Final exam PHYS 10310 Spring 2007

MC 1. B

MC2. Friction force prevents block from sliding down:

$f_s = mg \sin\theta$ to compensate component gravitational force parallel to incline.

E

MC3. Friction always opposes motion so same energy is dissipated.

C

MC4. Conservation of momentum:

$|m_g \cdot v_g| = |m_b\, v_b| \Rightarrow \quad v_b = v_g \frac{m_g}{m_b} = 3\,m/s \cdot \frac{20}{30} = 2\,m/s$

B

MC5. $F = G\frac{m_e m_p}{r^2} = 6.67 \cdot 10^{-11}\,Nm^2/kg^2 \cdot \frac{9.11 \cdot 10^{-31}\,kg \cdot 1.67 \cdot 10^{-27}\,kg}{(0.0529 \cdot 10^{-9}\,m)^2} = 3.6 \cdot 10^{-47}\,N$ C

MC6. Think mass on spring: $F = -k\Delta x$ D

MC7. $Q = \frac{2\pi}{|\Delta E/E|_{cycle}} = \frac{2\pi}{0.03} = 209$ E

MC8. A

II a) Conservation of energy:

$$\frac{1}{2} m v_i^2 = \frac{1}{2} m (v_i \cos 60^\circ)^2 + mg\,\Delta y$$

$\Downarrow$

$$\Delta y = \frac{v_i^2}{2g}(1 - \cos^2 60^\circ) = \frac{v_i^2}{2g} \sin^2 60^\circ = \underline{\underline{9.79\ m}}\ (0)$$

b) Cons. energy again

$$\frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 - mgh$$

$\Downarrow$

$$v_f = \sqrt{v_i^2 + 2gh} = \sqrt{(16\ m/s)^2 + 2 \cdot 9.81\ m/s^2 \cdot 12\ m} = \underline{\underline{22.2\ m/s}}\ (1)$$

c) Projectile motion:

$$y = \tan\theta_0\ x - \frac{g x^2}{2(v_0 \cos\theta_0)^2}$$

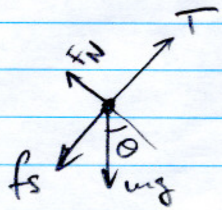
$\Downarrow$ (2) (3)

$$0.0766\ x^2 - 1.732\ x - 12\ m = 0$$

$\Downarrow$

$$x = \frac{1.732 \pm \sqrt{1.732^2 - 4 \cdot 0.0766 \cdot (-12)}}{2 \cdot 0.0766} = -5.56\ m\ ;\ \underline{\underline{28.16\ m}}$$

III Free-body diagram:




$$T - f_s - mg \sin\theta = 0$$

$$F_N - mg \cos\theta = 0 \Rightarrow F_N = mg \cos\theta$$

$$f_s = \mu_s F_N = \mu_s mg \cos\theta$$

a) $f_s = 0.4 \cdot 3\,\text{kg} \cdot 9.81\,\text{m/s}^2 \cdot \cos 20^\circ = \underline{\underline{11.06\,\text{N}}}$ downhill!

b) Tension must be provided by spring force:

$$T = f_s + mg \sin\theta = mg(\mu_s \cos\theta + \sin\theta) = k\,\Delta x$$

$$\Downarrow$$

$$\Delta x = \frac{mg}{k}\left(\mu_s \cos\theta + \sin\theta\right) = \frac{3\,\text{kg} \cdot 9.81\,\text{m/s}^2}{200\,\text{N/m}}\left(0.4 \cos 20^\circ + \sin 20^\circ\right)$$

$$= \underline{\underline{10.6\,\text{cm}}}\ (o)$$

c) Elastic potential energy:

$$U_s = \frac{1}{2} k\,\Delta x^2 = \underline{\underline{1.12\,\text{J}}}$$

IV a) Conservation of momentum:

$$2\,kg \cdot 8\,m/s = 2\,kg \cdot (-1\,m/s) + 5\,kg \cdot v_f$$

$$\Downarrow$$

$$v_f = \frac{2}{5}\left(8\,m/s - (-1)\,m/s\right) = \underline{\underline{3.6\,m/s}}$$

b) $$\Delta E = K_i - K_f = \frac{1}{2} \cdot 2\,kg \cdot (8\,m/s)^2 - \left(\frac{1}{2} \cdot 2\,kg \cdot (-1\,m/s)^2 + \frac{1}{2} \cdot 5\,kg \cdot (3.6\,m/s)^2\right)$$

$$= \underline{\underline{30.6\,J}}$$

c) Collision clearly inelastic since $\Delta E \neq 0$ but not perfectly inelastic since two carts does not have common final velocity!

$$e = -\frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}} = -\frac{3.6 - (-1)}{0 - 8} = 0.575.$$

a) Conservation of energy:

$$\Delta U = \Delta K \Rightarrow 30\,kg \cdot gh - 20\,kg \cdot gh = \frac{1}{2}(30\,kg + 20\,kg)v^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}(30\,kg + 20\,kg)v^2 + \frac{1}{2}\cdot\frac{1}{2}m\,r^2\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}(30\,kg + 20\,kg + \frac{1}{2}\cdot 5\,kg)v^2$$

$$v = \sqrt{\frac{(30-20)\,kg \cdot 9.81\,m/s^2 \cdot 2\,m}{\frac{1}{2}(30\,kg + 20\,kg + \frac{1}{2}\cdot 5\,kg)}} = \underline{2.73\,m/s} \quad (a)$$

$$\omega = v/r = \frac{2.73\,m/s}{0.1\,m} = \underline{0.273\,rad/s}$$

b) 1D kinematics:

$$v^2 - v_0^2 = 2a(x - x_0) \qquad x_0 = 0 \quad v_0 = 0$$

$$\Downarrow$$

$$a = \frac{v^2}{2h} \qquad t = v/a = v/(v^2/2h) = \frac{2h}{v} = \frac{2\cdot 2\,m}{2.73\,m/s} = \underline{1.46\,s}$$

c) Forces on 20 kg & 30 kg blocks:

$$T_{20} - 20\,kg \cdot g = 20\,kg \cdot a \qquad 30\,kg \cdot g - T_{30} = 30\,kg \cdot a$$

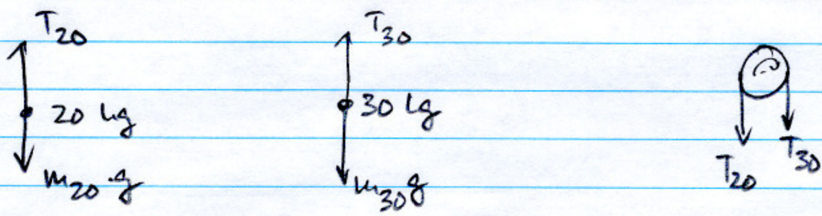
$$\Downarrow \qquad \Downarrow$$

$$T_{20} = 20\,kg\,(g + a) \qquad T_{30} = 30\,kg\,(g - a)$$

$$a = \frac{v^2}{2h} = \frac{(2.73\,m/s)^2}{2\cdot 2\,m} = 1.869\,m/s^2 \Rightarrow T_{20} = \underline{233.6\,N}$$

$$T_{30} = \underline{238.2\,N}$$

I. Alternative (traditional) solution:




N II: $T_{20} - m_{20} g = m_{20} a$ $\qquad$ a pos. ↑

$m_{30} g - T_{30} = m_{30} a$ $\qquad$ a pos. ↓

$(T_{30} - T_{20}) r = I \alpha$ $\qquad$ $\alpha = a/r$ $\qquad$ $I = \frac{1}{2} m r^2$.

Solve:

$T_{20} = m_{20}(g+a)$ $\qquad$ $T_{30} = m_{30}(g-a)$

$\left(m_{30}(g-a) - m_{20}(g+a)\right) r = \frac{1}{2} m r^2 \, a/r^2 \Rightarrow \left(m_{30} + m_{20} + \frac{1}{2} m\right) a = (m_{30} - m_{20}) g$

$\Rightarrow a = \dfrac{m_{30} - m_{20}}{m_{30} + m_{20} + \frac{1}{2} m} \cdot g = 1.869\ m/s^2$

a) 1D Kinematics:

$v^2 - v_0^2 = 2a(x - x_0) \Rightarrow v = \sqrt{2 \cdot a \cdot 2m} = 2.73\ m/s$ $\qquad$ $\underline{\omega = \frac{v}{r} = 27.3\ \frac{rad}{s}}$

b) $x = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \underline{\underline{1.46\ s}}$

c) $T_{30} = 30\ kg \cdot (9.81\ m/s^2 - 1.869\ m/s^2) = \underline{\underline{238.2\ N}}$

$T_{20} = 20\ kg \cdot (9.81\ m/s^2 + 1.869\ m/s^2) = \underline{\underline{233.6\ N}}$

VI a) Mass on a spring:

$$\omega = \sqrt{k/m} \Rightarrow m = k/\omega^2 = \frac{k}{(2\pi f)^2} = \frac{0.2\,N/m}{(2\pi \cdot 1.35\,Hz)^2} = \underline{\underline{2.78\,g}}$$

b) Escape velocity using mass & radius of Mars:

$$v_{esc} = \sqrt{2GM/R} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11}\,Nm^2/kg^2 \cdot 6.421 \cdot 10^{23}\,kg}{3397\,km}} = \underline{\underline{5021\,m/s}}$$

c) Gravitational field:

$$g_M = G\frac{M}{R^2} = 6.67 \cdot 10^{-11}\,Nm^2/kg^2 \cdot \frac{6.421 \cdot 10^{23}\,kg}{(3397\,km)^2} = \underline{\underline{3.71\,m/s^2}} \quad (c)$$

d) Thrust is given by

$$F_{Th} = Ru$$

If this is just to overcome Martian gravity we must have

$$F_{Th} = mg_M \Rightarrow Ru = mg_M \Rightarrow R = \frac{mg_M}{u} = \frac{20\,kg \cdot 3.71\,m/s^2}{2.3\,km/s}$$

$$= \underline{\underline{32.3\,g/s}}$$

I. a) Free fall:

$$s = \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \cdot 60\,m}{9.81\,m/s^2}} = \underline{\underline{3.50\,s}} \quad (0)$$

b) More 1D kinematics:

$$v = gt = 9.81\,m/s^2 \cdot 3.50\,s = \underline{\underline{34.3\,m/s}} \quad (1)$$

c) Doppler effect:

$$f_r = f_s \cdot \frac{v \pm u_r}{v \mp u_s} \qquad v = 340\,m/s \qquad u_s = 34.3\,m/s \qquad u_r = 0$$

$$= 440\,Hz \cdot \frac{340}{340 + 34.3} = \underline{\underline{400\,Hz}}$$

d) This is time to drop in (a) + time for sound to travel back up shaft:

$$t = 3.50\,s + \frac{60\,m}{340\,m/s} = \underline{\underline{3.67\,s}}$$