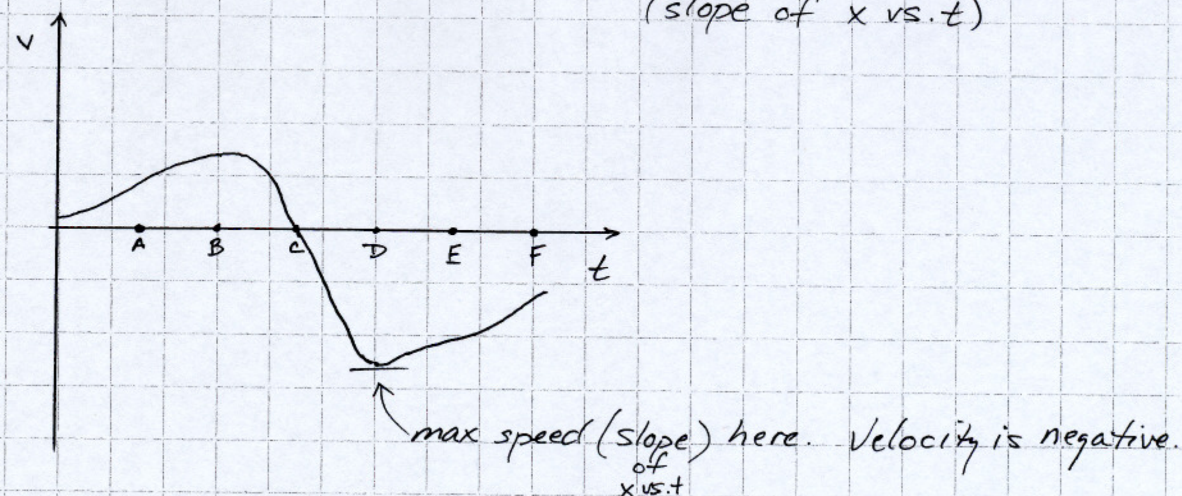
Fall 2007 Physics 10310 Exam I Solutions

I. Multiple Choice Questions

1. (D) To help us with #2, draw a velocity vs. time graph: (slope of x vs. t)

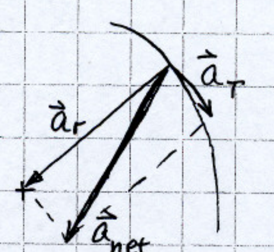


2. (C) $|\vec{a}|$ is biggest at point C.
   ↳ slope of v vs. t.

3. (A) $D = \frac{1}{2}gt^2, \quad D' = \frac{1}{2}g(2t)^2 = 4 \cdot \frac{1}{2}gt^2 = 4D$

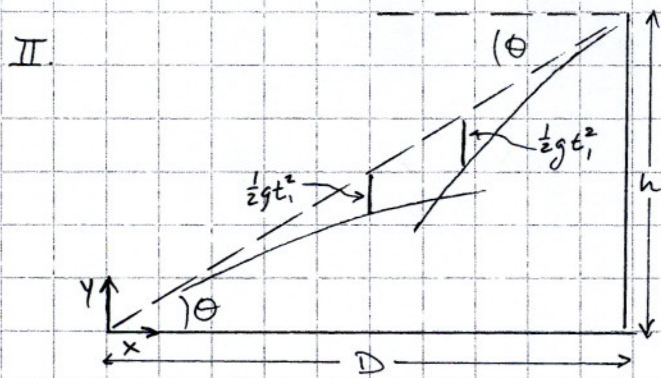
4. (B) $v_f = v_{0y} - gt = 0; \quad gt = v_{0y} = v_0 \sin 60°, \quad t \approx 2.7 \text{ sec}$

5. (D) The ball has a tangential and a radial component of acceleration:



Problems

II.



a.) Since the cannonballs have identical horizontal velocities they will collide at the center point if they arrive there at the same time. Since they both fall away from the diagonal line at the same rate, they are guaranteed to be at the same height if they are fired simultaneously.

Each cannonball has

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t \underbrace{- \tfrac{1}{2} g t^2 \hat{j}}$$

because of this term, the vertical distance below the line $\vec{v}_0 t$ is the same for both.

b.) A: $x_A = v_{0xA} t = v_0 \cos\theta\, t$; $y_A = y_{0A} + v_{0yA} t - \frac{1}{2} g t^2$

(using coordinate axes as marked above.) $= v_0 \sin\theta\, t - \frac{1}{2} g t^2$

B: $x_B = x_{0B} + v_{0xB} t = D - v_0 \cos\theta\, t$; $y_B = y_{0B} + v_{0yB} t - \frac{1}{2} g t^2$

$= h - v_0 \sin\theta\, t - \frac{1}{2} g t^2$

c.) The cannonballs will hit at $x = D/2$ if they are fired simultaneously since they both have the same magnitude of velocity in the x-direction. We can find the time of the collision, then use this to find the height.

$x_A = v_0 \cos\theta\, t = D/2$, $\boxed{t = \frac{D}{2 v_0 \cos\theta}}$

Now, put this into y_A: $y_A = v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$$= v_0 \sin\theta \left(\frac{D}{2 v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{D}{2 v_0 \cos\theta}\right)^2$$

$$\boxed{y_A = \frac{D}{2} \tan\theta - \frac{1}{8} \cdot \frac{g D^2}{v_0^2 \cos^2\theta}}$$

Aside: since $\tan\theta = h/D$ we can rewrite this using $D\tan\theta = h$ and $D^2 = h^2\cos^2\theta/\sin^2\theta$

$$y_A = \frac{h}{2} - \frac{g h^2}{8 v_0^2 \sin^2\theta}.$$

They always hit *below* $h/2$ unless $v_0 \to \infty$.

III. a.) Either convert $30\,m/s$ to $108\,km/hr$, in which case

$$V = \sqrt{108^2 + 60^2} = 124\ km/hr$$

or

convert $60\,km/hr$ to $16.7\,m/s$, in which case

$$v = \sqrt{(16.7)^2 + (30)^2} = 34.3\ m/s$$

Either way, ouch!

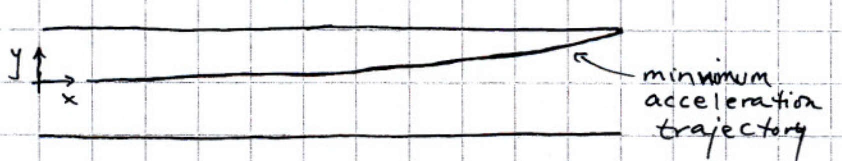
b.) Find out how long it takes the turnip to hit travelling sideways, then find out how far the car travelled forward in that same time:

turnip: $x = vt$, $5 = 30(t)$, $t = 1/6\ s.$

car: $y = vt$, $y = (16.7)(1/6) = \underline{2.8\ \text{meters}}$

IV. a.) Choose axes so $x_0 = 0$, $y_0 = 0$ at center of magnet entrance:

$$x = v_0 t$$
$$y = \frac{1}{2}at^2$$

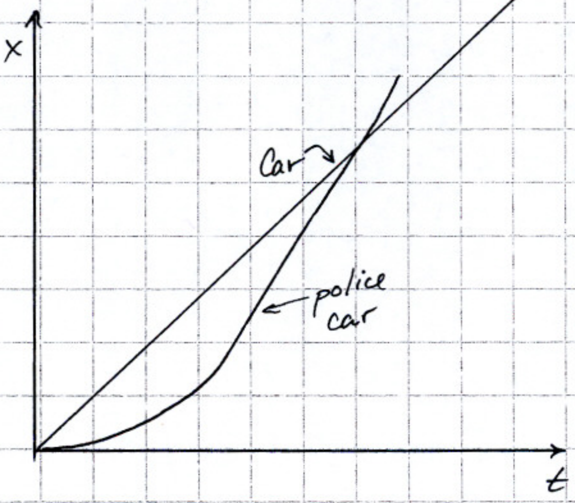


b.) The minimum acceleration trajectory is shown on the drawing; the beam just hits the corner of the magnet at $y = 0.15\,m$ when $x = 2.0\,m$. This occurs, of course, after the beam has crossed the whole magnet,

so $x = v_0 t$, $t = \frac{x}{v_0} = \frac{2}{2\times10^8} = 1\times10^{-8}\ sec$

$y_f = \frac{1}{2}at^2 \Rightarrow 0.15 = \frac{1}{2}a(1\times10^{-8})^2$, $\underline{a = 3\times10^{15}\,m/s^2}$

V. a.)



- car is linear
- police car has a quadratic part followed by linear with same slope as the end of the acceleration period.

b.) $x_{car} = (34.7\ m/s)t$

police car's motion is broken into two parts:

1.) acceleration to final velocity : takes some time t_1 :

$$v_f = v_0 + at_1 \quad , \quad v_f = 52.8\ m/s = (2.2)t_1 \quad , \quad t_1 = 24\ sec.$$

During this time, he travels $x_{pol.} = \frac{1}{2}at^2$
$= \frac{1}{2}(2.2)(24)^2 = 633.6\ m$

2.) motion at constant velocity, until he catches the speeder.

If we define $t = 0$ at the start of the problem,

- $x_{pol.} = x_0 + v_0 t' = 633.6 + (52.8)(t - 24)$
  distance from first part
- $x_{car} = (34.7)t \quad , \quad x_{car} = x_{pol}$ gives:

$633.6 = (18.1)t \quad , \quad \boxed{t = 35\ sec.}$

c.) Both cars travel same distance : $x_{car} = vt = (34.7)(35) = \underline{1215\ m}$

V b) alternate solution to final part:

take $t = 0$ when police car stops accelerating after 24 seconds. At this time, the speeding car has already traveled $x_{car} = (34.7)(24) = 832.8$ m.
$= v_0 \cdot t$

Now: $x_{car} = 832.8 \text{ m} + v_0 t = 832.8 + (34.7) t$

$x_{police} = 633.6 \text{m} + v_{0_p} t = 633.6 + (52.8) t$

$x_{car} = x_{pol} \Rightarrow (832.8 - 633.6) = [(52.8) - 34.7] t$

$199.2 = (18.1) t$, $t = 11$ sec. for this part of the chase.

total time is $t_{tot} = 24 + 11 = \underline{35 \text{ sec}}$.