ACMS 80870: Topics in Statistics: Geometry and Statistics

Instructor: Lizhen Lin (**Crowley 203A**) Email: lizhen.lin@nd.edu

Class meets: **9:35am-10:50am TTH** Classroom: TBD

Office hours: by appointment

Useful books of references:
(1) Nonparametric Inference on Manifolds: With Applications to Shape Spaces by Abhishek Bhattacharya and Rabi Bhattacharya, *IMS monograph #2*

(2) An introduction to optimization on smooth manifolds
http://www.nicolasboumal.net/book

Geometry and Statistics (course description)

This course will provide an overview on the role and of geometry (e.g., differential geometry, Riemannian geometry, information geometry) in the broad area of statistics. General topics of interests include: (1) statistics on manifolds, which extends statistical models and foundational theory to manifold-valued data which are now common encountered in modern science and engineering; (2) manifold learning which aims to extract the lower-dimensional structure of manifold of the potentially higher-dimensional data. This is different from topic (1) on statistics on manifolds described above, in which the geometry or manifold is **known**, while in manifold learning one aims to learn the typical unknown lower-dimensional manifold with data observed in some higher-dimensional ambient space; (3) information geometry, which aims to explore the geometry of statistical models for learning and inference;

A tentative outline of the specific topics:

Part I: Statistics on manifolds

(1) An overview on the basics of differential geometry and Riemannian geometry
(2) Frechet means on general metrics spaces and manifolds; Empirical means; Extrinsic means; Intrinsic means;
(3) Central limit theorems on manifolds;
(4) Statistics on specific manifolds: the sphere (directional statistics), shape spaces, Grassmannians, the space of positive definite matrices
(5) Density estimation, regression and classification on manifolds
(6) General optimization problems on manifolds

Class presentations.

Part II: manifold learning

(7) Dimension reduction tools
(8) Model based methods for manifold learning
(9) Minimax rates for manifolds learning

Class presentations.

Part III: Information geometry

(10) The Fisher-information metric and geometry of statistical models
(11) Natural gradient descent algorithms
(12) Applications to geometric MCMC

Class presentations.

Exams and grades

There will be NO in-class exams. There will be NO homework assignments. Grades will be based on attendance and in-class presentations.