Multiple Choice Solutions

1. (C) At any given position of the block there are two forces acting on it: 1) gravity (down) and 2) the Normal force (towards the center).

2. (A) Maximum static friction force is $\mathcal{F}_s = \mu_s mg = 0.5 \times 50 \times 9.8 = 245$ N. Since this exceeds F, the block will remain stationary.

3. (B) By conservation of energy, $-\Delta U = \Delta KE + \Delta E_{therm}$ where the last term is the increase in thermal energy (heat) due to air resistance (think Space Shuttle reentry). Since $\Delta E_{therm} > 0,\ -\Delta U > \Delta KE$.

4. (D) $KE = \frac{1}{2}mv^2$. If $v' = 1.2v$, then $KE' = \frac{1}{2}m(v')^2 = 1.44 \times K$.

4. (B) Conservation of energy: $E^0_{e^-} + E^0_{e^+} = 3E_\gamma$.

$$E^0_{e^-} = E^0_{e^+} = \frac{9.109 \times 10^{-31} \times (2.998 \times 10^8)^2}{1.602 \times 10^{-19}} = 511 \text{ keV}.$$

$E_\gamma = 2 \times 511/3 = 341$ keV.

II a) $l_A = \frac{1}{4} 2\pi r_A = \frac{\pi}{2} \cdot 70\text{ m} = \underline{\underline{109.96\text{ m}}}$ (0)

$l_B = \frac{\pi}{2} \cdot 40\text{m} + 2 \cdot 20\text{ m} = \underline{\underline{102.83\text{ m}}}$ (1)

b) Maximum speed corresponds to centripetal force equal to maximum static friction force:

$$m a_c = \mu_s m g \Rightarrow a_c = \frac{v^2}{r} = \mu_s g \Rightarrow v = \sqrt{\mu_s g r}$$

$$v_A = \sqrt{0.95 \cdot 9.81\text{ m/s}^2 \cdot 70\text{ m}} = \underline{\underline{25.54\text{ m/s}}} \text{ (2)}$$

$$v_B = \sqrt{0.95 \cdot 9.81\text{ m/s}^2 \cdot 40\text{ m}} = \underline{\underline{19.31\text{ m/s}}} \text{ (3)}$$

c) Time to drive is given by $t = \frac{l}{v}$:

$$t_A = 109.96\text{ m} / 25.54\text{ m/s} = \underline{\underline{4.305\text{ s}}}$$

$$t_B = 102.83\text{ m} / 19.31\text{ m/s} = \underline{\underline{5.326\text{ s}}}$$

Consequently trajectory is faster despite being longer.

III a) The friction force btw. the two blocks must be static if they are not slipping. Maximum static friction force is:

$$f_{s,max} = \mu_s mg = 0.3 \cdot 2kg \cdot 9.81\,m/s^2 = 5.886\ N$$

This corresponds to an acceleration of upper block of:

$$a = \frac{f_{s,max}}{m} = \mu_s g = 2.943\,m/s^2.$$

Since F accelerates both blocks this becomes:

$$F_1 = (2kg + 4kg) \cdot 2.943\,m/s^2 = \underline{\underline{17.66\ N}}$$

b) Since 24 N > 17.66 N blocks are now slipping on one another!

On top block: $f_k = \mu_k mg = 0.2 \cdot 2kg \cdot 9.81\,m/s^2 = \underline{\underline{3.924\ N}}$ to the right

On bottom block: $f_k = \underline{\underline{3.924}}$ to the left (N III).

c) Use N II:

for top block: $a = f_k/m = \mu_k g = 0.2 \cdot 9.81\,m/s^2 = \underline{\underline{1.962\,m/s^2}}$ to the right

for ~~bottom~~ bottom block: $a = (F - f_k)/m = \frac{(24 - 3.924)N}{4kg} = \underline{\underline{5.019\,m/s^2}}$ to the right

IV. a) Work by const. force:

$$W = \vec{F} \cdot \Delta \vec{s} = 50N \cdot 0.1m = \underline{5\ J}$$

b) Potential energy of spring:

$$U_s = \frac{1}{2} k \Delta x^2 = \frac{1}{2} \cdot 800\ N/m \cdot (0.1m)^2 = \underline{\underline{4\ J}}$$

c) By energy conservation

$$K = W - U_s = 1\ J \quad \text{(there is no friction)}$$

Velocity then is:

$$K = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 1J}{4kg}} = \underline{\underline{0.707\ m/s}}$$

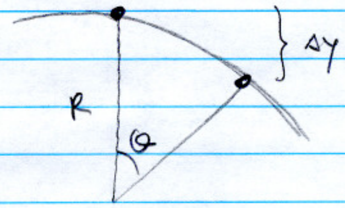
d) At maximum elongation box is @ rest. So by conservation of energy:

$$W = \frac{1}{2} k \Delta x^2 \quad \text{but now} \quad W = F \Delta x$$

$$\Downarrow$$

$$F \Delta x = \frac{1}{2} k \Delta x^2 \Rightarrow \Delta x = 2 F/k = 2 \cdot \frac{50N}{800N/m} = \underline{\underline{12.5\ cm}}$$

V. a) The change in potential energy is:



$$\Delta U = mg\,\Delta y = mgR(\cos\theta - 1)$$

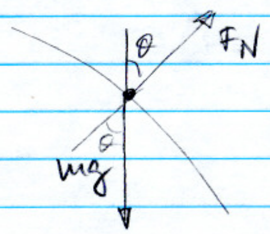
$$= 0.020\,kg \cdot 9.81\,m/s^2 \cdot 0.2\,m\,(\cos 40^\circ - 1)$$

$$= \underline{\underline{-9.18\,mJ}}$$

b) Conservation of energy:

$$\Delta K = -\Delta U = \frac{1}{2}mv^2 \Rightarrow \quad v = \sqrt{\frac{2\cdot(-\Delta U)}{m}} = \sqrt{\frac{2\cdot 9.18\,mJ}{0.02\,kg}} = \underline{\underline{0.958\,m/s}}$$

c) Free-body diagram:



Net force directed towards center of sphere:

$$mg\cos\theta - F_N$$

This must equal centripetal force

$$F_c = ma_c = m\frac{v^2}{R} = \frac{\cancel{m}}{R}\cdot\frac{-2\Delta U}{\cancel{m}} = \frac{2}{R}\cdot mgR(1-\cos\theta)$$

$$= 2mg(1-\cos\theta)$$

Combining we get:

$$mg\cos\theta - F_N = 2mg(1-\cos\theta)$$

$$\Downarrow$$

$$F_N = mg(3\cos\theta - 2) = 0.02\,kg\cdot 9.81\,m/s^2\,(3\cos 40^\circ - 2)$$

$$= \underline{\underline{58.5\,mN}}$$

d) Particle looses contact when $F_N = 0$! Using result from (c) we find that this corresponds to

$$3\cos\theta - 2 = 0 \Rightarrow \quad \cos\theta = 2/3 \qquad \theta = \underline{\underline{48.2^\circ}}$$