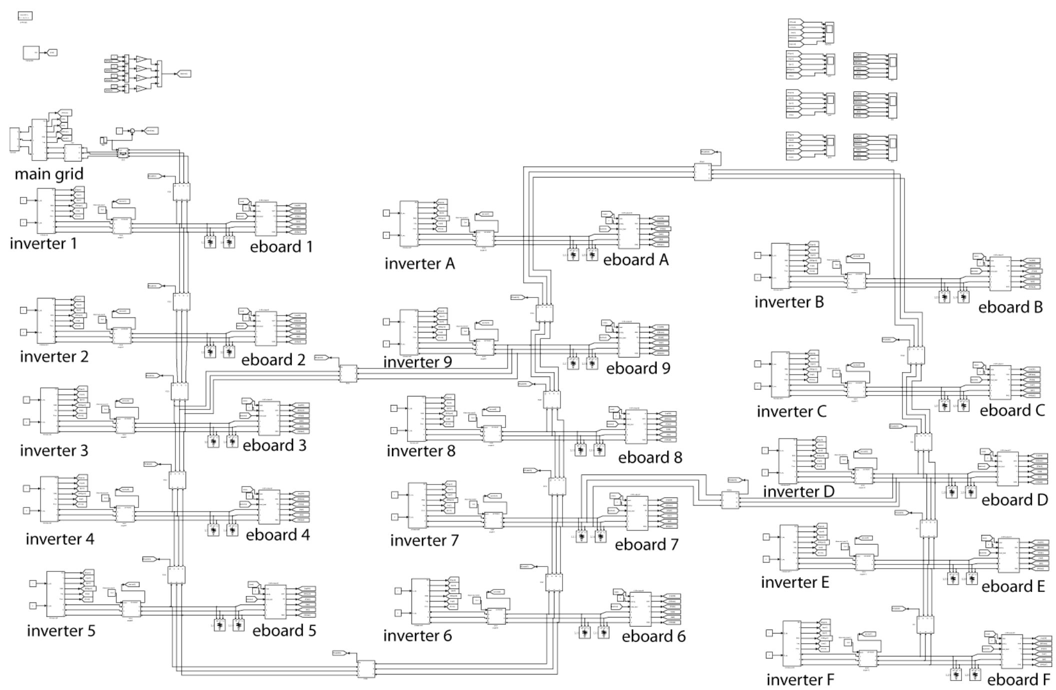
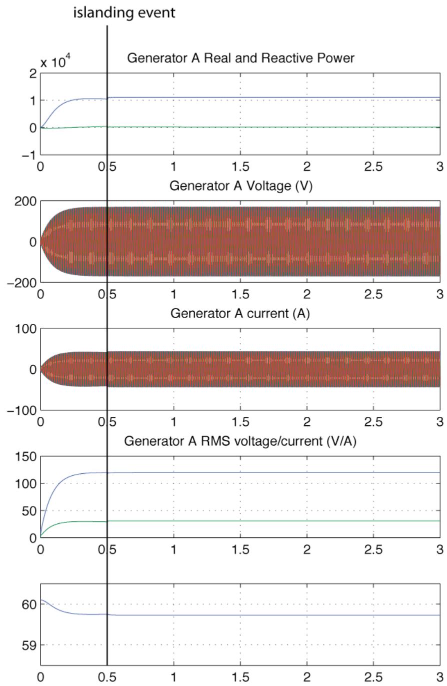
MONTHLY PROGRESS REPORT	
Contractor Name: University of Notre Dame (Michael Lemmon)	
Contractor Address: Office of Research, 940 Grace Hall, Notre Dame, IN 46556	
Contract/Purchase Order No. W9132T-10-C-0008 (prime contract no.)	Task Order No.
Project Title: Design and Simulation of Intelligent Control Architecture for Military Microgrids	
Period Covered: June 1 2011 – July 1, 2011	
POC/COR (Reference Paragraph 5 of the SOW):	
Achievements (Describe by task. Add additional tasks, if needed.): task numbers refer to tasks in Odyssian's original contract	
Task II: Model and Simulate Intelligent Microgrid Built 3-phase simulation with 15 microsources to see if system stability scales well with the number of sources.	
Task III: Distributed Control Algorithm Development Began a more formal stability analysis of the microgrid droop controllers.	
Task VI: Develop Wireless Communication No activity	
Task VII: Develop Wireless Distributed Control No activity	
Problems Encountered (Describe by task. Add additional tasks, if needed):	
Task II: None	
Task III: None	
Task VI: None	
Task VII: None	
Open Items (List items that require action by the Contractor or the Government):s No open items	

Summary Assessment and Forecast (Provide an overall assessment of the work and a forecast of contract completion):

A “large-scale” 3-phase of an inverter-based microgrid was built and tested. 15 of ND’s simPower models for the UWM microsources were interconnected into the mesh topology shown below. Each microsource had a capacity of 15 kW with P_{req} = 0.3 p.u. and E_{req}=1.0 p.u. A total of 12 kW was attached to each generator bus, with 4 kW of load being an e-board bus that could be automatically shed if the frequency drooped below 59.5 Hz.



The simulation results are shown in the attached figure. The simulation starts with all sources (including the main grid) connected. At 0.5 seconds the microgrid islands. As can be seen from the plots, the voltage magnitude and frequency are preserved through the islanding event. These results suggest that the fast microsource inverter controls will retain their stability over a wide range of grid topologies.



A more complete attempt to understand the scaling of the microgrid droop controllers was done through formal analysis.

Consider a power system consisting of n power buses. Let P_i denote the real power generated at bus i and Q_i denote the reactive power generated at bus i. Let Y_{ij} denote the admittance between buses i and j. Let $|Y_{ij}|$ denote the modulus of Y_{ij} and let θ_{ij} denote the angle of that complex quantity. We'll assume that $Y_{ii} = 0$ for all i.

Let $|V_i|$denote the magnitude of the voltage at bus i and δ_i be the relative phase of that voltage with respect to a known reference. The real and reactive power exported from bus i may be written as

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\delta_j - \delta_i + \theta_{ij}) \tag{1}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\delta_j - \delta_i + \theta_{ij}) \tag{2}$$

We assume that the "control" variables are the voltage magnitude, $|V_i|$, and angle δ_i for each i. We further assume that these controls are chosen according to a droop rule such that

$$|V_i| = \int_0^t (Q_{\text{set},i} - Q_i(\tau)) m_Q d\tau \tag{3}$$

$$\delta_i = \int_0^t (P_{\text{set},i} - P_i(\tau)) m_P d\tau \tag{4}$$

where m_Q and m_P are droop gains and $Q_{\text{set},i}$ and $P_{\text{set},i}$ are desired reactive and real power setpoints.

We examine the stability of the linearized equations about the setpoints. The state equations for this system are as follows,

$$
\begin{aligned}
\frac{dP_i}{dt} &= \sum_{j=1}^{n} \frac{d|V_i|}{dt}|V_j||Y_{ij}|\cos(\delta_j - \delta_i + \theta_{ij}) + \sum_{j=1}^{n}|V_i|\frac{d|V_j|}{dt}|Y_{ij}|\cos(\delta_j - \delta_i + \theta_{ij}) \\
&\quad - \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\sin(\delta_j - \delta_i + \theta_{ij})\left(\frac{d\delta_j}{dt} - \frac{d\delta_i}{dt}\right) \\
\frac{dQ_i}{dt} &= -\sum_{j=1}^{n} \frac{d|V_i|}{dt}|V_j||Y_{ij}|\sin(\delta_j - \delta_i + \theta_{ij}) - \sum_{j=1}^{n}|V_i|\frac{d|V_j|}{dt}|Y_{ij}|\sin(\delta_j - \delta_i + \theta_{ij}) \\
&\quad - \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\cos(\delta_j - \delta_i + \theta_{ij})\left(\frac{d\delta_j}{dt} - \frac{d\delta_i}{dt}\right) \\
\frac{d|V_i|}{dt} &= m_Q(Q_i^* - Q_i(t)) \\
\frac{d\delta_i}{dt} &= m_P(P_I^* - P_i(t))
\end{aligned}
$$

Let P_i^*, $|V_i^*|$, δ_i^*, and Q_i^* denote the values obtained from a load flow analysis of the power system. Let $\Delta P_i(t) = P_i^* - P_i(t)$ and $\Delta Q_i(t) = Q_i^*$. Since $\frac{d\Delta P_i}{dt} = \frac{dP_i}{dt}$ and $\frac{d\Delta Q_i}{dt} = \frac{dQ_i}{dt}$, we can linearize the above equation about these set points to obtain

$$
\begin{aligned}
\frac{d\Delta P_i}{dt} &= \sum_{j=1}^{n} m_Q \Delta Q_i |V_j^*||Y_{ij}|\cos(\delta_j^* - \delta_i^* + \theta_{ij}) + \sum_{j=1}^{n}|V_i^*| m_Q \Delta Q_j |Y_{ij}|\cos(\delta_j^* - \delta_i^* + \theta_{ij}) \\
&\quad - \sum_{j=1}^{n}|V_i^*||V_j^*||Y_{ij}|\sin(\delta_j^* - \delta_i^* + \theta_{ij})\left(\Delta P_j - \Delta P_i\right) m_P \\
\frac{d\Delta Q_i}{dt} &= -\sum_{j=1}^{n} m_Q \Delta Q_i |V_j^*||Y_{ij}|\sin(\delta_j^* - \delta_i^* + \theta_{ij}) - \sum_{j=1}^{n}|V_i^*| m_Q \Delta Q_j |Y_{ij}|\sin(\delta_j^* - \delta_i^* + \theta_{ij}) \\
&\quad - \sum_{j=1}^{n}|V_i^*||V_j^*||Y_{ij}|\cos(\delta_j^* - \delta_i^* + \theta_{ij})\left(\Delta P_j - \Delta P_i\right) m_P \\
\frac{d|V_i|}{dt} &= m_Q \Delta Q_i \\
\frac{d\delta_i}{dt} &= m_P \Delta P_i
\end{aligned}
$$

To show the structure of this more easily, we rewrite the above equations as

$$\begin{aligned}\frac{d\Delta P_i}{dt} &= \sum_{j=1}^{n} G_{ij}\Delta Q_i + \sum_{j=1}^{n} G_{ij}\Delta Q_j + \sum_{j=1}^{n} \overline{H}_{ij}\Delta P_i - \sum_{j=1}^{n} \overline{H}_{ij}\Delta P_j \\ \frac{d\Delta Q_i}{dt} &= -\sum_{j=1}^{n} \overline{G}_{ij}\Delta Q_i - \sum_{j=1}^{n} \overline{G}_{ij}\Delta Q_j + \sum_{j=1}^{n} H_{ij}\Delta P_i - \sum_{j=1}^{n} H_{ij}\Delta P_j\end{aligned}$$

where

$$\begin{aligned}G_{ij} &= m_Q|V_i^*||Y_{ij}|\cos(\delta_j^* - \delta_i^* + \theta_{ij}) \\ \overline{G}_{ij} &= m_Q|V_i^*||Y_{ij}|\sin(\delta_j^* - \delta_i^* + \theta_{ij}) \\ H_{ij} &= m_P|V_i^*||V_j^*||Y_{ij}|\cos(\delta_j^* - \delta_i^* + \theta_{ij}) \\ \overline{H}_{ij} &= m_P|V_i^*||V_j^*||Y_{ij}|\sin(\delta_j^* - \delta_i^* + \theta_{ij})\end{aligned}$$

In matrix-vector form this system of equations can be written as

$$\frac{d}{dt}\begin{bmatrix}\Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_n \\ \Delta Q_1 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_n\end{bmatrix} = \left[\begin{array}{cccc|cccc} \sum_j \overline{H}_{1j} & -\overline{H}_{12} & \cdots & -\overline{H}_{1n} & \sum_j G_{1j} & G_{12} & \cdots & G_{1n} \\ -\overline{H}_{21} & \sum_j \overline{H}_{2j} & \cdots & -\overline{H}_{2n} & G_{21} & \sum_j G_{2j} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\overline{H}_{n1} & -\overline{H}_{n2} & \cdots & \sum_j \overline{H}_{nj} & G_{n1} & G_{n2} & \cdots & \sum_j G_{nj} \\ \hline \sum_j H_{1j} & -H_{12} & \cdots & -H_{1n} & -\sum_j \overline{G}_{1j} & -\overline{G}_{12} & \cdots & -\overline{G}_{1n} \\ -H_{21} & \sum_j H_{2j} & \cdots & -H_{2n} & -\overline{G}_{21} & -\sum_j \overline{G}_{2j} & \cdots & -\overline{G}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -H_{n1} & -H_{n2} & \cdots & \sum_j H_{nj} & -\overline{G}_{n1} & -\overline{G}_{n2} & \cdots & -\sum_j \overline{G}_{nj} \end{array}\right]\begin{bmatrix}\Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_n \\ \Delta Q_1 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_n\end{bmatrix}$$

The system matrix is in a 2 by 2 block structure. Each block is diagonally dominant, so there will always be a selection of the gains that assure this matrix is Hurwitz (stable). Note that this is a preliminary analysis, but it applies to both strong and weak distribution networks; a situation that can be expected in a microgrid.

The preceding analysis follows the heuristics discussed in [Engler05][1] and appears to corroborate the stability analysis done in [Venkataramanan07][2] for chain microgrids. The difference in this analysis is that no special assumptions have been placed on the interconnection topology.

In spite of the positive message asserted through this analysis, there are significant simplifying assumptions that cannot be addressed using the small-signal linearization outlined above. The major observation is that the droop controllers have saturation nonlinearities that should be addressed using nonlinear stability techniques. The second

[1] A. Engler, *Applicability of Droops in Low Voltage Grids,* International Journal of Distributed Energy Resources, volume 1, number 1, pages 3-15, 2005.
[2] G. Venkataramanan and M. Illindala, *Small signal dynamics of inverter interfaced distributed generation in a chain-microgrid,* Power Engineering Society General Meeting, pages 1-6, 2007.

observation is that this analysis does not take into account the impact that frequency or voltage dependent load-shedding may have on overall system stability. In particular, since loads are shed in discrete groups, we again have a nonlinear type of feedback that cannot be addressed using the linearization methods. A more complete study of the scaling properties of the proposed hierarchical control scheme (i.e., one that includes the nonlinearities in the droop and load-shedding controllers) will probably require a more complete nonlinear analysis of the overall system.