2005 Final. 10310 ①

MC1. $\lambda = 4\ m$

$T = 1/2\ s$

$v = \lambda f = \lambda/T = 8\ m/s$ Ⓐ

MC2. $m\frac{v^2}{R} = G\frac{mM}{R^2}$

$\Downarrow$

$\left(\frac{2\pi R}{T}\right)^2 = G M/R$

$\Downarrow$

$T^2 = 4\pi^2 \frac{R^3}{GM}$ $T = 2\pi\sqrt{\frac{R^3}{GM}}$ Ⓔ

MC3. $\underbrace{m_1 v_0 = m_1 v_1 + m_2 v_2 \quad \& \quad \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$

$\cancel{\frac{1}{2} m_1 v_0^2} = \frac{1}{2} m_1 \left(v_0 - \frac{m_2}{m_1} v_2\right)^2 + \frac{1}{2} m_2 v_2^2 = \cancel{\frac{1}{2} m_1 v_0^2} - m_2 v_0 v_2 + \frac{1}{2}\frac{m_2^2}{m_1} v_2^2 + \frac{1}{2} m_2 v_2^2$

$\frac{1}{2} \cancel{m_2} \left(\frac{m_2}{m_1} + 1\right) v_2 = \cancel{m_2} v_0$ $v_2 = 2 \cdot \frac{m_1}{m_1 + m_2} v_0$ $0 < v_2 < v_0$

$v_1 = v_0 - \frac{m_2}{m_1} v_2 = v_0 \left(1 - \frac{m_2}{\cancel{m_1}} \cdot 2 \frac{\cancel{m_1}}{m_1 + m_2}\right)$

$= v_0 \cdot \frac{m_1 + m_2 - 2m_2}{m_1 + m_2} = \frac{m_1 - m_2}{m_1 + m_2} v_0$ $0 > v_1$ Ⓔ

MC4. $\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - \frac{1}{2} m v_{esc}^2 = \frac{1}{2} m \left(9 v_{esc}^2 - v_{esc}^2\right) = 8 \frac{1}{2} m v_{esc}^2$

$v_f = \sqrt{8}\ v_{esc} = \sqrt{8} \cdot 11.2\ km/s = 31.7\ km/s$ Ⓓ

MC5. $mv_i = (m+M)\,v_f$ $v_f = \frac{m}{m+M} v_i$ $a = \mu_k g$

$v^2 = 2ax \Rightarrow$ $x = \left(\frac{m}{m+M}\right)^2 \frac{v_i^2}{2\mu_k g} = \left(\frac{0.018}{4.668}\right)^2 \frac{(725\,m/s)^2}{2\cdot 0.35\cdot 9.81\,m/s^2} = 1.14\,m$ (A)

MC6. $\vec{L}$ along $\hat{z}$. To decrease $|\vec{L}|$ $\vec{\tau}$ must be along $\hat{T}$ (A)

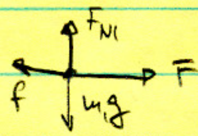
MC7. $\underbrace{a_c = \frac{v^2}{R} = \mu_s g \;\;\&\;\; v = \omega R}$

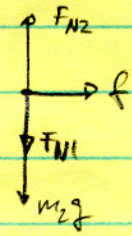
$\omega^2 R = \mu_s g \Rightarrow$ $R = \frac{\mu_s g}{\omega^2} = \frac{0.2\cdot 9.81\,m/s^2}{(1.4\,rad/s)^2} = 1\,m$ (B)

MC8. $K = \frac{1}{2}mv^2 > 0$ Excludes 1, 3, 4

K oscillates with period $T/2 \to 5$ (E)

II. a)





b) $F - f = m_1 a_1$ & $F_{N1} - m_1 g = 0$

c) $f = m_2 a_2$ & $F_{N2} - F_{N1} - m_2 g = F_{N2} - (m_1 + m_2) g = 0$

d) $f = F - m_1 a_1 = \mu_k m_1 g$

$$\mu_k = \frac{F - m_1 a_1}{m_1 g} = \frac{320\,N - 60\,kg \cdot 3\,m/s^2}{60\,kg \cdot 9.81\,m/s^2} = \underline{\underline{0.238}} \quad (a)$$

e) $$a_2 = f/m_2 = \frac{F - m_1 a_1}{m_2} = \frac{320\,N - 60\,kg \cdot 3\,m/s^2}{100\,kg} = \underline{\underline{1.4}}\ m/s^2$$

III. a) $$\bar{g} = -G\frac{M}{R^2} = -G\frac{4\pi/3\,\rho R^3}{R^2} = -\frac{4\pi}{3}G\rho R = -\frac{4\pi}{3}\cdot 6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}\cdot 1.9\frac{10^{-3}kg}{(10^{-2}m)^3}\cdot 2.631\cdot 10^{6}\,m$$

$$= \underline{\underline{-1.4}}\ m/s^2 \quad (a)$$

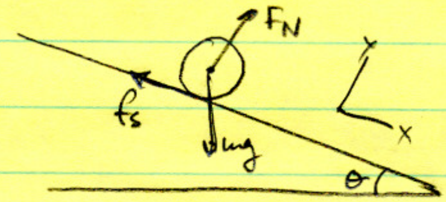
b) $$\bar{g}(r) = -\frac{4\pi}{3}G\rho r = \bar{g}(R)\frac{r}{R}$$

c) $$F = m\bar{g}(R)\,r/R = -kr \quad \Rightarrow \quad k = -m\bar{g}(R)/R$$

$$\omega = \sqrt{k/m} = \sqrt{\frac{-\bar{g}(R)}{R}} = \sqrt{\frac{-1.4\,m/s^2}{2.631\cdot 10^6\,m}} = 7.29\cdot 10^{-4}\ rad/s$$

$$f = \frac{\omega}{2\pi} = \underline{\underline{1.16}}\cdot 10^{-4}\ Hz$$

IV. a)



b) $mg\sin\theta - f_s = ma$

c) $\tau = I\alpha \Rightarrow f_s \cancel{R} = \frac{2}{3} m R^{\cancel{2}}\, a/\cancel{R} \Rightarrow f_s = \frac{2}{3} ma$

d) $ma + f_s = ma\left(1 + \frac{2}{3}\right) = mg\sin\theta$

$\Downarrow$

$a = \frac{3}{5} g \sin\theta = \frac{3}{5}\cdot 9.81\,m/s^2 \cdot \sin 15^\circ = \underline{1.52\ m/s^2}$ (d)

e) $f_s = \frac{2}{3} ma = \frac{2}{3}\cdot\frac{3}{5} mg\sin\theta = \frac{2}{5} mg\sin\theta = \frac{2}{5}\cdot 0.6\,kg \cdot 9.81\,m/s^2 \cdot \sin 15^\circ = \underline{\underline{0.61\ N}}$

V. a) $E_n = \left(1 - (\Delta E/E)_{cycle}\right)^n E_0 = \frac{1}{2} E_0$

$n \ln\left(1 - \left(\frac{\Delta E}{E}\right)_{cycle}\right) = -\ln 2$ $\qquad n = -\frac{\ln 2}{\ln 0.95} = 13.5$ cycles

b) $Q = \frac{2\pi}{(\Delta E/E)_{cycle}} = \frac{2\pi}{0.05} = 40\pi = \underline{\underline{125.7}}$

c) $\Delta\omega = \omega_0/Q = \frac{2\pi f_0}{2\pi/0.05} = 0.05 f_0 = 0.05 \cdot 120$ rad/s $= 6$ rad/s

VI.

a) Cons. energy: $mg(\ell + \Delta x)\sin\theta = \frac{1}{2}k\Delta x^2$

$$\overset{50}{\frac{1}{2}k}\Delta x^2 - \overset{9.81}{mg\sin\theta}\cdot\Delta x - \overset{39.24}{mg\ell\sin\theta} = 0$$

$$\Delta x = \frac{9.81\,N \pm \sqrt{(9.81\,N)^2 + 4\cdot 50\,N/m\cdot 39.24\,Nm}}{2\cdot 50\,N/m} = -0.793\,m;\ \underline{\underline{0.989\,m}}\ (6)$$

b) Again: $mg(\ell+\Delta x)\sin\theta - \mu_k mg\cos\theta(\ell+\Delta x) = \frac{1}{2}k\Delta x^2$

$$\overset{50}{\frac{1}{2}k}\Delta x^2 - \overset{6.41\,(7)}{mg(\sin\theta - \mu_k\cos\theta)}\Delta x - \overset{25.65\,(2)}{mg\ell(\sin\theta - \mu_k\cos\theta)} = 0$$

$$\Delta x = \frac{6.41\,N \pm \sqrt{(6.41\,N)^2 + 4\cdot 50\,N/m\cdot 25.65\,Nm}}{2\cdot 50\,N/m} = -0.655\,m;\ \underline{\underline{0.783\,m}}$$

VII

a) $I = P_{av}/A$

$$P_{av} = IA = 4\pi I R^2 = 4\pi\cdot 10^{-3}\,W/m^2\cdot(100\,m)^2 = \underline{\underline{126\,W}}\ (6)$$

b) $I_{30} = \frac{P_{av}}{A} = I_{100}\left(\frac{100\,m}{30\,m}\right)^2 = 1.11\cdot 10^{-2}\,W/m^2$

$$\beta = 10\,dB\log I_{30}/I_0 = 10\,dB\log 1.11\cdot 10^{10} = \underline{\underline{100.5\,dB}}$$