# Subspace Optimization in Centralized Non-Coherent MIMO Radar

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**Abstract**

We consider the problem of subspace optimization for centralized non-coherent MIMO radar based on various measures such as capacity, diversity, and probability of detection. In subspace centralized non-coherent MIMO radar (SC-MIMO), a subset of stations is selected based on channel knowledge or channel statistics to reduce system complexity while simultaneously attempting to optimize the performance of the reduced-dimension centralized MIMO radar system. The radar transmitters are assumed to be sufficiently separated (e.g., at different locations) to yield spatially white channel transfer gains and are assumed to operate on a non-interference basis through time-division or frequency-division multiplexing. Detection optimization for the SC-MIMO system in a Neyman Pearson sense is found to be equivalent to selecting the subspace that maximizes the Frobenius norm of the corresponding channel matrix. Information-theoretic measures for capacity and diversity are also applied to the problem of subspace selection. Channels with temporal coherence times that are long relative to the radar system's latencies and channels with coherence times that are short relative to the radar system's latencies are considered. In the former case, metrics are based upon instantaneous channel estimates, whereas in the latter case, average channel estimates are used. Numerical analyses are conducted to illustrate the use of the metrics for optimizing system performance.

Keywords: MIMO radar, detection, capacity, diversity, subspace optimization, channel matrix

## 1 Introduction

Distributed MIMO radar is a research area that has received increasing attention lately, see, e.g., [5, 3, 10]. A salient feature of distributed radar is its ability to simultaneously engage a target from multiple aspect angles. While the legacy radar is confined to viewing the target from a single aspect angle at any given time instant, the distributed radar utilizes waveforms from spatially diverse stations to illuminate the target and detect reflected target energy from multiple aspect angles, taking advantage of aspect-dependent RCS to significantly improve the ability to detect and track targets. The benefit over a single station implementation comes at the cost of increased system complexity including more demanding inter-station communications for data fusion and coordination among the stations. Given a multiplicity of stations, down-selecting the number of stations used in processing provides one mechanism to reduce system complexity, where the stations are selected in a manner that optimizes the performance for the number of resources that are dedicated to the task. We refer to this architecture as subspace centralized non-coherent MIMO radar (SC-MIMO). The SC-MIMO radar architecture, exemplified in Figure 1, is characterized by the optimized selection of a subset of spatially diverse radar stations and joint processing of the received signals from this subset at a common fusion center. The transmitters are assumed to be sufficiently separated to yield spatially white channel transfer gains and are assumed to operate on a non-interference basis through time- or frequency multiplexing, which facilitates both the separation of the signals at the receivers and the application of associated Doppler compensation tapering for signal conditioning.

Subspace optimization measures in SC-MIMO are explored to optimize system performance in terms of probability of detection, information-theoretic capacity, and channel diversity, where optimized system performance in each of these senses is achieved by selecting the subspace that maximizes measures associated with the MIMO channel matrix. Information-theoretic metrics such as capacity and diversity are considered because of their ability to characterize MIMO channels in a manner that could potentially be exploited by an SC-MIMO system. For the case of SC-MIMO radar detection performance, joint detection optimization in a Neyman-Pearson sense with noncoherent square-law processing is shown to be equivalent to maximizing the Froebenius norm of the SC-MIMO radar channel matrix. The channel capacity measure is optimized by maximizing the determinant of the channel matrix [6, 12]. Diversity can be optimized by evaluating correlations between the elements of the channel matrix [8]. These subspace optimization measures are applied in the case of slowly changing channels wherein the channel changes can readily be tracked and utilized by the radar system for optimal subspace selection. Optimization measures are also applied in the case of channels that change faster than can be tracked and exploited by the radar system. In this case, average channel estimates, rather than instantaneous channel estimates, are employed in the optimization strategies. The channel capacity measure in this case is achieved by selecting the sub space that maximizes the sum of the eigenvalues associated with the corresponding channel matrix. In relation to the waveform design approaches addressed in [2], these metrics represent alternative approaches to optimizing the performance of a MIMO radar system.

The remainder of the paper is organized as follows. The system model, including the MIMO radar channel matrix $\mathbf{G}$ is introduced in Section 2. Using this model, Section 3 provides a theoretical development of target detection optimization in centralized MIMO radar. Subspace architectures are considered in Section 4 and the Frobenius norm of the subspace channel matrix is identified as a measure for optimizing detection probability in SC-MIMO. Optimization in channels with long coherence times (i.e., such that the radar can estimate the instantaneous channel and exploit this knowledge in the subspace selection) are addressed, and a numerical example is given that illustrates the impact of the subspace dimension of the detection performance. Information-theoretic optimization measures for capacity and diversity are also introduced. In Section 5, adaptation of the measures for channels with short coherence times are addressed. In these dynamic channels, the radar cannot adequately track and utilize the instantaneous channel estimate and must instead resort to exploiting average channel estimates. Results illustrating the potential for joint optimization of detection probability, capacity and diversity are presented based upon the revised metrics. Section 6 contains the conclusions of the paper.

## 2 System Model

A general system model, in accordance with Figure 1, is given below:

$$\mathbf{z}(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_K(t) \end{pmatrix} = \begin{pmatrix} g_{11}(t) & \dots & g_{1K}(t) \\ g_{21}(t) & \dots & g_{2K}(t) \\ \vdots & \vdots & \vdots \\ g_{K1}(t) & \dots & g_{KK}(t) \end{pmatrix} \begin{pmatrix} s_1(t-\tau_1) \\ s_2(t-\tau_2) \\ \vdots \\ s_K(t-\tau_K) \end{pmatrix} + \mathbf{w}(t) \tag{1}$$

$$= \mathbf{G}(t)\mathbf{S}(t) + \mathbf{w}(t) \tag{2}$$

where

$$z_l(t) = \sum_{k=1}^{K} z_{lk}(t) = \sum_{k=1}^{K} g_{lk}(t) s_k(t - \tau_k) \quad l = 1, 2, \ldots, K \quad k = 1, 2, \ldots, K \tag{3}$$

are the received waveforms at each receiver, $z_{lk}$ is the received signal component at the $l^{th}$ receive station from the $k^{th}$ transmit station, $g_{lk}$ is the complex channel transfer gain from the $k$-th transmit station to the $l$-th receive station, $s_k(t)$ is the orthogonal unit norm waveforms from the $k^{th}$ transmit station at time $\tau_k$, and $\mathbf{w}(t)$ is additive Gaussian noise. Note that $\mathbf{G} \in C^{K \times K}$ is not necessarily symmetric (for example if the radar stations transmit on different frequencies) and that each element $g_{lk}$ in $\mathbf{G}$ is proportional to the square root of the target's RCS and will depend on the target aspect angle relative to both the transmit station and the receive station (see Figure 1). In general, $g_{lk}$ will also be a function of the directional transmit and receive antenna gains, propagation losses, and other link budget parameters. We assume that the stations are deployed in a manner that results in spatially white, but temporally colored, channel transfer gains.

# 3 Target Detection Optimization in Centralized MIMO Radar

We present formulations based on Neyman-Pearson detection for the centralized detection approach. The development is similar to one found in [4], although we assume the utilization of either short term statistics or long term averages, leading to non-central chi-square distributions for the alternative hypothesis, $\mathcal{H}_1$. We also employ weighted noncoherent detection, where the contributions from the radar stations are normalized relative to the measured noise level at each station. This serves to accommodate asymmetric noise, such as intentional jamming, that may be present in the RF environment. We also link the optimal detection solution with the maximization of a channel matrix norm, giving a mechanism for subspace optimization by expeditiously selecting a subset of radar stations employed in detection processing.

Assuming that the waveform orthogonality is preserved, the outputs of the bank of matched filters

$$\hat{z}_{lk} \triangleq \int z_l(\tau) s_k^*(t - \tau) \mathrm{d}\tau \tag{4}$$

can be expressed as

$$\begin{array}{lcll} H_0 & : & \hat{z}_{lk} & = n_{lk} \\ H_1 & : & \hat{z}_{lk} & = g_{lk} + n_{lk} \end{array} \quad l = 1, 2, \cdots, K \; ; \quad k = 1, 2, \cdots, K \tag{5}$$

where $n_{lk}$ is the matched filter output's noise component, which is assumed to be zero-mean complex Gaussian with variance $\sigma^2_{n_{lk}}$. The channel transfer gain $g_{lk}$ is a complex random variable that is approximately constant over the coherence time. Appropriate Doppler compensation of the received signal is assumed to have been employed at each receiver. Doppler compensation involves compensating for Doppler shift imparted on the received signal. Methods such as those discussed in [9, 14, 13, 15, 16] are relevant. Doppler compensation in the noncoherent MIMO radar case is enabled by the assumption of orthogonal waveforms. The orthogonal property can be achieved through various tactics, including time multiplexing (e.g., where each transmitter is assigned to a time slot) or frequency multiplexing (e.g., where each transmitter is assigned to a different frequency). This stands in contrast to the methods of [1] in which the radar waveforms occupy the same bandwidth and where pseudo-orthogonality is achieved through waveform design. The proposed model exploits short term statistics associated with the bistatic returns, leading to returns (represented by $g_{lk}$) that exhibit negligible variability over the coherence time. Normalizing each output with respect to the noise level (which may be measured, e.g., through CFAR techniques), the normalized outputs are

$$\begin{array}{lcll} H_0 & : & z_{lk} & = \tilde{n} \\ H_1 & : & z_{lk} & = \tilde{g}_{lk} + \tilde{n} \end{array} \quad l = 1, 2, \cdots, K \; ; \quad k = 1, 2, \cdots, K \tag{6}$$

where the noise components are zero-mean Gaussian with variance $\sigma^2$. The probability density function (pdf) of $z_{lk}$ under each hypothesis is:

$$p_Z(z_{lk}) \sim \begin{cases} \mathbb{C}\mathcal{N}(0, \sigma^2) & \text{under} \quad H_0 \\ \mathbb{C}\mathcal{N}(\tilde{g}_{lk}, \sigma^2) & \text{under} \quad H_1 \end{cases} \tag{7}$$

where $\mathbb{C}\mathcal{N}(\mu, \sigma^2)$ denotes a complex normal distribution with mean $\mu$ and variance $\sigma^2$. The test static is obtained by noncoherently combining the normalized filter outputs. The corresponding distributions of the test statistic $y = \|z\|^2$ for each hypothesis is given by:

$$y = \|z\|^2 = \begin{cases} \frac{\sigma^2}{2}\chi^2_{2K^2} & H_0 \\ \\ \frac{(s^2+\sigma^2)}{2}\chi'^2_{2K^2} & H_1 \end{cases} \tag{8}$$

where $\chi^2_d$ denotes a chi-square random variable having $d$ degrees of freedom [11] and $\chi'^2_d$ denotes a noncentral chi-square random variable [11] having $d$ degrees of freedom and a noncentrality parameter

$$s^2(\mathbf{G}) = \sum_{i=1}^{K}\sum_{j=1}^{K}\tilde{g}^2_{ij}. \tag{9}$$

For optimal detection in the Neyman-Pearson (NP) sense, the relations between the probability of false alarm, $P_{fa}$, the probability of detection, $P_d$, and the threshold are governed by NP Theorem. The probability of false alarm is given by

$$P_{fa} = \int_{\delta}^{\infty} p(y|H_0)\mathrm{d}y = 1 - F_{\chi^2_{2K^2}}(\frac{2\delta}{\sigma^2}) \tag{10}$$

For a desired $P_{fa}$ and with the knowledge of $\sigma^2$, $\delta$ may be set using

$$\delta = \frac{\sigma^2}{2}F^{-1}_{\chi^2_{2K^2}}(1 - P_{fa}) \tag{11}$$

where $F^{-1}_{\chi^2_{2K^2}}$ is the inverse cumulative distribution function of a chi-square random variable with $2K^2$ degrees of freedom. Note that $\delta$ is independent of the MIMO radar channel gains. The probability of detection is given by

$$P_d = \int_{\delta}^{\infty} p(y|H_1)\mathrm{d}y \tag{12}$$

$$= 1 - F_{\chi'^2_{2K^2}}\left(\frac{2\delta}{(s^2(\mathbf{G}) + \sigma^2)}\right) \tag{13}$$

$$= 1 - F_{\chi'^2_{2K^2}}\left(\frac{\sigma^2}{(s^2(\mathbf{G}) + \sigma^2)}F^{-1}_{\chi^2_{2K^2}}(1 - P_{fa})\right) \tag{14}$$

where $F_{\chi'^2_{2K^2}}$ is the cumulative distribution function of a noncentral chi-square random variable with degree $2K^2$.

## 4 Subspace Optimization for Centralized Detection: The Case of Short Term Statistics

The concept of subspace optimization for centralized MIMO radar processing may be addressed using results from the above theoretical development. Subspace optimization assumes that a set of radar stations are available for organization into a MIMO radar system. Rather than utilizing all of the resources for a given detection problem, the premise is that it might be more desirable from a resource utilization perspective to employ only a subset of these resources for a given detection problem, where the resources are carefully selected to ensure that detection performance does not suffer substantially. We assume a centralized topology, and hence the central processor needs access to all of the K statistics in order to identify the subspace that will suitably optimize performance. A centralized approach based upon short-term statistics can readily be achieved in systems where information sharing between nodes does not introduce latencies that are on the order of the temporal decorrelation times of the channel transfer gain statistics. Where latencies associated with information sharing exceeds a major fraction of the temporal decorrelation times, subspace optimization based on short term statistics will be marginalized. In this case, optimizations based upon longer-term average statistics (as described in Sections 5) would be more appropriate. A downselection approach has the benefit of reducing the overall complexity of the problem in terms of required resources, the backhaul communications for coordination and passing of data (once a subspace is assigned), and also the signal processing at each radar station and at the fusion center. In this section, short term statistics are assumed. This approach would be representative of implementations with low-latency inter-station communications. In the next section, longer term statistics that exploit average channel characteristics are addressed.

For the specific case of short-term statistics, the following theorem and the subsequent corollary define the mechanism for optimizing selection of the radar stations for subspace centralized detection.

**Theorem 4.1.** *Let $\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_L$ represent a set of $L$ $N \times N$ channel matrix realizations. Given a constant false alarm rate, $P_{fa}$, and assuming a constant noise variance $\sigma^2$ at the receivers, the channel matrix realization with the largest Froebenius norm yields the largest probability of detection in the Neyman-Pearson sense.*

*Proof.* For identical $\sigma^2$ at all receivers, and a constant prescribed $P_{fa}$, the threshold $\delta$ is given by (11) and is constant. Using $\delta$ with (9) in (15) where N is used in place of K and $\mathbf{G}_\alpha$ in place of $\mathbf{G}$ (to enable indexing among the different channel realizations), and recognizing that the cumulative distribution function is nonincreasing with increasing $s^2(\mathbf{G}_\alpha)$ leads to the conclusion that $P_d$ is monotonically nondecreasing with $s^2(\mathbf{G}_\alpha)$, and hence the maximum $s^2(\mathbf{G}_\alpha)$ leads to the highest $P_d$. But the Froebenius norm of each channel realization, $G_\alpha$, with $\alpha \in \{1, \ldots, L\}$, is given by

$$\|\mathbf{G}_\alpha\|_F = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} |\tilde{g}_{\alpha,ij}|^2} = \sqrt{s^2(\mathbf{G}_\alpha)} = \sqrt{Tr\left[\mathbf{G}_\alpha^H \mathbf{G}_\alpha\right]} = \sqrt{\sum_{\ell=1}^{N} \lambda_{\alpha,\ell}} \tag{15}$$

where the $\tilde{g}_{\alpha,ij}$ are the normalized channel transfer gains of $\mathbf{G}_\alpha$ for $, i, j = 1, \ldots, N$, the $\lambda_{\alpha,\ell}$ are the eigenvalues of $\mathbf{G}_\alpha^H \mathbf{G}_\alpha$, and the dependence of $s^2$ on the channel realization $\mathbf{G}_\alpha$ is expressed. Therefore it follows that the channel realization yielding the largest Froebenius norm maximizes $s^2(\mathbf{G}_\alpha)$ and hence the probability of detection in the Neyman-Pearson sense. As evidenced in (16), this metric corresponds to a maximization of the sum of the eigenvalues of $\mathbf{G}_\alpha^H \mathbf{G}_\alpha$ over the index range $\alpha = 1, \ldots, L$. □

From a systems perspective, this result implies that maximum system performance generally cannot be achieved if each radar station independently attempts to maximize its own monostatic return (e.g., through beamsteering, maneuvers, etc.). Rather, improved performance is more readily achieved if the radar stations take on a global perspective in their responses in order to maximize the sum of the reflected energy received by all of the sensors. We also note that the requirement for identical $\sigma^2$ implies that the stations influenced by noise in the environment, such as from a local interferer, will incur channel power transfer attenuations in the signal processing that are inversely proportional to the variance of the noise, i.e., all contributing signals are normalized to have unit variance.

**Corollary 4.2.** *Under similar assumptions as in Theorem 4.1, but with $K > N$ stations, the subset of N stations yielding the channel matrix with the largest Froebenius norm provides the maximum probability of detection in the Neyman Pearson sense, assuming a constant false alarm rate.*

*Proof.* Define $L = \binom{K}{N}$ as the number of possible N×N channel realizations. Application of Theorem 4.1 then proves the corollary. □

Computational simulations were employed to evaluate the relative performance of subspace configurations. Figure 2 depicts results using (15) and (16) that illustrate the detection performance of the subspace optimization approach as a function of the SNR (defined here as $|\mathbf{G}^H\mathbf{G}|^2/\sigma^2$ where $\mathbf{G} \in R^{N\times N}$ and $K$ is the number of available stations) and the subset size. For these simulations, a topology of $K = 6$ stations was assumed, and subspace sizes of two, three and four radar stations were considered using a false alarm probability of $P_{fa} = 10^{-5}$. The channel gains assumed in the investigation were drawn from a $\mathcal{CN}(0,1)$ distribution. Ten channel realizations were averaged to obtain the results. The figure indicates performance advantages of K-choose-N topologies over topologies with N fixed stations. The results also illustrate the performance dependence on the subset size.

The understanding that the Neyman-Pearson formulation for subspace optimization in centralized MIMO radar is equivalent to optimizations based on the Frobenius norm of the MIMO radar channel state matrix invites consideration of other channel-based information-theoretic measures to optimize MIMO radar system performance in some sense. We consider here optimization metrics that have been reported for MIMO system capacity [6, 12] and MIMO system diversity [8]. The information-theoretic formulations are based upon the channel product $\mathbf{G}^H\mathbf{G}$ and the correlation matrix derived from the elements of the channel matrix $\mathbf{G}$, respectively. Diversity applies to long-term statistics and is projected to be useful when assigning MIMO resources over long periods of time, i.e., when resource assignment updates are infrequent. Capacity metrics apply to both short-term and longer-term statistics and provide a measure of the ability of the MIMO system to convey information. This attribute might be instrumental in applications where radar transmissions serve a dual purpose of conveying communications information while also providing illumination signals for radar detection.

## 4.1 Capacity Measure

The (theoretical) capacity of a MIMO communications system with a corresponding channel matrix $\mathbf{G}_\alpha \in C^{N\times N}$ is determined from

$$C = log\left(\left|\mathbf{I} + \frac{P}{N_o}\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right|\right) \tag{16}$$

where $P$ is the signal power and $N_o$ is the power spectral density of the noise. Given a set of L N×N channel realizations, $G_1, G_2, \ldots, G_L$, it is evident that channel capacity is maximized by selecting the channel matrix $G_{max} \in \{G_1, G_2, \ldots, G_L\}$ for which the determinant $|\mathbf{G}_\alpha^H\mathbf{G}_\alpha|$ is maximized over $\alpha$, where $\alpha = 1, 2, \ldots, L$. This is equivalent to selecting the channel realization that maximizes the product of the eigenvalues of $\mathbf{G}_\alpha^H\mathbf{G}_\alpha$ over $\alpha$ since

$$\left|\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right| = \prod_{\ell=1}^{N}\lambda_{\alpha,\ell} \tag{17}$$

Therefore, this strategy maximizes the product of the eigenvalues of $\mathbf{G}_\alpha^H \mathbf{G}_\alpha$ over $\alpha$ instead of the sum of the eigenvalues, leading to a different form of subspace optimization. The capacity measure yields detection performance levels that are less than or equal to that for the Frobenious norm measure. The degradation will generally depends upon the specific channel matrix realization. This form of optimization may be useful to MIMO radar configurations that attempt to employ radar signals that double as communications signals (e.g., for sharing information between the radar systems). For example, a weighted combination of the sum of the eigenvalues and the product of the eigenvalues of $\mathbf{G}_\alpha^H \mathbf{G}_\alpha$ might be employed to ensure that both radar and communications functions could be productively employed. For the case of the single channel realization represented in Table 1, the detection performance and the channel capacity for each of the six-choose-four configurations are indicated in Figure 3. The results illustrate the potential tradeoff that occurs when trying to jointly maximize the detection performance and the MIMO communications capacity. For example the $2^{nd}$ subset yields maximum capacity with suboptimal detection performance, whereas the $11^{th}$ subset yields maximum probability of detection but with suboptimal capacity. While the capacity and the probability of detection metrics are not necessarily optimized by the same subspace, it is evident that subpaces leading to good results for one also tend to yield reasonable results for the other. This would be expected given the inherent relationship between the sum of eigenvalues and the product of eigenvalues.

Table 1: Scaled Channel Gains Employed in Computational Studies

| Channel Gains (columns are Tx Stations, rows are Rx Stations) | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | -0.62 + 0.51i | 1.37 - 0.20i | -0.02 - 1.90i | -0.13 + 0.17i | -0.31 - 0.12i | -0.02 - 1.86i |
| 2 | -0.60 - 0.05i | -1.61 - 0.63i | 1.68 - 2.15i | 0.27 - 0.48i | 2.12 - 1.43i | 0.17 - 0.25i |
| 3 | -0.53 + 0.27i | -0.73 + 1.33i | 1.15 - 0.71i | 0.02 - 1.55i | 2.52 + 1.42i | 1.13 + 0.17i |
| 4 | -0.59 + 0.72i | -1.48 + 0.51i | 1.03 - 0.39i | -0.26 + 0.03i | 1.39 - 1.11i | -0.40 + 0.13i |
| 5 | 0.69 - 0.46i | -0.32 + 0.47i | -0.99 - 0.38i | -0.10 - 1.61i | 0.94 + 0.54i | -0.85 + 0.68i |
| 6 | -0.45 - 0.11i | 0.52 - 1.07i | -0.89 + 1.29i | 1.48 + 1.43i | -0.91 - 0.72i | 0.03 - 0.09i |

### 4.2 Diversity Measure

A second subspace optimization approach that also finds a basis in wireless communications is one that maximizes diversity, where the diversity is measured by [8]

$$\Psi(\mathbf{R}_\alpha) = \left( \frac{tr\mathbf{R}_\alpha}{\|\mathbf{R}_\alpha\|_F} \right)^2 \tag{18}$$

where $\mathbf{R}_\alpha = E[vec(\mathbf{G}_\alpha) vec(\mathbf{G}_\alpha)^H] \in C^{N^2 \times N^2}$ is a correlation matrix between the elements of $\mathbf{G}_\alpha$ and $vec(\mathbf{X})$ is the column stacking operation. This approach is expected to provide robust detection performance through long-term diversity when severe time-varying channel fading is prevalent and the latencies encountered in subspace formation and coordination among the stations exceeds the coherent time of the fading channels. Figure 4 illustrates the magnitude of the system diversities that are achievable in subspaces of dimension $\mathcal{R}^2$ , $\mathcal{R}^3$, and $\mathcal{R}^4$ as a function of the interstation temporal correlations in channels with coherence times less than system latencies. Because temporal diversity estimation requires longer term statistics, we provide comparative examples of diversity performance in the next section.

## 5 Subspace Optimization: Longer Term Statistics

In this section, the case of long-term channel statistics is considered. Under these conditions, the system does not exploit knowledge of the instantaneous channel matrix. Rather, it exploits knowledge of the average channel matrix product $E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]$ and the correlation matrix $\mathbf{R}_\alpha = E\left[vec(\mathbf{G}_\alpha)vec(\mathbf{G}_\alpha)^H\right]$. The need to resort to longer term statistics would be appropriate in scenarios involving radar stations with high intercommunications and processing latencies such as might be experienced with widely-spaced radar stations. We shall examine subspace optimization based upon 1) probability of detection using the Frobenius norm of the average channel estimates, 2) the capacity based on the average channel estimates, and 3) the diversity based on the correlation matrix $\mathbf{R}_\alpha$, where these matrices take on the following form for a $2 \times 2$ subspace MIMO radar configuration:

$$\mathbf{R}_\alpha = \begin{pmatrix} |g_{\alpha,1,1}|^2 & g^*_{\alpha,1,1}g_{\alpha,1,2}\rho_{\alpha,1,2} & g^*_{\alpha,1,1}g_{\alpha,2,1}\rho_{\alpha,1,3} & g^*_{\alpha,1,1}g_{\alpha,2,2}\rho_{\alpha,1,4} \\ g^*_{\alpha,1,2}g_{\alpha,1,1}\rho_{\alpha,2,1} & |g_{\alpha,1,2}|^2 & g^*_{\alpha,1,2}g_{\alpha,2,1}\rho_{\alpha,2,3} & g^*_{\alpha,1,2}g_{\alpha,2,2}\rho_{\alpha,2,4} \\ g^*_{\alpha,2,1}g_{\alpha,1,1}\rho_{\alpha,3,1} & g^*_{\alpha,2,1}g_{\alpha,1,2}\rho_{\alpha,3,2} & |g_{\alpha,2,1}|^2 & g^*_{\alpha,2,1}g_{\alpha,2,2}\rho_{\alpha,3,4} \\ g^*_{\alpha,2,2}g_{\alpha,1,1}\rho_{\alpha,4,1} & g^*_{\alpha,2,2}g_{\alpha,1,2}\rho_{\alpha,4,2} & g^*_{\alpha,2,2}g_{\alpha,2,1}\rho_{\alpha,4,3} & |g_{\alpha,2,2}|^2 \end{pmatrix} \tag{19}$$

and

$$E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right] = \begin{bmatrix} |g_{\alpha,1,1}|^2 + |g_{\alpha,2,1}|^2 & g^*_{\alpha,1,1}g_{\alpha,1,2}\rho_{\alpha,1,2} + g^*_{\alpha,21}g_{\alpha,2,2}\rho_{\alpha,3,4} \\ g^*_{\alpha,1,2}g_{\alpha,1,1}\rho_{\alpha,2,1} + g^*_{\alpha,2,2}g_{\alpha,2,1}\rho_{\alpha,4,3} & |g_{\alpha,1,2}|^2 + |g_{\alpha,2,2}|^2 \end{bmatrix} \tag{20}$$

where the $g_{\alpha,i,j}$ represent the average channel transfer gains associated with $G_\alpha$ and $\rho_{\alpha,i,j} = \rho_{\alpha,j,i}$ corresponds to the correlation between the $i^{th}$ and$j^{th}$ channel elements in $vec(G_\alpha)$. Extensions to larger configurations can be similarly computed.

## 5.1 Probability of Detection Based on Average Channel Estimates

Use of long term statistics is applicable to operation in time-varying channels. To circumvent the difficulties faced in computing the average probability of detection for each subset over these time-varying fading channels, it is proposed instead to employ a computationally simpler metric based on the average channel product. This strategy results in replacing the channel matrix product $\mathbf{G}_\alpha^H\mathbf{G}_\alpha$ with its expected value, $E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]$. Thus, the proposed measure for estimating detection performance uses (15) and (16), but where $Tr\left[E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]\right]$ is used instead of $Tr\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]$ in the computation of $s^2(\mathbf{G}_\alpha)$. The resulting detection metric, $\hat{P}_d$ is

$$\hat{P}_d(G_\alpha) = 1 - F_{\chi'^2_{2K^2}}\left(\frac{\sigma^2}{(Tr\left[E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]\right] + \sigma^2)}F^{-1}_{\chi^2_{2K^2}}\left(1 - P_{fa}\right)\right) \tag{21}$$

## 5.2 Capacity Measure Based on Average Channel Estimates

We adopt a multiplexing gain metric proposed in [7] to estimate average capacity based upon the average channel product estimate $E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]$. The metric is given by

$$C = Tr\left[\mathbf{D}\left(\sigma^2\mathbf{I} + \mathbf{D}\right)^{-1}\right] \tag{22}$$

where $\mathbf{D}$ is a diagonal matrix containing the eigenvalues of $E\left[\mathbf{G}_\alpha^H\mathbf{G}_\alpha\right]$. Note that this measure is based on an optimization of the sum of the eigenvalues and hence should yield a strong correlation with the $P_d$ measure. Whereas application of the metric defined in (18) requires tracking the instantaneous channels, the application of the metric in (23) is less stringent and instead requires tracking average channel statistics, which change more slowly. Hence, while the performance of this approach is moderated due to averaging, so are the update requirements, lending the approach to practical implementation in highly variable channels.

### 5.3 Example Performance Estimates for Longer Term Statistics

In this subsection, the relative performance as defined in (19), (22), and (23) are computed. For the computations, the channel gains in Table 1 are employed along with randomized correlation coefficients uniformly distributed over the closed interval [0 1] to generate $E\left[\mathbf{G}_{\alpha}^{H}\mathbf{G}_{\alpha}\right]$ and $\mathbf{R}_{\alpha}$. The resulting averages are then employed to compute metrics for each of the channel realizations corresponding to the radar station subsets (assuming a six-choose-four downselection process). The resulting measures for each subset are plotted in Figure 5. Joint optimization of the subspace can be achieved through optimization with respect to all three metrics, or with respect to either the $P_d$ vs capacity, the capacity vs diversity, or the $P_d$ vs diversity projections. Note the high correlation that is evident between the capacity and the probability of detection measures.

## 6 Conclusions

Operationally, SC-MIMO radar involves the selection and utilization of a subset of available radar stations to reduce system complexity while attempting to optimize the performance of a centralized MIMO radar system. Subspace optimization criteria based on the MIMO channel matrix have been proposed, where the optimization is characterized in terms of probability of detection, communications capacity, and channel diversity. Subspace selection to optimize Neyman-Pearson detection statistics in SC-MIMO was found to be equivalent to the selection of the subspace channel matrix yielding the maximum Frobenius norm, which is equivalent to maximizing the sum of the eigenvalues of the channel matrix product $\mathbf{G}^{H}\mathbf{G}$. The subspace size can be adapted to channel conditions to limit resource utilization while meeting prescribed performance levels. A diversity metric was also discussed that is useful for robust performance in fading channels. Achievable diversities for systems employing this measure are reported as a function of the inter-station temporal correlations. Instantaneous capacity measures drawn from information theory are also employed. The measures were applied to channels with long coherence times that enabled the radar system to exploit instantaneous channel estimates and to channels that changed more quickly than could be tracked or exploited by the radar system. In this latter case, average channel estimates, rather than instantaneous channel estimates, were applied to estimate the optimization measures. When channel averaging is employed, the capacity measure reduces to a measure that is proportional to the sum of the eigenvalues of the channel matrix product, and therefore is highly correlated to the detection metric. The analysis assumes the availability of six radar stations from which subsets were selected for detection processing and illustrates that joint optimization based upon all three metrics or optimization based upon a pair of measures using projections onto either the $P_d$ vs. Capacity plane, the $P_d$ vs. Diversity plane, or the Capacity vs. Diversity plane is possible.

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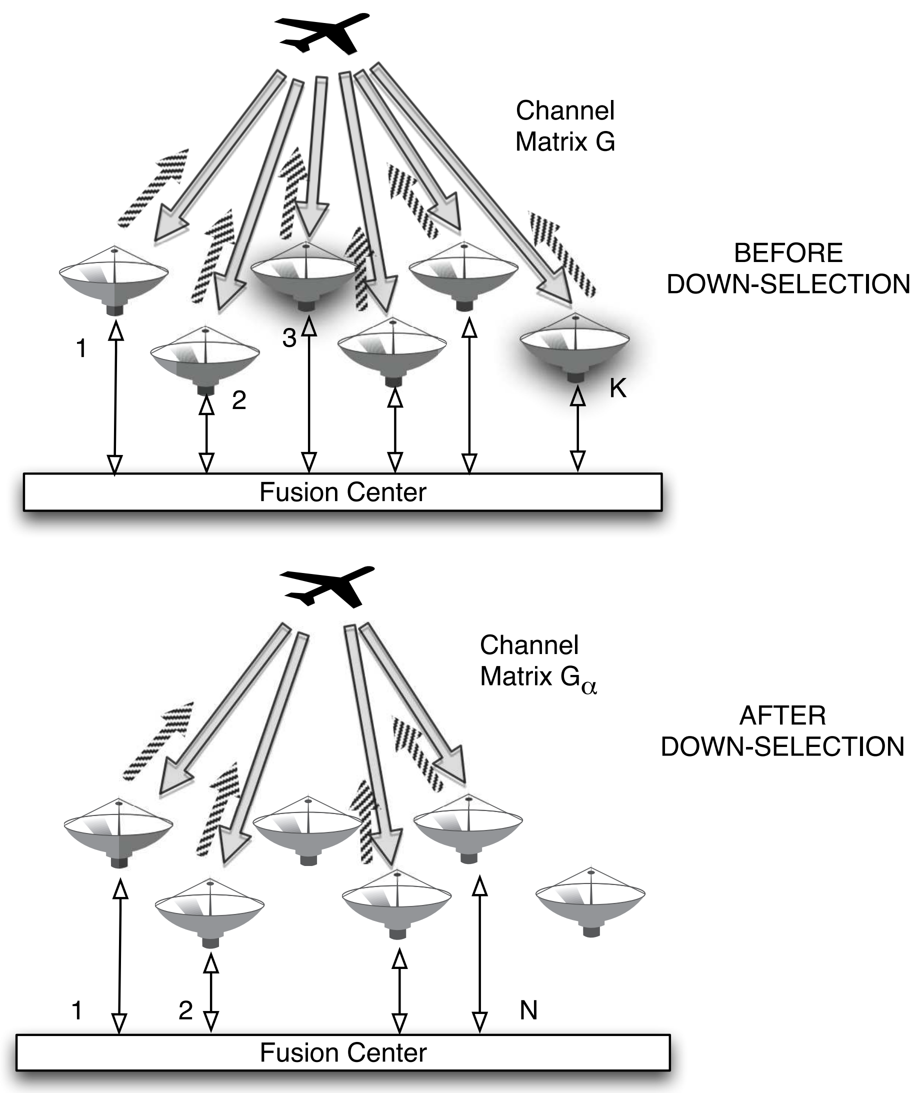


Figure 1: Example of a Subspace Centralized MIMO Radar Architecture

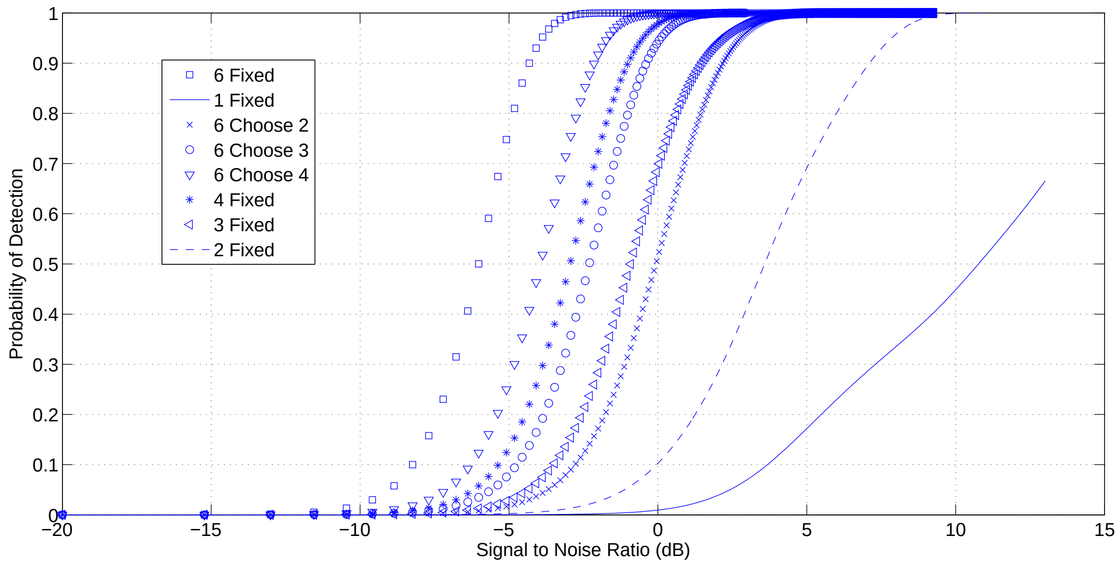


Figure 2: Detection Performance for Subspace MIMO Radar using the Frobenius Norm with $P_{fa} = 10^{-5}$.

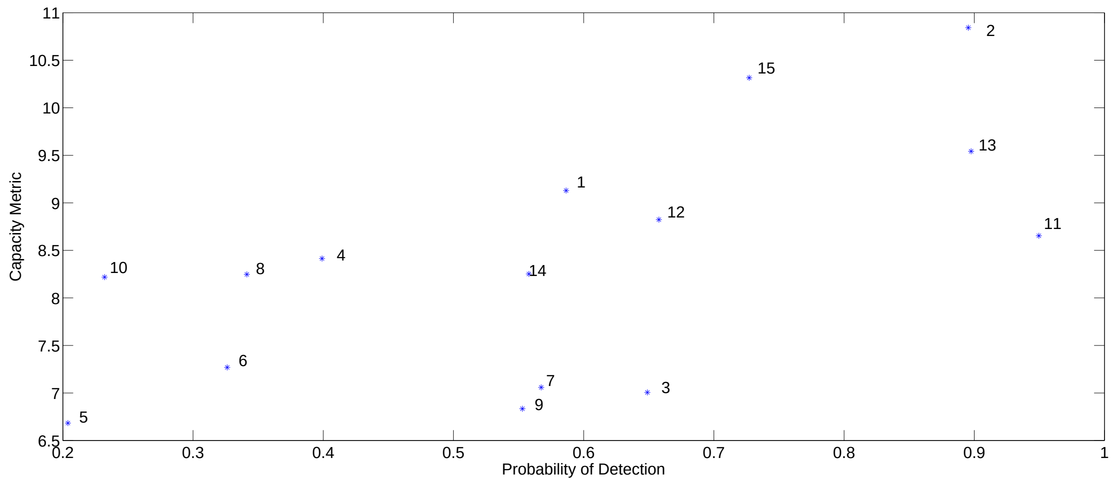


Figure 3: Probability of Detection versus Capacity for each of the 4-Station Subsets

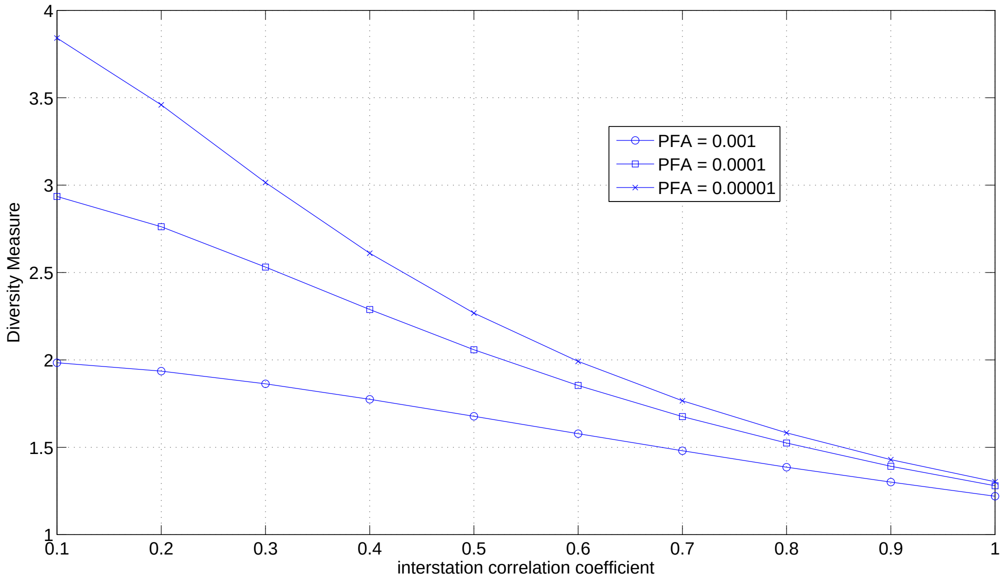


Figure 4: Diversity Measure as a Function of the Interstation Temporal Correlation. All interstation correlations are assumed to be identical.

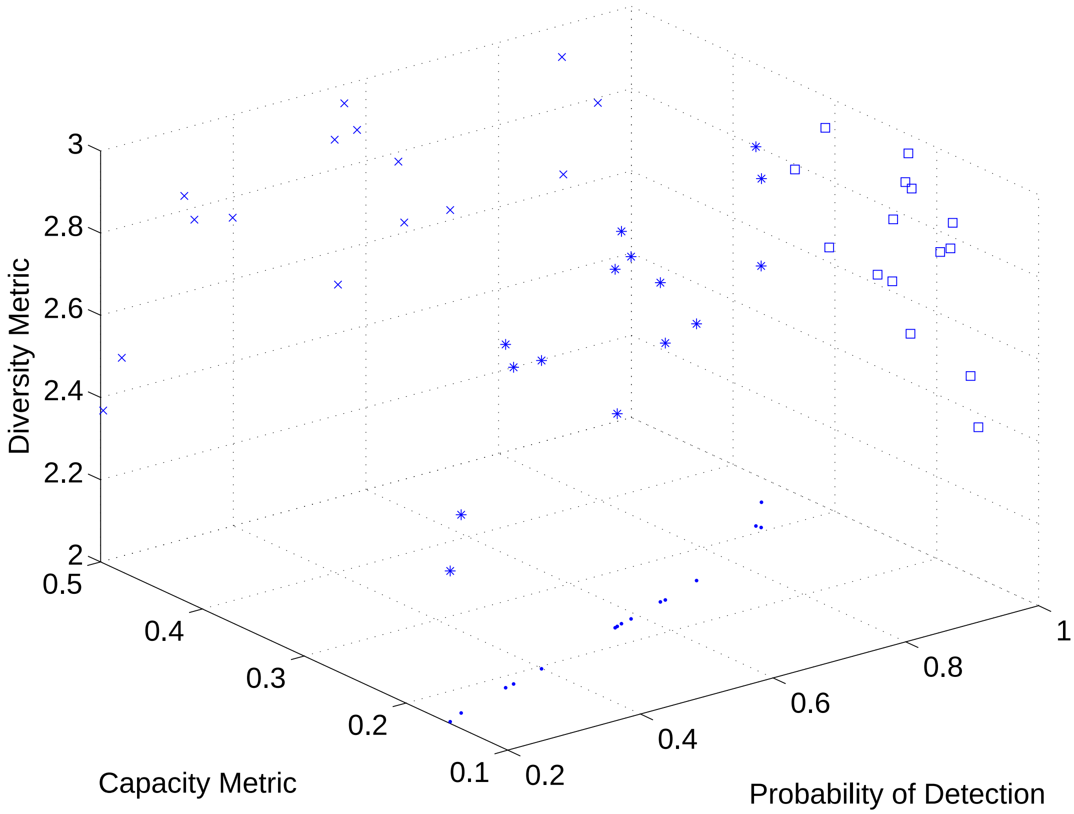


Figure 5: Comparative Performance of Station Configurations. ($*$) correspond to the 3-dimensional results. ($\bullet$) corresponds to the projection onto the $P_d$ versus Capacity plane. ($\times$) corresponds to the projection onto the $P_d$ versus Diversity plane. ($\square$) corresponds to the projection onto the Capacity versus Diversity plane.