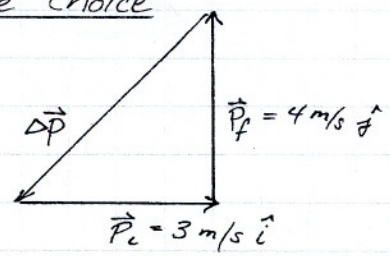
I. Multiple Choice

1. (b)



$\Delta\vec{p} = \vec{P}_f - \vec{P}_i$, $= \vec{J} = \vec{F}\Delta t$

$|\Delta\vec{p}| = |\vec{F}|\Delta t$, $5 = F(0.5)$

$\underline{10N = F}$

2. (b) $mv_i + MV_i = (m+M)V_f \Rightarrow -(500kg)(2\,m/s) + (2000)(5\,m/s) = 9000\,kg\,m/s$

$9000 = 2500V_f$, $V_f = 3.6\ m/s \Rightarrow KE_f = \frac{1}{2}(M+m)V_f^2 = \frac{1}{2}(2500)(3.6)^2 = 16,200\ J$

$KE_i = \frac{1}{2}MV_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}(2000)(5)^2 + \frac{1}{2}(500)(2)^2 = 26,000\ J$

$KEf/KE_i = \underline{0.623}$

3. (e) two ways: $\alpha = \frac{\Delta\omega}{\Delta t} = -\frac{2\pi}{10}$, $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$, $0 = (2\pi)^2 - 2\left(\frac{2\pi}{10}\right)\Delta\theta$

$4\pi^2 = \frac{4\pi}{10}\Delta\theta$, $\underline{\Delta\theta = 10\pi}$

or: $\theta_f = \omega_0 t + \frac{1}{2}\alpha t^2 = (2\pi)(10) - \frac{1}{2}\left(\frac{2\pi}{10}\right)(100) = 20\pi - 10\pi = \underline{10\pi}$

4. (d) Use the Parallel Axis Theorem: $I = I_{cm} + Mh^2$, where h is the distance to the new axis. Since $R_2 > R_1$,

$I_2 = I_{cm} + MR_2^2 > I_1 = I_{cm} + MR_1^2 > I_{cm}$

5. (d) $P = \tau\cdot\omega = 50,000 \times \left(\frac{100\ rev.}{min}\cdot\frac{1\ min}{60\ sec}\cdot\frac{2\pi}{1\ rev.}\right) = \underline{523\ kW}$

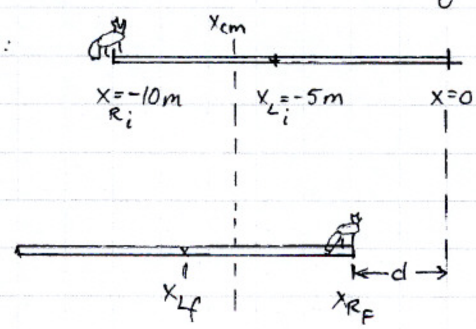
or, $P = F\cdot v$, @ 0.5 m radius, a torque of 50,000 N requires a force of 100,000 N. The angular velocity is 10.47 rad/s, which implies a linear velocity of 5.23 m/s at the edge of the winch. $F\cdot v = (100,000)(5.23) = \underline{523\ kW}$

Problems

II. There are a couple of different ways to solve this problem

Center of Mass (Isolated System) → C of M remains in same place.

Initially:



Finally

$$x_{cm} = \frac{m_R x_{R_i} + M_L x_{L_i}}{m_R + M_L}$$

$$= \frac{(25)(-10) + (100)(-5)}{125} = -6m$$

$x_{R_f} = -d$, $x_{L_f} = -(5+d)$

$$x_{cm} = -6m = \frac{25(-d) + 100\cdot(-(5+d))}{125}$$

$250 = 125d \Leftarrow 750 = 25d + 500 + 100d$

$\underline{d = 2m}$ → answer to (b)

Then, for (a) we know the raccoon moves with $u = 0.1\,m/s$ with respect to the log, so it takes him $(0.1)t = 10m$, $t = 100$ sec to walk the length of the log. In that time, the log moves $\underline{-2m}$, so, $v_{log}(100) = -2$, $\underline{v_{log} = -.02\,\hat{\imath}\,m/s}$ → (a)

Conservation of Momentum: (a.) $\vec{P}_{tot} = 0 \Rightarrow m\vec{v}_{R,s} + M\vec{V}_{L,s} = 0$, where

$\vec{V}_{R,s} = \vec{V}_{R,L} + \vec{V}_{L,s}$ (velocity of raccoon w.r.t. shore is the velocity of raccoon w.r.t. log plus vel. log w.r.t. shore).

So, $m\vec{v}_{R,s} = -M\vec{V}_{L,s}$, $\vec{V}_{R,s} = -\frac{M}{m}\vec{V}_{L,s} \Rightarrow -\frac{M}{m}\vec{V}_{L,s} = \vec{V}_{R,L} + \vec{V}_{L,s}$

$-\vec{V}_{L,s}\left(1 + \frac{M}{m}\right) = \vec{V}_{R,L}$, $\vec{V}_{L,s} = \frac{-\vec{V}_{R,L}}{1 + \frac{M}{m}} = -\frac{1}{5}\vec{V}_{R,L} = \underline{-0.02\,m/s}\ \hat{\imath}$

(b.) It takes the raccoon $v \cdot t = d \Rightarrow t = \frac{d}{v} = \frac{10}{0.1} = 100$ sec to travel down the log. If the log moves at $-0.02\,m/s$, then it will move $\underline{\underline{-2m}}$ during this time.

Problems, (cont.)

III a.) Conservation of Momentum – Inelastic Collision

$$mv_i = (M+m)v_f, \qquad v_f = \frac{m}{m+M}v_i = \frac{0.005}{20.005}(1000\ \mathrm{m/s}) = 0.25\ \mathrm{m/s}$$

b.) Conservation of Energy: $KE_i = U_f \Rightarrow \frac{1}{2}(m+M)v_f^2 = \frac{1}{2}kx^2$

$$\tfrac{1}{2}(20.005)(0.25)^2 = \tfrac{1}{2}(500)x^2, \quad x = 0.05\ \mathrm{m} = 5\ \mathrm{cm}$$

c.) Work-Energy Theorem: $E_f - E_i = W_{ext}$, $W_{ext} = W_{F_k} = -F_k \cdot x$

(Friction does negative work)

$$U_f - \cancel{U_i}^{\,0} + \cancel{KE_f}^{\,0} - KE_i = -F_k \cdot x, \quad \tfrac{1}{2}kx^2 - \tfrac{1}{2}(m+M)v^2 = -F_k \cdot x$$

$$\tfrac{1}{2}kx^2 + F_k \cdot x - \tfrac{1}{2}(m+M)v^2 = 0$$

$\Rightarrow$ calculate numerical coefficients, use quadratic equation to solve for x.

$$F_k = \mu_k N = \mu_k Mg = 39.2\ \mathrm{N}, \quad \tfrac{1}{2}(m+M)v^2 = 0.625\ \mathrm{J}$$

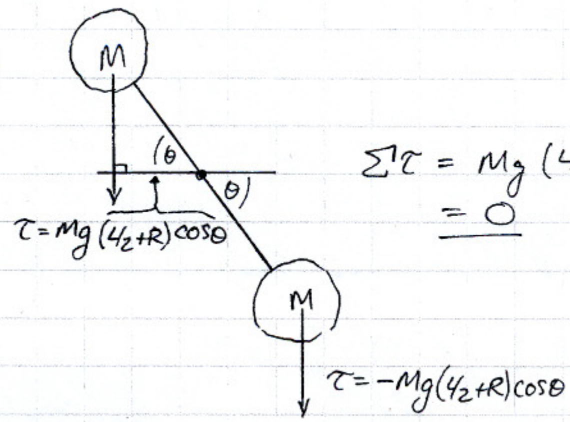
So, $250x^2 + 39.2x - 0.625 = 0$, $\quad x = \dfrac{-39.2 \pm \sqrt{(39.2)^2 + 4(250)(0.625)}}{500}$

$$x = \frac{-39.2 \pm 46.5}{500} \xrightarrow{\text{choose } \oplus} \underline{= 0.015\ \mathrm{m}}$$

$$\underline{= 1.5\ \mathrm{cm}}$$

IV a.) $I = \frac{1}{12}mL^2 + 2\left(\underbrace{\tfrac{2}{5}MR^2 + M(R+L/2)^2}_{\text{Parallel Axis Theorem}}\right)$

b.)



$$\Sigma\tau = Mg(L/2+R)\cos\theta - Mg(L/2+R)\cos\theta + \overset{\text{bar}}{0}$$

$$\underline{= 0}$$

(c) $\tau = R_\perp F = (44)(L/2+R) = \underline{8.8\ \mathrm{Nm}}$

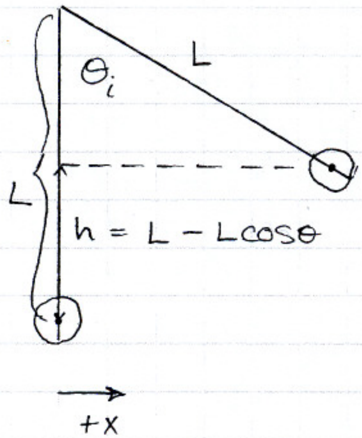
d.) $\tau = I\alpha$, $\quad 8.8\ \mathrm{Nm} = I(10\ \mathrm{rad/s^2})$, $\quad I = 0.88$

$$I = \tfrac{1}{12}mL^2 + 2M\left(\tfrac{2}{5}R^2 + (R+L/2)^2\right) = 0.00067 + M(.088) = 0.88$$

$$\boxed{M = 10\ \mathrm{kg}}$$

Problems, (cont.)

V a.) Use Conservation of Energy to find the bob's velocity at the bottom of its swing, given the starting angle.



$$m_p g (L - L\cos\theta_i) = \frac{1}{2} m_p v_f^2$$

$$v_f^2 = gL(1-\cos\theta_i), \quad v_f = \sqrt{gL(1-\cos\theta_i)}$$

$$= \underline{-3.54 \text{ m/s } \hat{i}}$$

b.) Use conservation of Energy again on the upswing:

$$\frac{1}{2} m_p v_i^2 = m_p g L (1 - \cos\theta_f)$$

$$v_i^2 = gL(1-\cos\theta_f), \quad \underline{v_i = 0.4 \text{ m/s } \hat{i}}$$

c.) For an elastic collision, we can relate the velocities by:

$$v_i + v_f = V_i + V_f$$, let v correspond to the block, V :: bob

$$v_f = -3.54 + 0.4 = \underline{-3.14 \text{ m/s } \hat{i}}$$

d.) Using conservation of momentum:

$$m_b v_f + m_p V_f = m_p V_i \Rightarrow m(-3.14) + m_p(0.4) = m_p(-3.54)$$

$$m(-3.14) = (0.4)(-3.94)$$

$$\boxed{m = 0.5 \text{ kg}}$$