PHYS 10310 2006 Exam III

MC1. $M a_{cm} = m_1 a_1 + m_2 a_2 = m_1 \frac{F}{m_1} + m_2 \cdot 0 = F.$

$a_{cm} = \frac{F}{M} = \frac{F}{m_1 + m_2}$ (B)

MC2. $mgh = \frac{1}{2} m v_0^2 \qquad v_0 = \sqrt{2gh}$ cons. energy

$m v_0 = 2m v_1 \Rightarrow v_1 = v_0/2$ cons momentum.

$2mgH = \frac{1}{2}(2m)v_1^2 \qquad H = \frac{v_1^2}{2g} = \frac{v_0^2}{8g} = \frac{2gh}{8g} = \frac{h}{4}$ (B)

MC3. $E_{tr} = \frac{1}{2} M v_{cm}^2$

$E_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{2} M R^2 \cdot \left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{4} M v_{cm}^2$

$\frac{E_{rot}}{E_{tot}} = \frac{\frac{1}{4} M v_{cm}^2}{\frac{3}{4} M v_{cm}^2} = \frac{1}{3}.$ (A)

MC4. $\tau = I \alpha \qquad \alpha = \frac{\tau}{I} = \frac{Fd}{I}$

$\alpha' = \frac{F 2d}{I} = 2\alpha$ (B)

MC5 large disc has larger I and hence larger L. (C)

II.

a) System initally @ rest.

$$m_g v_g + m_b v_b = 0 \qquad\qquad v_b - v_g = 400\,\text{m/s}$$

$$m_g v_g + m_b\left(v_g + 400\,\text{m/s}\right) = \left(m_g + m_b\right) v_g + m_b \cdot 400\,\text{m/s} = 0$$

$$v_g = -400\,\text{m/s} \cdot \frac{m_b}{m_b + m_g} = -400\,\text{m/s} \cdot \frac{0.02}{12.02} = \underline{\underline{-0.666\,\text{m/s}}}$$

b) System center-of-mass still @ rest: $\underline{\underline{v_g = 0}}$

c) Time of flight: $t = \dfrac{L}{v_b} = \dfrac{1.5\,\text{m}}{400\,\text{m/s}} = 0.00375\,\text{s}$

$$\Delta x_g = v_g t = \cancel{k} = -0.666\,\text{m/s} \cdot 3.75\,\text{ms} = \underline{\underline{2.50\,\text{mm}}}$$

III. $T_1 - m_1 g = m_1 a$ $\qquad$ $T_1 = m_1(g + a)$

$-T_2 + m_2 g = m_2 a$ $\qquad$ $T_2 = m_2(g - a)$

$(T_2 - T_1) r = I\alpha = I \frac{a}{r}$ $\qquad$ $T_2 - T_1 = \frac{1}{2} m r^2 \frac{a}{r^2} = \frac{1}{2} m a.$

a) Combine:

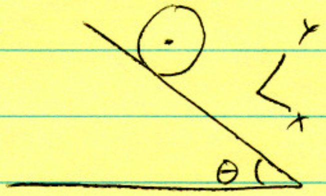
$$m_2(g-a) - m_1(g+a) = \frac{1}{2} m a$$

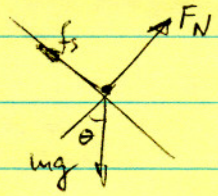
$$a(m_1 + m_2 + \frac{1}{2} m) = (m_2 - m_1) g$$

$$a = \frac{m_2 - m_1}{m_1 + m_2 + \frac{1}{2} m} g = \underline{\underline{0.095\ m/s^2}}$$

b) $\underline{\underline{T_1 = 4.952\ N}}$ $\qquad$ $T_2 = \underline{\underline{4.955\ N}}$

IV





NII: $mg\sin\theta - f_s = ma$

$F_N - mg\cos\theta = 0$

$f_s r = I\alpha = \frac{2}{3} m r^2 \frac{a}{r} = \frac{2}{3} m r a \Rightarrow f_s = \frac{2}{3} m a$

a) Solve:

$mg\sin\theta - \frac{2}{3} ma = ma \Rightarrow g\sin\theta = \frac{5}{3} a$

$a = \frac{3}{5} g \sin\theta$

b) $f_s = \frac{2}{3} m \cdot \frac{3}{5} g \sin\theta = \frac{2}{5} mg\sin\theta$

c) f_s can not be greater than $f_{s,max}$:

$\frac{2}{5} mg\sin\theta \le \mu_s F_N = \mu_s mg\cos\theta$

$\tan\theta_m = \frac{5}{2}\mu_s$

V

a) $\bar{L} = \bar{r} \times \bar{p}$ but everything @ right angles:

$$L = mrv_0 = 0.1\,kg \cdot 0.8\,m \cdot 14\,m/s = \underline{\underline{1.12\ kg\,m^2/s}}$$

b) $K = \frac{1}{2} I\omega^2 = \frac{1}{2} mr^2 \left(\frac{v}{r}\right)^2 = \frac{1}{2} mv^2 = \underline{\underline{9.8\ J}}$

c) $T = F_c = m\frac{v^2}{r} = \underline{\underline{24.5\,N}}$

d) Ang. momentum conserved:

$$r_0 m v_0 = \frac{r_0}{2} m v' \qquad v' = 2v_0$$

$$K' = \frac{1}{2} m v'^2 = 4K_0 = 4 \cdot 9.8\,J = 39.2\ J$$

$$W = \Delta K = 3K_0 = \underline{\underline{29.4\ J}}$$