots \circ \tilde{W}_j\left(\left|\tilde{\alpha}_{j+1}(k)\right|\right) \qquad (13)$$

where $v_0(k)$ and $\omega_0(k)$ are the speed and angular velocity of the first vehicle in the chain, and

$$\tilde{W}_j(\cdot) := \frac{1}{\left(1 + M_{L_j}(k)\right)K_{\alpha_j}(k)} W_j(\cdot),\ j = 1, \ldots, N-2.$$

Proof: Consider the first leader-follower pair, the Proposition IV.1 implies

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0\left(\left|\tilde{\alpha}_1(k)\right|\right).$$

Since

$$\left(1 + M_{L_1}(k)\right)K_{\alpha_1}(k)\left|\tilde{\alpha}_1(k)\right| \leq W_1\left(\left|\tilde{\alpha}_2(k)\right|\right)$$

holds due to inequality (11), then

$$\max\left\{\left|v_0(k)\right|, \left|\omega_0(k)\right|\right\} \leq W_0 \circ \tilde{W}_1\left(\left|\tilde{\alpha}_2(k)\right|\right).$$

Repeating above procedure leads to the final conclusion (13). ■

A. Almost Sure Convergence of Inter-Vehicle Distance L_j

In this section, we present the first main result of this paper involving the almost sure convergence of inter-vehicle distance. First, the following lemma provides a sufficient condition on the controller gain K_{L_j}, under which one can show $L_j(t)$ converges at an exponential rate to an invariant set $\Omega_{\text{inv, j}}$ centered at the desired inter-vehicle distance L_{d_j}, for $j = 1, 2, \ldots, N-1$ regardless of the change on channel state.

Lemma IV.5: Let the hypothesis of Proposition IV.1 hold, consider the system in (9), (10) with the selected controller gain $\{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$. If $K_{L_j}(k) > (W_j(|\tilde{\alpha}_j(k)|)/\rho(L_{d_j} - d))$ and $L_j(0) > d$, then for any sample path, $L_j(t) \geq d$ for all $t \in \mathbb{R}_+$ and there exists a finite time $\overline{T} > 0$ such that $L_j(t)$ enters and remains in the set

$$\Omega_{\text{inv, j}} \equiv \left\{L_j \in \mathbb{R}_+ \left| \left|L_j - L_{d_j}\right| \leq \frac{W_j\left(\left|\tilde{\alpha}_j(k)\right|\right)}{\rho K_{L_j}(k)}\right.\right\}$$

for all $t \geq \overline{T}$ and any $\rho \in (0, 1]$.

Proof: Consider the function $V(L_j) = (1/2)(L_j - L_{d_j})^2$ and closed-loop state (10). Taking the directional derivative of V over time interval $[\tau_k, \tau_{k+1})$ one obtains

$$\begin{aligned}
\dot{V}(L_j) &= -K_{L_j}\left(L_j - L_{d_j}\right)^2 + \left(L_j - L_{d_j}\right) \cdot v_{j-1}\cos\alpha_j \\
&\leq -K_{L_j}(1-\rho)\left(L_j - L_{d_j}\right)^2 - \rho \cdot K_{L_j}\left(L_j - L_{d_j}\right)^2 \\
&\quad + \left|L_j - L_{d_j}\right| W_j\left(\left|\tilde{\alpha}_j(k)\right|\right)
\end{aligned}$$

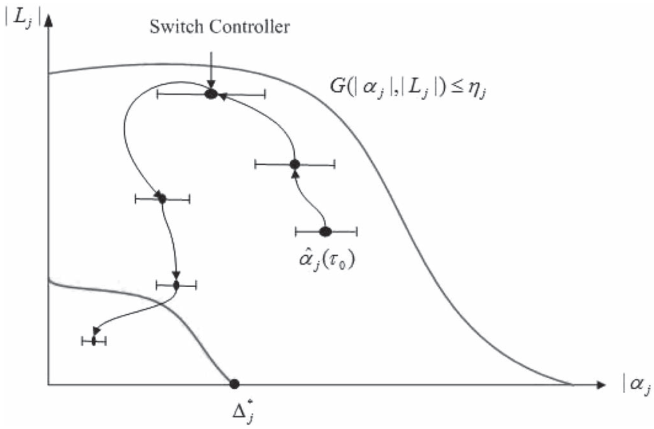

Fig. 3. Partition of formation state space.

for any $\rho\in(0,1]$. The last inequality holds because of Proposition IV.1. When $|L_j - L_{d_j}| \ge W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}$, the following dissipative inequality holds:

$$\begin{aligned}\dot{V}(L_j) \le & -K_{L_j}(1-\rho)\left(L_j - L_{d_j}\right)^2 \\ = & -2K_{L_j}(1-\rho)V(L_j). \qquad (14)\end{aligned}$$

This implies that $V(L_j(t))$ is exponentially decreasing when the state $L_j(t)$ is outside the set $\Omega_{\mathrm{inv,j}}$. Since $L_j(0) > d$, the feasible region outside the invariant set is $L_j(t) \ge (W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)) + L_{d_j} > d$. By inequality (14), it is clear that $L_j(t)$ converges to the set $\Omega_{\mathrm{inv,j}}$ in finite time and $L_j(t) > d$ for all time since for any $L_j \in \Omega_{\mathrm{inv,j}}$, it satisfies

$$L_j \ge -\frac{W_j\left(|\tilde{\alpha}_j(k)|\right)}{\rho K_{L_j}} + L_{d_j} > d.$$

Since the time interval $[\tau_k, \tau_{k+1})$ is selected arbitrarily, the conclusion holds for any $k \in \mathbb{Z}_+$. ■

Remark IV.6: Note that d is the distance from the center of the vehicle to the front of the vehicle. As shown in Fig. 1, $L_j(t) > d$ means that the two vehicles do not collide with each other.

Corollary IV.7: Consider closed-loop system in (9), (10), let the hypotheses of Proposition IV.1 and Lemma IV.5 hold. If the bearing angle α_j is almost surely convergent to α_{d_j} with $W_{j-1}(0) = 0,\ j = 1, 2, \ldots, N-1$, then the separation distance L_j almost surely converges to L_{d_j}.

Proof: From Lemma IV.5, one knows that the inter-vehicle separation converges to a invariant set with size of $W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)$. With $W_{j-1}(0) = 0,\quad j = 1, 2, \ldots, N-1$, and $\lim_{k\to\infty} \Pr\{\alpha_j(k) \to \alpha_{d_j}\} = 1$, it is easy to show that the event $\lim_{k\to\infty}(W_j(|\tilde{\alpha}_j(k)|)/\rho K_{L_j}(k)) = 0$ occurs with probability one as time goes infinity, i.e., the separation $L_j(t)$ almost surely converges to L_{d_j}. ■

B. Almost Sure Asymptotic Stability and Practical Stability for Bearing Angle α_j

This section provides the second main result of this paper that assures almost sure asymptotic stability and almost sure practical stability for the bearing angle α_j. Fig. 3 shows the basic idea and results. Two types of sets are depicted in Fig. 3 with one enclosed by the blue solid curve, and the other one enclosed by the red dashed curve. The set enclosed by the blue solid curve represents the partition generated by inequality $G(|\alpha_j|, |L_j|) \le \eta_j$ with associated threshold $\eta_j \in (0,1)$, which is shown in Lemma IV.9. The area enclosed by the red dashed curve characterizes the target set where the system trajectory will converge to almost surely. The size of the target set is characterized by Δ_j^*. The almost sure asymptotic stability result is interpreted as a special case when the target set contains only origin.

The main result states that the bearing angle α_j will almost surely converge to the target set if the system trajectory enters and remains in the set enclosed by the blue solid curve. To assure the invariance of the set enclosed by the blue solid curve, we adopt a switching control strategy to reconfigure the control gain for each leader-follower pair. Fig. 3 shows one possible evolution of the system trajectory α_j and L_j with the switching strategy. We use black dots to represent the estimates of the bearing angle $\hat{\alpha}_j(\tau_k)$ at each sampling time τ_k. A bar is used to characterize the uncertainty interval with the estimate $\hat{\alpha}_j(\tau_k)$ as its center. The length of bar can be viewed as an upper bound of the quantization error $|\alpha_j(\tau_k) - \hat{\alpha}_j(\tau_k)|$, and increases as the channel condition decreases. Therefore, the basic idea for switching is that when the system trajectory approaches the blue set's boundary with an increasing uncertainty length, an appropriate controller is re-selected to assure that the stochastic variation on the uncertainty length satisfies a super-martingale inequality, which guarantees the convergence of system states to the target set with probability one.

To be more specific about the main result, first, a dynamic quantization method is used to show that the quantization error $|\alpha_j(\tau_k) - \hat{\alpha}_j(\tau_k)|$ can be bounded by a recursively constructed sequence (Lemma IV.8). Then, a sufficient condition is presented to select controllers, under which the sequence (Lemma IV.9) and bearing angle estimate (Lemma IV.11) satisfy super-martingale like inequalities. Finally, the super-martingale inequality condition leads to the proof of almost sure asymptotic stability (Theorem IV.12) and practical stability (Theorem IV.14) for the bearing angle α_j.

Recall that $\{\alpha_j(k^-), U_j(k)\}_{k=0}^{\infty}$ characterizes the quantizer's state at each time instance τ_k. The following lemma gives a recursive construction for this sequence such that the quantization error remains bounded by some function of $U_j(k)$ for all $k \ge 0$. This bound is used to switch controllers to assure almost sure performance. Note that the technique used to prove the following lemma follows the pattern in dynamic quantization [6].

Lemma IV.8: Consider the closed-loop system in (9), (10), given the transmission time sequence $\{\tau_k\}_{k=0}^{\infty}$, and controller pairs $\{K_{L_j}(k), K_{\alpha_j}(k)\}_{k=0}^{\infty}$. Let $T_k = \tau_{k+1} - \tau_k$, let the hypothesis of Proposition IV.1 and Lemma IV.5 hold, the quantizer's initial state $\{\hat{\alpha}_j(0), U_j(0)\}$ is known to both leader and follower, and the initial state $\alpha_j(0) \in [-U_j(0), U_j(0)]$, $U_j(0) \le \pi/2$. If the sequence $\{\alpha_j(k^-), U_j(k)\}_{k=0}^{\infty}$ is constructed by the following recursive equation:

$$U_j(k+1) = B_j(k)T_k + 2^{-R_j(k)}U_j(k) \qquad (15)$$

$$\hat{\alpha}_j(k+1^-) = \left(\hat{\alpha}_j(k^+) - \alpha_{d_j}\right)e^{-K_{\alpha_j}(k)T_k} + \alpha_{d_j} \qquad (16)$$

where

$$B_j(k) = \max\left\{\frac{1}{\min\{L_{j\min}, L_j(k)\}}, 1\right\} W_{j-1}(|\tilde{\alpha}_j(k)|)$$

$$L_{j\min} = \left[-\tilde{L}_j(k) + \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)}\right] e^{-K_{L_j}(k)T_k} + L_{d_j} - \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)}$$

$$\tilde{L}_j(k) = L_{d_j} - L_j(k)$$

then the bearing angle $\alpha_j(k)$ for all $j = 1, 2, \ldots, N-1$ generated by system (9), (10) can be bounded as

$$\left|\alpha_j(k) - \hat{\alpha}_j(k^+)\right| \leq \overline{U}_j(k) \tag{17}$$

where $\overline{U}_j(k) = 2^{-R_j(k)} U_j(k)$ and $R_j(k)$ is the number of bits received over the time interval $[\tau_k, \tau_{k+1})$.

Proof: Let $e_j(t) = \alpha_j(t) - \hat{\alpha}_j(t)$ denote the estimation error. By inequality $d|e_j|/dt \leq |de_j/dt|$, the dynamic of $e_j(t)$ over time interval $[\tau_k, \tau_{k+1})$ is bounded by

$$\begin{aligned}\frac{d|e_j|}{dt} &\leq \left|\begin{bmatrix} -\frac{\sin\alpha_j}{L_j} & 1\end{bmatrix}\begin{bmatrix} v_{j-1} \\ \omega_{j-1}\end{bmatrix}\right| \\ &\leq \left(\frac{1}{|L_j|} + 1\right)\left|\begin{bmatrix} v_{j-1} \\ \omega_{j-1}\end{bmatrix}\right| \\ &\leq \left(\frac{1}{|L_j|} + 1\right) W_{j-1}(|\tilde{\alpha}_j(k)|).\end{aligned} \tag{18}$$

The last inequality holds because of Proposition IV.1. Since $\dot{L}_j \geq K_{L_j}(k)(L_{d_j} - L_j) - |v_{j-1}| \geq K_{L_j}(k)(L_{d_j} - L_j) - W_{j-1}(|\tilde{\alpha}_j(k)|)$, using Gronwall-Bellman inequality over $[\tau_k, \tau_{k+1})$ yields

$$L_j(t) \geq \left[L_j(\tau_k) - \left(L_{d_j} - \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)}\right)\right] e^{-K_{L_j}(k)(t-\tau_k)} + L_{d_j} - \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)}\Bigg).$$

Since $L_{d_j} \geq (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k))$ and $L_j(t) > d$ from Lemma IV.5, we know $\inf_{\tau_k \leq t < \tau_{k+1}} L_j(t)$ is obtained at either $t = \tau_k$ or $t = \tau_{k+1}$

$$L_j(t) \geq \inf_{\tau_k \leq t < \tau_{k+1}} L_j(t) = \min\{L_{j\min}, L_j(\tau_k)\} \tag{19}$$

where $L_{j\min} = [-\tilde{L}_j(k) + (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k))]e^{-K_{L_j}(k)T_k} + (L_{d_j} - (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k)))$. By inequality (19), (18) is rewritten as

$$\frac{d|e_j|}{dt} \leq \left(\frac{1}{\min\{L_{j\min}, L_j(\tau_k)\}} + 1\right) W_{j-1}(|\tilde{\alpha}_j(k)|).$$

Solving above differential inequality, we have

$$|e_j(t)| \leq \underbrace{\left(\frac{1}{\min\{L_{j\min}, L_j(\tau_k)\}} + 1\right) W_{j-1}(|\tilde{\alpha}_j(k)|)}_{B_j(k)} \times (t - \tau_k) + |e_j(\tau_k)|.$$

For $t \to \tau_{k+1}$, one can get $|e(k+1^-)| \leq B_j(k)T_k + |e_j(k)|$. And assume that $|e_j(k)| \leq \overline{U}_j(k)$, then $|e(k+1^-)| \leq B_j(k)T_k + \overline{U}_j(k)$. We know that

$$\begin{aligned}\left|e(k+1^+)\right| &\leq 2^{-R_j(k+1)}\left|e(k+1^-)\right| \\ &\leq 2^{-R_j(k+1)}\left(B_j(k)T_k + \overline{U}_j(k)\right).\end{aligned}$$

From (15) and $\overline{U}_j(k+1) = 2^{-R_j(k+1)}U_j(k+1)$, we have $|e(k+1^+)| \leq \overline{U}_j(k+1)$. The (16) holds by simply considering the solution to the ODE $\dot{\tilde{\alpha}}_j = -K_{\alpha_j}\tilde{\alpha}_j$ with initial value $\tilde{\alpha}_j = \alpha_{d_j} - \hat{\alpha}_j(k^+)$. ■

With Lemma IV.8, the following lemma provides a sufficient condition on the selection of controller gains that assures a super-martingale like property for the sequence $\{U_j(k)\}_{k=0}^{+\infty}$, $j = 1, 2, \ldots, N-1$.

Lemma IV.9: Consider the closed-loop system in (9), (10). Let

$$G(|\alpha_j|, |L_j|) = e^{-h(|\alpha_j|,|L_j|)\gamma(|\alpha_j|,|L_j|)}\left(1 + h(|\alpha_j|, |L_j|)\gamma(|\alpha_j|, |L_j|)\right)$$

be a non-negative, monotonically increasing function with respect to $|\alpha_j|$ and $|L_j|$ respectively. If there exists a sequence of controller gains $\{K_{L_j}(k), K_{\alpha_j}(k)\}_{k=0}^{\infty}$ with $K_j(k) = \{K_{L_j}(k), K_{\alpha_j}(k)\} \in \mathscr{K}_j$ for all $k \in \mathbb{Z}$ such that the Proposition IV.1 and following inequality hold for any $\eta_j \in (0, 1)$

$$\begin{aligned}&G\left(\overline{\alpha}_j(k+1), \overline{L}_j(k+1)\right) \leq \eta_j \\ &\overline{\alpha}_j(k+1) = \left|-\tilde{\alpha}_j(k)e^{-K_{\alpha_j}(k)T_k} + \alpha_{d_j}\right| + B_j(k)T_k + \overline{U}_j(k) \\ &\overline{L}_j(k+1) = L_{d_j} + \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)} - \left[\tilde{L}_j(k) + \frac{W_{j-1}(|\tilde{\alpha}_j(k)|)}{K_{L_j}(k)}\right]e^{-K_{L_j}(k)T_k}\end{aligned} \tag{20}$$

then

$$\mathbb{E}\left[\overline{U}_j(k+1)|\overline{U}_j(k)\right] \leq \eta_j\overline{U}_j(k) + \eta_j B_j(k)T_k, \qquad \forall k \in \mathbb{Z}_+. \tag{21}$$

Proof: Consider the sequence $\{\overline{U}_j(k)\}_{k=0}^{\infty}$ that satisfies (15) in Lemma IV.8, using the argument in [16], one has $\mathbb{E}[\overline{U}_j(k+1)|\overline{U}_j(k)] \leq G(|\alpha_j(k+1)|, |L_j(k+1)|)(B_j(k)T_k + \overline{U}_j(k))$. Let $G(|\alpha_j(k+1)|, |L_j(k+1)|) \leq \eta_j$, we have final conclusion (21) hold. In order to select the controller gain $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ for the time interval $[\tau_k, \tau_{k+1})$, the selection decision is made based only on the information at time instant τ_k. Thus, we further bound the state $|\alpha_j(k+1)|$ and $|L_j(k+1)|$ by considering $|e_j(k+1^-)| = |\alpha_j(k+1^-) - \hat{\alpha}_j(k+1^-)| \leq B_j(k)T_k + \overline{U}_j(k)$. Since $\alpha_j(k+1) = \alpha_j(k+1^-)$, we have

$$\begin{aligned}|\alpha_j(k+1)| &\leq \left|\hat{\alpha}_j(k+1^-)\right| + B_j(k)T_k + \overline{U}_j(k) \\ &\leq \left|\alpha_{d_j} - \left(\alpha_{d_j} - \hat{\alpha}_j(k)\right)e^{-K_{L_j}(k)T_k}\right| \\ &\quad + B_j(k)T_j + \overline{U}_j(k) \\ &\triangleq \overline{\alpha}_j(k+1).\end{aligned}$$

Similarly, one also has $|L_j(k+1)| \leq \overline{L}_j(k+1) = (L_{d_j} + (W_{j-1}(|\tilde{\alpha}_j(k)|)/K_{L_j}(k)))(1 - e^{-K_{L_j}(k)T_k}) + L_j(k)e^{-K_{L_j}(k)T_k}$

that is shown in Proposition IV.1. Since the function $G(|\alpha_j(k+1)|, |L_j(k+1)|)$ is a monotonically increasing function w.r.t $|\alpha_j(k+1)|$ and $|L_j(k+1)|$, and then if $G(\overline{\alpha}_j(k+1), \overline{L}_j(k+1)) \le \eta_j$, we have $G(|\alpha_j(k+1)|, |L_j(k+1)|) \le \eta_j$, then the final conclusion holds. ■

Remark IV.10: Function $G(\alpha_j, L_j)$ in condition (20) is directly related to the EBB model, and it generates a partition of the formation state space as shown in Fig. 3. Each partition associates with a threshold η_j that characterizes the convergent rate for the uncertainty set. The aim of switching control strategy is to guarantee that the condition (20) holds with a selected η_j.

Similar to Lemma IV.9, the following lemma shows that the sequence of the estimate of bearing angle $\{\tilde{\alpha}_j(k)\}_{k=0}^{\infty}$ for $j = 1, 2, \ldots, N-1$ satisfies a super-martingale like property as sequence $\{\overline{U}_j(k)\}_{k=0}^{\infty}$ does.

Lemma IV.11: Consider the system in (9), (10), given a sequence of controller pair $\{K_{L_i}(k), K_{\alpha_i}(k)\}_{k=0}^{\infty}$ with each $\{K_{L_i}(k), K_{\alpha_i}(k)\}$ selected at time instants $\{\tau_k\}_{k=0}^{\infty}$ and $\{K_{L_i}(k), K_{\alpha_i}(k)\} \in \mathscr{K}_i$. Let $K^*_{\alpha_i} = \min\{K_{\alpha_i} | K_{\alpha_i} \in \mathscr{K}_i\}$ and let $\mathscr{I}_k$ denote the information available at time instant τ_k, then we have

$$\mathbb{E}\left[|\tilde{\alpha}_i(k+1)| \,|\mathscr{I}_k\right] \le e^{-K^*_{\alpha_i} T_k} |\tilde{\alpha}_i(k)| + \left(B_i(k)T_k + \overline{U}_i(k)\right)(1 - 2^{-\bar{R}_i}).$$

Proof: Consider the time interval $[\tau_k, \tau_{k+1})$, by (9), we know that $\dot{\hat{\alpha}}_j = K_{\alpha_j}(k)(\alpha_{d_j} - \hat{\alpha}_j(t))$ with initial value $\hat{\alpha}_j(\tau_k)$. Therefore, let $\tilde{\alpha}_j(k) = \alpha_{d_j} - \hat{\alpha}_j(k)$, we have $\tilde{\alpha}_j(k+1^-) = e^{-K_{\alpha_j}(k)T_k}\tilde{\alpha}_j(k)$. Let $E_j(k+1) = \tilde{\alpha}_j(k+1) - \tilde{\alpha}_j(k+1^-)$, then $\tilde{\alpha}_j(k+1) = e^{-K_{\alpha_j}(k)T_k}\tilde{\alpha}_j(k) + E_j(k+1)$. Let $K^*_{\alpha_j} = \min\{K_{\alpha_j} | K_{\alpha_j} \in \mathscr{K}_j\}$, then

$$|\tilde{\alpha}_j(k+1)| \le e^{-K^*_{\alpha_j} T_k} |\tilde{\alpha}_j(k)| + |E_j(k+1)| \tag{22}$$

The term $|E_j(k+1)|$ can be bounded as $|E_j(k+1)| \le (B_j(k)T_k + \overline{U}_j(k))(1 - 2^{-R_j(k+1)})$. Taking the conditional expectation on both sides of inequality (22) with respect to the information $\mathscr{I}_k$ available at time instant τ_k and using above bound on $|E(k+1)|$ yield $\mathbb{E}[|\tilde{\alpha}_j(k+1)||\mathscr{I}_k] \le e^{-K^*_{\alpha_j}T_k}|\tilde{\alpha}_j(k)| + (B_j(k)T_k + \overline{U}_j(k))(1 - 2^{-R_j(k+1)})$. Since $R_j(k) \le \overline{R}_j$ for all $k \in \mathbb{Z}_+$, the final conclusion holds. ■

With Lemma IV.9 and IV.11, we proceed to state the main theorem of almost sure asymptotic stability as follows,

Theorem IV.12: Consider closed-loop system in (9), (10). Let the hypothesis of Lemma IV.9 hold, suppose there exists a positive constant value ε_j such that

$$B_j(k) = \max\left\{\frac{1}{\min\{L_{j\min}, L_j(k)\}}, 1\right\} W_{j-1}(|\tilde{\alpha}_j(k)|) \le \varepsilon_j |\tilde{\alpha}_j(k)|$$

for all $k \in \mathbb{Z}_+$, if

$$\max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, (\eta_j + 1 - 2^{-\bar{R}_j})\varepsilon_j T_k + e^{-K^*_{\alpha_j}T_k}\right\} \le \delta \tag{23}$$

where $\delta \in (0,1)$. Then the system state of bearing angle α_j almost surely asymptotically converges to α_{d_j} for $j = 1, 2, \ldots, N-1$.

Proof: We prove the almost sure convergence of α_i by proving $\lim_{k\to\infty} \mathbb{E}[\overline{U}_i(k) + \tilde{\alpha}_i(k)] \to 0$. Since $\alpha_i = \hat{\alpha}_i + e_i$, then $|\alpha_i(k) - \alpha_{d_i}(k)| \le |\hat{\alpha}_i(k) - \alpha_{d_i}| + \overline{U}_i(k)$. By Lemmas IV.9 and IV.11, one has $\mathbb{E}[\overline{U}_i(k+1) + \tilde{\alpha}_i(k+1)] \le \delta_i \mathbb{E}[\overline{U}_i(k) + \tilde{\alpha}_i(k)]$ with $\delta_i \in (0,1)$, if inequality (23) holds. Then, it is clear that $\lim_{k\to\infty}\mathbb{E}[|\alpha_i(k) - \alpha_{d_i}(k)|] \to 0$. Using Markov inequality, we have $|\alpha_i(k) - \alpha_{d_i}(k)| \to 0$ almost surely, i.e., the bearing angle sequence $\{\alpha_i(k)\}$ almost surely converges to α_{d_i}. Because the state trajectory has no finite escape within each time interval $[\tau_k, \tau_{k+1}),\ \forall\, k \in \mathbb{Z}_+$. Then, the system state of bearing angle $\alpha_i(t)$ is almost surely convergent to α_{d_i}. ■

Remark IV.13: The condition $B_j(k) \le \varepsilon_j|\tilde{\alpha}_j(k)|$ is equivalent to $W_{j-1}(\tilde{\alpha}_i(k)) \le \varepsilon_i|\tilde{\alpha}_j(k)|$ since $L_j(t) > d > 1$ for $t \in \mathbb{R}_+$.

Almost sure practical stability is a weaker safety notion than almost sure asymptotic stability, and it allows the bearing angles to fluctuate within a reasonable safe set. Theorem IV.14 provides a sufficient condition to assure almost sure practical stability for bearing angle $\alpha_j(t),\ j = 1, 2, \ldots, N-1$.

Theorem IV.14: Consider closed-loop system in (9), (10). Let the hypothesis of Lemma IV.9 hold, for given positive values Δ^*_j, $j = 1, 2, \ldots, N-1$, if there exists a controller pair $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ with $\eta_j(k)$ such that

$$B_j(k) \le \frac{1 - r_j}{J_j} \min\left\{\Delta^*_j, |\tilde{\alpha}_j(k)| + \overline{U}(k)\right\},\ j = 1, 2, \ldots, N-1 \tag{24}$$

with $r_j < 1$ where

$$r_j = \max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, e^{-K^*_{\alpha_j}T_k}\right\} \tag{25}$$

$$J_j = (\eta_j + 1 - 2^{-\bar{R}_j})T_k. \tag{26}$$

Then the bearing angle α_j of leader-follower pair i almost surely converges to a compact set defined by $\Omega_j = \{\alpha_j(t) : |\alpha_j(t) - \alpha_{d_j}| \le \Delta^*_j\}$.

Proof: By Lemmas IV.9 and IV.11, one has

$$\begin{aligned}\mathbb{E}\left[|\tilde{\alpha}_j(k+1)| + \overline{U}_j(k+1)|\mathcal{I}_k\right] &\le \max\left\{\eta_j + 1 - 2^{-\bar{R}_j}, e^{-K^*_{\alpha_j}T_k}\right\}\left(|\tilde{\alpha}_j(k)| + \overline{U}_j(k)\right) \\ &\quad + (\eta_j + 1 - 2^{-\bar{R}_j})T_k B_j(k).\end{aligned} \tag{27}$$

Let $V_j(k) = |\tilde{\alpha}_j(k)| + \overline{U}_j(k)$, and consider function $V_j(k)$ as a candidate Lyapunov function. It is clear that $V_j(k) \ge 0$ for any $k \in \mathbb{Z}_+$. Then, we can rewrite inequality (27) into $\mathbb{E}[V_j(k+1)|V_j(k)] \le \mathbb{E}[V_j(k+1)|\mathscr{I}_j(k)] \le r_j V_j(k) + J_j B_j(k)$. Furthermore, if the controller gains $\{K_{L_j}(k), K_{\alpha_j}(k)\}$ are selected to assure $r_j < 1$, we have $\mathbb{E}[V_j(k+1)|V_j(k)] \le V_j(k) - [(1 - r_j)V_j(k) - J_j B_j(k)]$. By condition (24), one can obtain

$$\begin{aligned}\mathbb{E}\left[V_j(k+1)|V_j(k)\right] &\le V_j(k) + (1 - r_j)\min\left\{\Delta^*_j - V_j(k), 0\right\} \\ &= V_j(k) - (1 - r_j)\max\left\{V_j(k) - \Delta^*_j, 0\right\}.\end{aligned} \tag{28}$$

From inequality (28), one can prove the bounded set $\hat{\Omega}_j = \{V_j(k) : V_j(k) \le \Delta^*_j\}$ is invariant with respect to system in (9) and (10) almost surely by considering

1) when $V_j(k) \le \Delta^*_j$, inequality (28) is reduced to $\mathbb{E}[V_j(k+1)|V_j(k)] \le V_j(k)$, which implies that sequence $\{V_j(k)\}$

is a super-martingale and remains in the set $\hat{\Omega}_j$ almost surely.

2) when $V_j(k) > \Delta_j^*$, $\exists \varepsilon > 0$ such that $\mathbb{E}[V_j(k+1)|V_j(k)] \leq V_j(k) - \varepsilon$. Clearly, the trajectory of $V_j(k)$ will asymptotically decrease until reaching the set $\hat{\Omega}_j$ almost surely.

This condition can be viewed as a stochastic version of the LaSalle Theorem in discrete time system. With condition (28), one can easily attain the almost sure convergence property for $V_j(k)$ with respect to set $\hat{\Omega}_j$, i.e., $\lim_{k\to+\infty} \Pr\{\sup_k V_j(k) \leq \Delta_j^*\} \to 1$. Since $|\alpha_j(k) - \alpha_{d_j}| \leq |\tilde{\alpha}_j(k)| + \overline{U}_j(k) = V_j(k)$, $|\alpha_j(k) - \alpha_{d_j}|$ converges to set Ω_j almost surely. Since the state trajectories remains bounded within each transmission time interval $[\tau_k, \tau_{k+1})$ for all $k \in \mathbb{Z}_+$. Therefore, we have $\lim_{t\to+\infty} \Pr\{\sup_t |\alpha_j(t) - \alpha_{d_j}| \leq \Delta_j^*\} \to 1$. ∎

Remark IV.15: Inequality (24) characterizes an upper bound on the propagated disturbance $B_j(k)$ under which the leader-follower pair j is almost sure practically stable. This upper bound is a increasing function of the size of target set Δ_j^*, the worst-case of bearing angle $|\tilde{\alpha}_j(k)| + \overline{U}(k)$, and a decreasing function of the ratio η_j.

Remark IV.16: Inequality (24) can be viewed as a distributed rule to select $\eta_j(k)$ to assure almost sure practical stability for each leader-follower pair. The selected $\eta_j(k)$ is used in Lemma IV.9 to switch controller.

The following corollary shows an explicit bound on the bearing angle under which it is almost surely convergent to a "safe" set $\Omega_j(\Delta_j^*)$. Such bound is a function of η_j and Δ_j^*.

Corollary IV.17: In Theorem IV.14, suppose $W_j(\tilde{\alpha}_j(k)) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$ holds with $g_j(\eta_j) := (1 - r_j)/\varepsilon_j J_j \geq 1$ and $r_j < 1$ where r_j and J_j are defined in (25). If

$$|\tilde{\alpha}_j(k)| + \overline{U}_j(k) \leq g_j(\eta_j)\Delta_j^* \tag{29}$$

then the bearing angle α_j almost surely converges to a bounded set $\Omega_j = \{\alpha_j(t) : |\alpha_j(t) - \alpha_{d_j}| \leq \Delta_j^*\}$.

Proof: From Theorem IV.14, we know that the sufficient condition to assure almost sure practical stability with set Ω_j is $B_j(k) \leq (1 - r_j/J_j)\min\{\Delta_j^*, |\tilde{\alpha}_j(k)| + \overline{U}(k)\}$. By condition $W_j(\tilde{\alpha}_j(k)) \leq \varepsilon_j |\tilde{\alpha}_j(k)|$, the above sufficient condition holds, if

$$\begin{aligned} |\tilde{\alpha}_j(k)| + \overline{U}_j(k) &\leq \frac{1 - r_j}{\varepsilon_j J_j} \min\left\{\Delta_j^*, |\tilde{\alpha}_j(k)| + \overline{U}(k)\right\} \\ &= g_j(\eta_j)\Delta_j^* \end{aligned}$$

holds. The equality holds because $g_j(\eta_j) := (1 - r_j)/\varepsilon_j J_j \geq 1$. Therefore, the conclusion holds. ∎

Remark IV.18: $g_j(\eta_j)$ is a monotonically decreasing function with respect to η_j, and it characterizes the size of the region from which the state almost surely converges to the set Ω_j with size Δ_j^*. The inequality (29) may be viewed as a partition of the physical state in the sense that small η_j gives rise to large contraction set.

V. Simulation Experiments

This section presents simulation experiments examining the resilience of our proposed switched controller to deep fades, and also demonstrates the benefits of using almost sure practical stability as a safety measurement over the traditional mean square stability.

A. Simulation Setup

In the simulation, we consider $N = 4$ vehicles that is cascaded in a string as shown in Fig. 1. Each leader-follower pair uses a two-state Markov chain model to simulate the fading channel between the leader and follower. The two-state Markov chain has two states with one representing the good channel condition and the other one representing the bad channel condition. Here, the "good channel state" simply means the transmitted bit is successfully received, while the "bad channel state" means the failure of receiving the bit.

Following the characterization of Makov chain model in [42], one can find that the conditional probability for good channel state is a monotonically decreasing function of $L_j(t)/\cos\alpha_j(t)$, while the conditional probability for bad channel state is a monotonically decreasing function of $\cos\alpha_j(t)/L_j(t)$. The explicit function form depends on the distribution of the channel gain. In this simulation, we use $p_{11} = e^{-3\times10^{-3}(L_j(t)/\cos\alpha_j(t))^2}$ to denote the conditional probability for the good channel state and $p_{22} = e^{-6\times10^{2}(\cos\alpha_j(t)/L_j(t))^2}$ to represent the conditional probability for the bad channel condition. The corresponding transition probabilities between these states are $1 - p_{11}$ and $1 - p_{22}$. Then, we use the EBB model in (6) to characterize the low bit region generated by the two-state Markov chain model. The corresponding functions in EBB model (6) are $h(\alpha_j, L_j) = \bar{R}_j e^{-3\times10^{-4}(L_j(t)/\cos\alpha_j(t))^2}$, $\gamma(\alpha_j, L_j) = e^{-4.5\times10^{-3}(L_j(t)/\cos\alpha_j(t))^2}$ with $\bar{R}_j = 2$ representing two bits that are transmitted at each sampling period.

The 100 ms sampling time that is consistent with the transmission frequency in V2V communication technology [10] is widely used in mobile robot system, is selected for each leader-follower pair ($j = 1, 2, 3$), i,e, $T_k = 0.1$ sec for all $k \in \mathbb{Z}_+$. The functions $W_{j-1}(\cdot)$ in Proposition IV.1 are selected to be linear functions $W_{j-1}(|\tilde{\alpha}_j(t)|) = a_j|\tilde{\alpha}_j(t)| + b_j$ with parameters being $a_1 = 0.1$, $b_1 = 0.01$; $a_2 = 0.8$, $b_2 = 2$; $a_3 = 1$, $b_3 = 4$. The value of the parameter sets are chosen to be increasing with respect to j to guarantee the feasibility of the controller selection for each leader-follower system.

In this simulation, we consider an interesting and realistic scenario that the fourth vehicle from far distance intends to join the other three closed-spaced vehicles. Hence, the initial states for three leader-follower pairs ($j = 1, 2, 3$) are $\alpha_1(\tau_0) = \pi/3$, $\alpha_2(\tau_0) = \pi/4$, $\alpha_3(\tau_0) = \pi/6$ and $L_1(\tau_0) = 7.1$ m, $L_2(\tau_0) = 7.1$ m, $L_3(\tau_0) = 99$ m. The initial uncertainties are $U_j(\tau_0) = \pi/6$. By switching controller pairs from sets $\mathscr{K}_j = \{(K_{L_j}, K_{\alpha_j}) : 0 < K_{L_j} \leq 100,\ 0 < K_{\alpha_j} \leq 100\}$, each leader-follower pair is required to achieve and maintain desired set-points $\alpha_{d_j} = 0$, $L_{d_j} = 2$ m, $j = 1, 2, 3$.

B. Simulation Results

A Monte Carlo method was used to verify that the system has almost surely practical stability when Proposition IV.1 and Theorem IV.14 hold. Each simulation example is run 100 times over a time interval from 0 to 10 seconds.

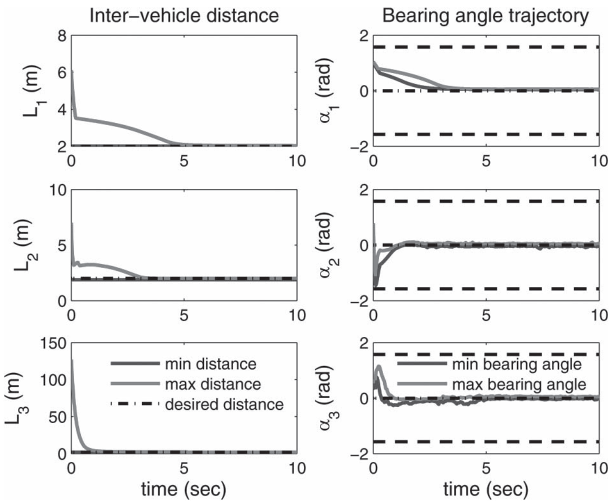

Fig. 4. The maximum and minimum value of separation L_j (m) and bearing angle α_j (rad) for leader-follower pair, $j = 1, 2, 3$.

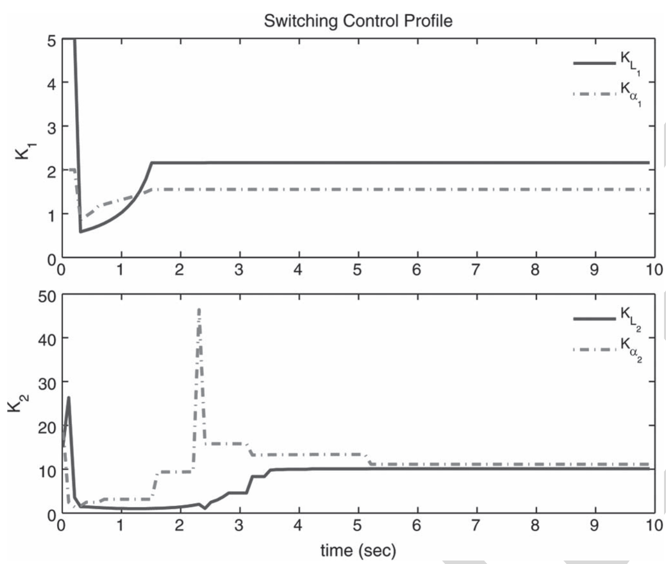

Fig. 5. One sample of switching controller profile for leader-follower pair 1 (Top) and 2 (Bottom): K_{L_j} and K_{α_j} are controller gains for the distance and bearing angle of leader-follower j, $j = 1, 2$.

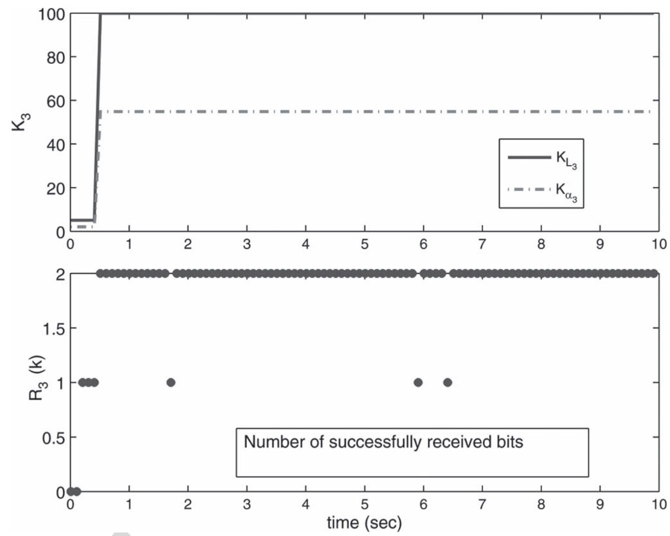

Fig. 6. One sample of switching profile (Top) and channel state (Bottom) for leader-follower pair 3: K_{L_3} and K_{α_3} are controller gains and $R_3(k)$ is the number of successfully received bits at each time interval $[\tau_k, \tau_{k+1})$.

In the first simulation, we select the controllers for each leader-follower pair from $\mathscr{K}_j$, $j = 1, 2, 3$ so that Proposition IV.1 and Theorem IV.14 hold at each time instant τ_k. Fig. 4 shows the maximum and minimum values of the system states L_j and α_j, $j = 1, 2, 3$ evaluated over all the 100 runs. The maximum value is marked by red lines and the minimum value is marked by blue lines. The two dashed lines in Fig. 4 represent the upper and lower bound for the relative bearing α, i.e., $|\alpha_j| \leq \pi/2$, which characterizes the safety region. We can see from Fig. 4 that the maximum and minimum values of the system states asymptotically converge to a bounded set containing the desired set-points $\alpha_{d_j} = 0$ and $L_{d_j} = 2$ m. This is precisely the behavior that one would expect if the system is almost sure practically stable. These results, therefore, seem to confirm our statement in Theorem IV.14.

Figs. 5 and 6 show one sample of switching controller profile and channel state for each leader-follower pair. The top plot in Fig. 5 shows the switching controller profile for the leader-follower pair 1 with red line marked as controller gain K_{α_1} and blue line as controller gain K_{L_1}. The bottom one is the switching controller profile for leader-follower pair 2 with the same marking rule. These plots show that the controller gains stay low at the first two seconds to avoid large disturbance to the bottom system, and then switch from low to high when the systems approach the equilibrium and are confident that the channel state will always stay good. The top plot in Fig. 6 is the switching controller profile for the leader-follower system 3 with same marking rule, and the bottom plot is the channel state $R_3(k)$ that characterizes the number of successfully received bits at each time interval. We can clearly see from the plots that the controller for system 3 starts with low gains to compensate the effect caused by a short string of zero bits at the beginning, and then switches from low gain to high gain when channel condition stays good. These results demonstrate that channel state indeed is used as a feedback signal to switch the controller.

In the second simulation, we studied the benefits of almost sure practical stability as a safety measurement over the traditional mean square stability. Traditional mean square stability requires the second moment of the system state converges to a positive constant value, but it does not put any constraint on the sample path which might potentially cause safety issues. For a fair comparison, the same simulation setup and parameters are applied in this simulation with the only difference being on the controllers. One type of controller used in this simulation is a mean square stabilizing controller, which is selected to guarantee mean square stability for each leader-follower pair. The other type of controller is the switching controller proposed in this paper to guarantee almost sure practical stability for each leader-follower pair. The switching control strategy uses the mean square stabilizing controller as its initial controller.

Fig. 7 shows a comparison of the maximum and minimum values of the bearing angle α_3 for leader-follower pair 3 with the switching controller case in the top plot and the mean square controller $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$

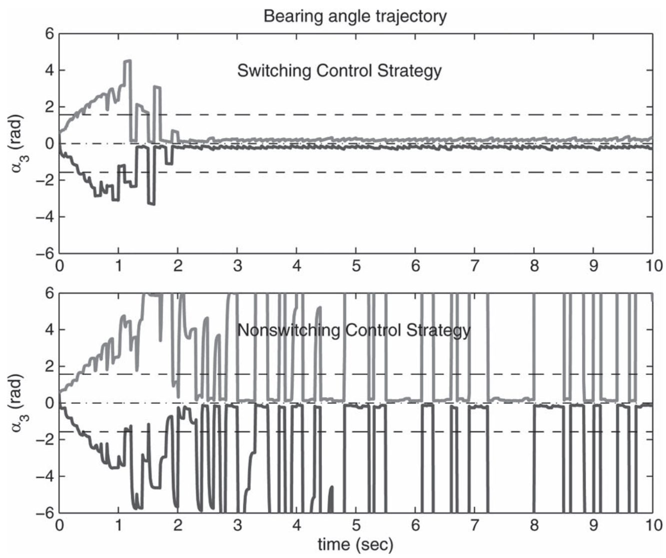

Fig. 7. The maximum and minimum system trajectory for leader-follower pair 3 with switching controller (Top) and non-switching controller pair (Bottom) $K_{L_3} = 2$ and $K_{\alpha_3} = 50$.

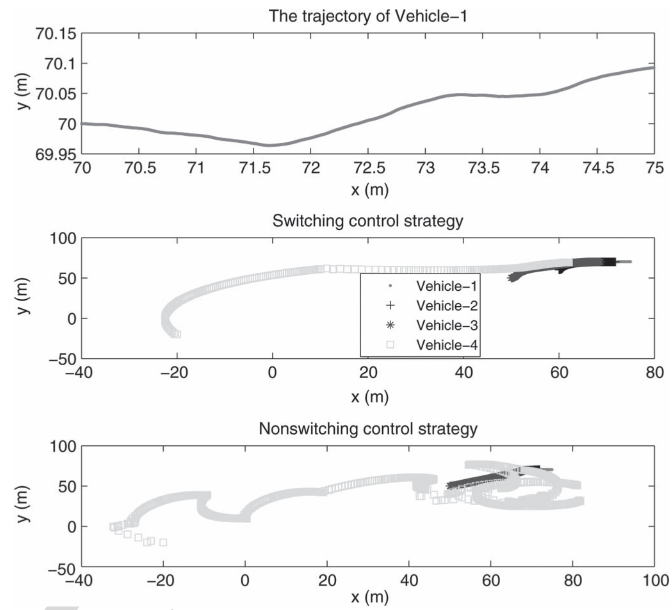

Fig. 8. The comparison of one sample run (10 seconds) of vehicles' trajectories generated by a switching control strategy (Middle) and a non-switching strategy (Bottom) with $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$. The top plot is the trajectory of the leading vehicle (Vehicle-1).

in the bottom plot. It is worth noting that (K_1, K_2, K_3) is just one of the many selections in our simulation. Because of the space limitations, we only use (K_1, K_2, K_3) as an example to demonstrate the results. It is clear from Fig. 7 that the system's sample path goes unbounded as time increases by using a mean square stabilizing controller, but it converges asymptotically to a bounded set by using a switching controller. These results suggest that the composition of mean square stable systems does not guarantee mean square stability for the whole system, while the composition of almost sure stable systems may still guarantee almost sure stability for the whole system.

Fig. 8 shows the comparison of one sample run of vehicles' trajectories in Euclidean space that are generated by the switching control strategy proposed in this paper and the non-switching strategy with controller gain $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$. The top plot of the figure is the leading vehicle's trajectory generated by a velocity profile $(v_1(t), \omega_1(t))$ which satisfies the condition in Corollary IV.4. The middle plot shows the trajectories of four vehicles that adopt the switching strategy where the red, black, blue and green dots represent the trajectories of leading vehicle (Vehicle-1), Vehicle-2, Vehicle-3 and Vehicle-4 respectively. It is clear from the plot that the leader-follower system almost surely converges to the specified formation. The bottom plot shows the result for non-switching control strategy using the mean square controller $K_1 = (5, 0.5)$; $K_2 = (5, 0.5)$; $K_3 = (2, 50)$ which exhibits significantly unsafe oscillatory behavior in Vehicle-4.

VI. Conclusion

This paper studies the almost sure safety property for a chain of leader-follower vehicular networked system in the presence of a V2V channel that exhibits exponentially bounded burstiness and varies as a function of vehicular state. The concept of almost sure safety is examined in terms of almost sure asymptotic stability and practical stability. Switching strategy is adopted to assure almost sure safety by adaptively reconfiguring local controller gains to the changes of channel state. Sufficient conditions are provided to decide which controller is placed in the feedback loop at each transmission time. As a result of the correlation between channel state and physical vehicular state, the sufficient conditions partition the vehicular state space into a set of regions in which controllers are designed to achieve almost sure safety. The simulation results of a four-vehicle leader-follower formation control are provided to support our theoretical analysis and illustrate the benefit of using almost sure practical stability as a safety measurement over traditional mean square stability.

It is important to note that this paper studies the effect of a V2V communication channel on the safety of leader-follower systems under the assumption that no measurement noise is present in the system. This assumption turns out to be a necessary and sufficient condition to assure almost sure stability due to negative results in [25]. One can only hope for a weaker notion of stochastic stability if state-independent noise is present in the system. Addressing this issue is beyond the scope of this paper and will be explored in our future work.

Appendix

Proof of Proposition IV.1: Consider the infinite norm of the control input given in (8)

$$\begin{aligned} &\left\| \begin{bmatrix} v_j(t) \\ \omega_j(t) \end{bmatrix} \right\| \\ &\le \left\| \begin{bmatrix} -\cos\phi_j & -L_j \sin\phi_j \\ -\frac{\sin\phi_j}{d} & \frac{L_j}{d}\cos\phi_j \end{bmatrix} \right\| \left\| \begin{bmatrix} K_{L_j}(k)\left(L_{d_j} - L_j\right) \\ K_{\alpha_j}(k)\left(\alpha_{d_j} - \hat{\alpha}_j\right) \end{bmatrix} \right\| \\ &\le (1 + |L_j(t)|) \max\left\{ K_{L_j}(k) \left|\tilde{L}_j(t)\right|, K_{\alpha_j}(k) \left|\tilde{\alpha}_j(t)\right| \right\} \end{aligned} \quad (30)$$

with $\tilde{L}_j(t) = L_{d_j} - L_j(t)$. The supreme of $|L_j(t)|$ over time interval $[\tau_k, \tau_{k+1})$ can be obtained by considering $\dot{L}_j(t) \leq K_{L_j}(k)(L_{d_j} - L_j)(t) + W_{j-1}(|\tilde{\alpha}_j(k)|)$. Using Gronwall Bellman theorem to solve above inequality and yield

$$\begin{aligned} L_j(t) \leq & L_j(k) e^{-K_{L_j}(k)(t-\tau_k)} \\ & + \left(L_{d_j} + \frac{W_{j-1}(|\alpha_j(k)|)}{K_{L_j}(k)}\right)\left(1 - e^{-K_{L_j}(k)(t-\tau_k)}\right) \\ \triangleq & \overline{L}_j(t). \end{aligned}$$

Assume $L_j(t) > 0$ (In Lemma IV.8, we prove that if controller gain $K_{L_j}(k)$ is selected sufficiently large, $L_j(t) > d > 0$ holds for all $t \geq 0$), and because $d\overline{L}_j/dt \geq 0$ or $d\overline{L}_j/dt < 0$ over interval $[\tau_k, \tau_{k+1})$. In other words, $\overline{L}_j(t)$ is a monotonically function over $[\tau_k, \tau_{k+1})$. Thus $\sup_{\tau_k \leq t < \tau_{k+1}} L_j(t)$ is obtained when $t = \tau_k$ or $t \to \tau_{k+1}$, i.e.,

$$L_j(t) = \max\left\{\overline{L}_j(\tau_k), \overline{L}_j(\tau_{k+1})\right\} \triangleq M_{L_j}(k). \tag{31}$$

Note that over time interval $[\tau_k, \tau_{k+1})$, one has $d|\tilde{L}_j(t)|/dt \leq K_{L_j}(k)|\tilde{L}_j(t)| + W_{j-1}(\tilde{\alpha}_j(k))$ thus

$$\begin{aligned} \sup_{\tau_k \leq t \leq \tau_{k+1}} K_{L_j}(k)\left|\tilde{L}_j(t)\right| = & K_{L_j}(k)\left|\tilde{L}_j(k)\right| e^{K_{L_j}(k)T_k} \\ & + W_{j-1}(|\tilde{\alpha}_j(k)|)\left(e^{K_{L_j}(k)T_k} - 1\right) \\ \triangleq & \tilde{L}_{j,\max}(k). \end{aligned} \tag{32}$$

By inequalities (31), (32), (30) can be further bounded

$$\left\|\begin{bmatrix} v_j(t) \\ \omega_j(t) \end{bmatrix}\right\|_\infty \leq \left(1 + M_{L_j}(k)\right)\max\left\{\tilde{L}_{j,\max}(k), K_{\alpha_j}(k)\left|\tilde{\alpha}_j(t)\right|\right\} \tag{33}$$

with $\tilde{\alpha}_j(t) = \alpha_{d_j} - \hat{\alpha}_j(t)$ satisfying $\dot{\tilde{\alpha}}_j = -K_{\alpha_j}(k)\tilde{\alpha}_j, t \in [\tau_k, \tau_{k+1})$ with initial value $\tilde{\alpha}_j(\tau_k)$. From the solution of the above ODE, it is obvious that $|\tilde{\alpha}_j(t)| < |\tilde{\alpha}_j(\tau_k)|$, then it is straightforward to show that if the condition (11) is satisfied, the inequality (12) holds. ■

Proof of Lemma III.3: Consider the case that the collection of random variables $\{X_{jl}(k)\}$ is i.i.d. and the probability of successfully decoding a packet is equal to the probability that the signal to noise ratio (SNR_{jl}) exceeds some fixed threshold γ_0 [39], i.e.,

$$\Pr\{X_{jl}(k) = 1\} = \Pr\{\text{SNR}_{jl} \geq \gamma_0\}.$$

The selection of the threshold γ_0 is often directly related to the communication system (e.g. modulation scheme). We assume a pre-selected γ_0 for a fixed communication system. For Raleigh fading, one can explicitly compute the successfully decoding probability as

$$\Pr\{\text{SNR}_{jl} \geq \gamma_0\} = e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \triangleq p(L_j, \alpha_j)$$

with $\overline{\gamma}(L_j, |\alpha_j|) = \mathbb{E}(P_{rec}(L_j, |\alpha_j|))/N_0$ where $P_{rec}(L_j, |\alpha_j|)$ and N_0 are powers of the receiving signal and noise respectively. According to directional antenna gain theory [2], one knows that $P_{rec}(L_j, |\alpha_j|)$ is a monotonically decreasing function with respect to $L_j \in (0, +\infty)$ and $|\alpha_j| \in [0, \pi/2]$ and so does $p(L_j, |\alpha_j|)$. Since $\{X_{jl}(k)\}$ is i.i.d., one has that $R_j(k) = \sum_{l=1}^{\overline{R}_j} X_{jl}(k)$ follows a binomial distribution with mean value $\mathbb{E}(R_j(k)) = \overline{R}_j p(L_j, |\alpha_j|)$. Using Chernoff inequality, one has

$$\begin{aligned} \Pr & \left\{R_j(k) \leq (1-\delta)\overline{R}_j p(L_j, |\alpha_j|)\right\} \\ & \leq e^{-\frac{\delta^2}{2}\overline{R}_j p(L_j, |\alpha_j|)}, \delta \in (0, 1). \end{aligned}$$

Let $h(L_j, |\alpha_j|) = \delta_d \overline{R}_j p(L_j, |\alpha_j|)$ for some $\delta_d \in (0, 1]$, then

$$\begin{aligned} & \Pr\{R_j(k) \leq (1-\delta)h(L_j, |\alpha_j|)\} \\ & = \Pr\left\{R_j(k) \leq (1-\delta)\delta_d \overline{R}_j p(L_j, |\alpha_j|)\right\} \\ & = \Pr\left\{R_j(k) \leq (1 - (1 - \delta_d + \delta_d\delta))\overline{R}_j p(L_j, |\alpha_j|)\right\} \\ & \leq e^{-\frac{(1-\delta_d+\delta_d\delta)^2}{2\delta_d\delta}\delta h(L_j, |\alpha_j|)} \end{aligned} \tag{34}$$

Let $\sigma = \delta h(L_j, |\alpha_j|)$, $\hat{\gamma}(\delta) = (1 - \delta_d + \delta_d\delta)^2/2\delta_d\delta$, then wen have

$$\Pr\{R_j(k) \leq h(|L_j|, |\alpha_j|) - \sigma\} \leq e^{-\hat{\gamma}(\delta)\sigma}$$

where $\sigma \in [0, h(|L_j|, \|\alpha_j\|))$. The last inequality holds due to Chernoff inequality. Taking the first derivative of $\hat{\gamma}(\delta)$ w.r.t δ, one has

$$\frac{d\hat{\gamma}}{d\delta} = \frac{\overbrace{(1 - \delta_d + \delta_d\delta)}^{>0}(\delta_d\delta - 1 + \delta_d)}{2\delta_d\delta^2}.$$

Clearly, given $0 < \delta_d < 1$, $\hat{\gamma}$ has the minimum value at $\delta^* = (1/\delta_d) - 1$, and $\hat{\gamma}(\delta^*) = 2(1 - \delta_d)$. One has a EBB characterization as follows:

$$\Pr\{R_j(k) \leq h(|L_j|, |\alpha_j|) - \sigma\} \leq e^{-2(1-\delta_d)\sigma}.$$

Consider the case that the collection of random variables $\{X_{jl}(k)\}$ is a two-state Markov process and for Rayleigh fading channels, the transition probability matrix $M(k)$ for a two-state Markov chain can be obtained by using the technique in [42], as seen in (35), as shown at the top of the next page, for $l = 2, 3, \ldots, N$ and c is the system parameter for a selected V2V wireless system and is sufficiently small to assure the transition probability is valid, i.e., within [0, 1]. Note that the function forms in (35) are particular for Rayleigh fading channels. One may not have explicit function form for other type of fading channel, but the fundamental relationship between physical state L_j, α_j and the fading function should remain the same. Given the transition probability in (35), the stationary distribution π_1 and π_0 can be obtained as

$$\begin{pmatrix} \pi_1 \\ \pi_0 \end{pmatrix} = \begin{pmatrix} e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \\ 1 - e^{-\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} \end{pmatrix}.$$

Let $\lambda_2(M)$ denote the second largest eigenvalue of transition matrix M, and it is easy to obtain $\lambda_2(M)$ as follows:

$$\lambda_2(M) = 1 - c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} - c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j, |\alpha_j|)}} - 1}$$

$$
\begin{aligned}
M(k) &= \begin{pmatrix} \Pr\left\{X_{jl}(k)=1 | X_{j(l-1)}(k)=1\right\} & \Pr\left\{X_{jl}(k)=1 | X_{j(l-1)}(k)=0\right\} \\ \Pr\left\{X_{jl}(k)=0 | X_{j(l-1)}(k)=1\right\} & \Pr\left\{X_{jl}(k)=0 | X_{j(l-1)}(k)=0\right\} \end{pmatrix} \\
&= \begin{pmatrix} 1 - c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}} & c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}-1} \\ c\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}} & 1 - c\frac{\sqrt{\frac{2\pi\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}}{e^{\frac{\gamma_0}{\overline{\gamma}(L_j,|\alpha_j|)}}-1} \end{pmatrix}
\end{aligned} \tag{35}
$$

With results in [8], we know that there also exists Chernoff type bound for finite Markov Chains. In particular, if the two-state Markov chain starts with its stationary distribution π_0 and π_1, then for $0 < \delta < 1$, we have

$$\Pr\left\{R_j(k) \le (1-\delta)\pi_1\overline{R}_j\right\} \le e^{-(1-\lambda_2(M))\delta^2\pi_1\overline{R}_j}. \tag{36}$$

The transformation used in inequality (34) can be applied to probability inequality (36). Let $h(L_j,\alpha_j) = \delta_d\overline{R}_j\pi_1(L_j,\alpha_j)$ for some selected $\delta_d \in (0,1]$, then

$$
\begin{aligned}
&\Pr\left\{R_j(k) \le (1-\delta)h(L_j,\alpha_j)\right\} \\
&\le e^{-(1-\lambda_2(M))\frac{(1-\delta_d+\delta_d\delta)^2}{\delta_d\delta}\delta h(L_j,\alpha_j)} \\
&\le e^{-f(L_j,|\alpha_j|)4(1-\delta_d)\delta h(L_j,\alpha_j)}
\end{aligned}
$$

where $f(L_j,|\alpha_j|) = c(\sqrt{2\pi(\gamma_0/\overline{\gamma}(L_j,|\alpha_j|))})/(1-e^{-(\gamma_0/\overline{\gamma}(L_j,|\alpha_j|))})$ with $c > 0$. It is easy to check that function $f(L_j,|\alpha_j|)$ is monotonically decreasing with respect to L_j and $|\alpha_j|$. Hence, one can always find corresponding EBB characterizations for both i.i.d and two-state Markov processes with monotonically decreasing function pairs $\{\delta_d\overline{R}_j p(L_j,\alpha_j), 2(1-\delta_d)\}$ and $\{\delta_d\overline{R}_j\pi_1(L_j,\alpha_j), 4(1-\delta_d)f(L_j,|\alpha_j|)\}$, respectively. ■

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