PHYS 10310 - Exam I   Spring '07

MC1. $[ML^2/T^2] = [k]\cdot[M]\cdot[L/T]^2 \Rightarrow [k] = 1$ i.e. dimensionless.

E

MC2. $a = \frac{d^2x}{dt^2}$ i.e. equal to change in slope. As slope decreases acceleration is negative. E

MC3. $v_{av} = \frac{\Delta x}{\Delta t}$. Best way to find velocity at time t is to use $\Delta x = x(t+1s) - x(t-1s)$ and divide by 2s:

$$v_{av} = \frac{x(t+1s) - x(t-1s)}{2s} = \frac{\frac{1}{2}a(t+1s)^2 - \frac{1}{2}a(t-1s)^2}{2s}$$

$$= \frac{a}{4s}\left(\cancel{t^2} + 2t\cdot 1s + \cancel{(1s)^2} - \cancel{t^2} + 2t\cdot 1s - \cancel{(1s)^2}\right) = at = v.$$

Hence $v_A(t=3s)$ is equal to v_B since $\Delta x_A(2-4) = \Delta x_B(2-4)$.

C

MC4. Current is irrelevant since heading is straight across (but of course boat will land further downstream)

$$t = \frac{x}{v} = \frac{80m}{1.6m/s} = 50\ m/s.$$ A

MC5. Since $\bar{a}=0$ the total net force on car must be zero. Breaking force (friction) must be uphill.

C

I. a) AB: $\Delta v = (15-5)\,m/s = 10\,m/s$ $\quad \Delta t = 3s$

$$a_{av} = \frac{10\,m/s}{3s} = \underline{\underline{3.33\,m/s^2}}$$

BC: $\Delta v = 0 \quad \underline{\underline{a_{av} = 0}}$

CE: $\Delta v = (-15-15)\,m/s = -30\,m/s \quad \Delta t = 4s$

$$a_{av} = \frac{-30\,m/s}{4s} = \underline{\underline{-7.5\,m/s^2}}$$

b) $x_A = 0$ this is starting position

$$\Delta x_B = \frac{v_B^2 - v_A^2}{2a_{AB}} = \frac{(15\,m/s)^2 - (5\,m/s)^2}{2\cdot 3.33\,m/s^2} = \underline{\underline{30\,m}}$$ motion w/const. acc.

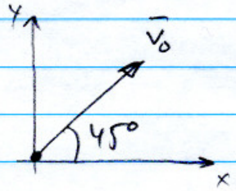
$$x_C = x_B + (x_C - x_B) = 30\,m + v_{BC}\,t_{BC} = 30\,m + 15\,m/s \cdot 3s = \underline{\underline{75\,m}}$$

$$x_D = x_C + (x_D - x_C) = 75\,m + \frac{v_D^2 - v_C^2}{2a_{CD}} = 75\,m + \frac{0-(15\,m/s)^2}{2\cdot(-7.5\,m/s^2)} = 75\,m + 15\,m = \underline{\underline{90\,m}}$$

$$x_E = x_D + (x_E - x_D) = 90\,m + \frac{v_E^2 - v_D^2}{2a_{ED}} = 90\,m + \frac{(-15\,m/s)^2 - 0}{2\cdot(-7.5\,m/s^2)} = 90\,m - 15\,m = \underline{\underline{75\,m}}$$

c) At $t = 8s$ $\quad v = 0$ (we cannot travel more slowly than that!)

III. a)



$$v_x = v_0 \cos 45° = 10\cdot\sqrt{2}\,m/s \cdot \frac{1}{\sqrt{2}} = \underline{\underline{10\,m/s}}$$

$$v_y = v_0 \sin 45° = 10\cdot\sqrt{2}\,m/s \cdot \frac{1}{\sqrt{2}} = \underline{\underline{10\,m/s}}$$

b) Several ways to do this. Easiest is probably to find range until ball is back @ initial height. At half range ball is @ max. height:

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

With $\theta = 45°$: $\sin\theta_0 = \cos\theta_0 = \frac{1}{\sqrt{2}}$, $\tan\theta_0 = \sin 2\theta_0 = 1$

$$= R/2 - \frac{g(R/2)^2}{2(v_0/\sqrt{2})^2} = R/2 - \frac{gR^2}{4v_0^2}$$

$$= \frac{v_0^2}{2g} - \frac{g v_0^4}{4g^2 v_0^2} = \frac{v_0^2}{g}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{v_0^2}{4g}$$

Add to this initial height $y_0 = 2m$ and we get

$$y_{max} = y_0 + \frac{v_0^2}{4g} = 2m + \frac{(\sqrt{2}\cdot 10\,m/s)^2}{4\cdot 9.81\,m/s^2} = \underline{\underline{7.097\,m}}$$

c) We must solve eq (1.10) w/ $y = -2m$:

$$y = x - \frac{gx^2}{v_0^2} \Rightarrow \frac{gx^2}{v_0^2} - x + y = 0$$

$$x = \frac{1 \pm\sqrt{1 - 4\cdot g/v_0^2\cdot y}}{2\,g/v_0^2} = \frac{v_0^2}{2g}\left(1 + \sqrt{1 - 4\,g/v_0^2\cdot y}\right)$$

We must choose "+" sign to get $x > 0$.

$$= \frac{200\,m^2/s^2}{2\cdot 9.81\,m/s^2}\left(1 + \sqrt{1 - 4\,\frac{9.81\,m/s^2}{200\,m^2/s^2}\cdot(-2m)}\right) = \underline{\underline{22.2\,m}}$$

III cont'd

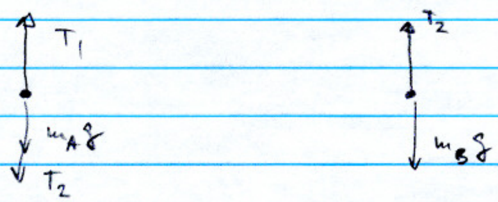
d) We must find height @ $x = 4$ m:

$$y = x - \frac{gx^2}{v_0^2} = 4\text{m} - \frac{9.81\,\text{m/s}^2 \cdot (4\text{m})^2}{200\,\text{m}^2/\text{s}^2} = 3.22\ \text{m}.$$

Add to this initial height $y_0 = 2\text{m}$ and total height at ball

$y(4\text{m}) = 5.22\text{m}$ -> it clears wall.

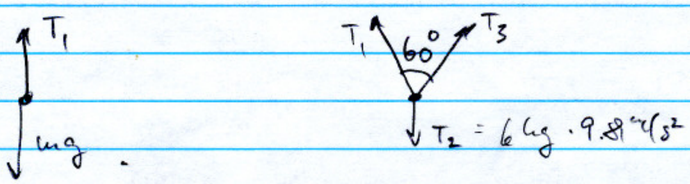
IV a)



b) NII: $T_2 - m_B g = m_B a$ => $T_2 = m_B (g + a) = 2\,\text{kg}\,(9.81\,\text{m/s}^2 + 1\,\text{m/s}^2) = \underline{\underline{21.6\ \text{N}}}$

c) NII: $T_1 - (T_2 + m_A g) = m_A a$ => $T_1 = T_2 + m_A (g + a) = T_2 + 1\,\text{kg}\,(9.81 + 1)\,\text{m/s}^2 = \underline{\underline{32.4\,\text{N}}}$

V. a)



b) From geometry T_1 & T_3 must both make an angle of 30° w.r.t. vertical.

I cont'd

$N\mathrm{II}$: $T_1 - mg = 0$ $\quad T_1 = mg$

$-T_1 \sin 30° + T_3 \sin 30° = 0 \Rightarrow T_1 = T_3$

$(T_1 + T_3) \cos 30° - T_2 = 0 \Rightarrow 2T_3 \cos 30° = T_2$

$T_2 = 6\,kg \cdot 9.81\,m/s^2 = \underline{\underline{58.9\,N}}$

$T_3 = \dfrac{T_2}{2\cos 30°} = \underline{\underline{34.0\,N}} = T_1$

c) $m = \dfrac{T_1}{g} = \dfrac{34.0\,N}{9.81\,m/s^2} = \underline{3.46\,kg}$