Physics 10310 Spring 2008 Final Exam Solutions

I. Multiple Choice

1. (b) To move the object, $\sum F_x = F - f_s = 0$, $\sum F_y = 0 = N - mg$,

$$\Rightarrow f_{s_{max}} = \mu_s N = \mu_s mg \Rightarrow F = \mu_s mg = f_s$$

Once it starts moving, $\sum F_x = F - f_k = ma_x$, $f_k = \mu_k N = \mu_k mg$

$$\Rightarrow f_s - f_k = ma_x;\ \mu_s mg - \mu_k mg = ma_x,\ a_x = g(\mu_s - \mu_k) = 0.98\,m/s^2$$

2. (d) $E_{tot} = U + KE \Rightarrow -2J = -6J + KE$, $KE = 4J$, $\frac{1}{2}mv^2 = 4$,

$$\Rightarrow \tfrac{1}{2}(2)v^2 = 4,\ v^2 = 4,\ v = 2\,m/s$$

3. (c) $0 = v_0^2 - 2gH$, $v_0 = \sqrt{2gH} \Rightarrow$ $y_1 = H - \frac{1}{2}gt^2$, $y_2 = 0 + v_0 t - \frac{1}{2}gt^2$ } they hit when $y_1 = y_2$

$$\Rightarrow H - \cancel{\tfrac{1}{2}gt^2} = v_0 t - \cancel{\tfrac{1}{2}gt^2},\quad t = \frac{H}{v_0} = \frac{H}{\sqrt{2gH}}$$

at this time, $y_1 = H - \frac{1}{2}gt^2 = H - \frac{1}{2}g\left(\frac{H^2}{2gH}\right) = H - \frac{1}{4}H = 3/4\,H.$

4. (a) Cons. of Momentum: $mv = (m+m)v_f$, $v_f = \frac{1}{2}v$.

Cons. of Energy: $\frac{1}{2}(\cancel{m+m})v_f^2 = (\cancel{m+m})gh$, $h = \frac{v_f^2}{2g} = \frac{v^2}{8g}$

5. (c) $\tau = I\alpha \Rightarrow F(L/2) = \frac{1}{12}ML^2\alpha$, $\alpha = \frac{6F}{ML}$, $\omega = \alpha t$, $v = \frac{L}{2}\omega = \frac{L}{2}\alpha t$

Now, $L \to L/2$, but v doesn't depend on L, so $v_{first} = \frac{3F}{m}t$

$v_{new} = v_{first}$

OR:

$$\tau = I\alpha_n \Rightarrow F(L/4) = \tfrac{1}{12}M\left(\tfrac{L}{2}\right)^2\alpha_n,\ \alpha_n = \frac{12F}{ML},\ V_n = \frac{L}{4}\omega_n = \frac{L}{4}\alpha t = \frac{3F}{m}t\ (\checkmark)$$

6. (a),(b) $A = 0.2\,m$, $\omega = \sqrt{\frac{k}{m}} = 7\,rad/s$. If $+x$ is to the right, we start @ $-A$ at $t=0$, so $\delta = \pi$. The problem doesn't state $+x$, though, so (b) is ok.

7. (e) or (a) Only (a) has units of frequency $(1/s)$, as you can see from $kg \times m/s^2 \times m/(kgm^2/s) = 1/s$. The correct frequency should have a factor of 2π, so (e) is truly correct.

8. (c) $KE = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega_{cm}^2$, $\omega_{cm} = \frac{v_{cm}}{R} \Rightarrow KE = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\frac{v_{cm}^2}{R^2}$

$$\Rightarrow KE = Mv_{cm}^2\left(\tfrac{1}{2} + \tfrac{1}{3}\right) = 3.75J$$

Problems

II. a.) $E_i = mgh$, $E_f = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, $\sum F_r = kx - mg = \frac{mv^2}{h}$

h = total height = total length of the stretched cord
x = distance cord is stretched

b.) Use centripetal force to subsitute for KE: $\frac{mv^2}{h} = kx - mg$,

$\frac{1}{2}mv^2 = \frac{kxh}{2} - \frac{mgh}{2}$, From Energy: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

Substitute:

$mgh = \frac{kxh}{2} - \frac{mgh}{2} + \frac{1}{2}kx^2 \Rightarrow \frac{3}{2}mgh = k\left(\frac{xh}{2} + \frac{x^2}{2}\right)$

Plugging in numbers, $132{,}300 = k(3750 + 1250)$, $\underline{k = 26.46 \ N/m}$

c.) Use either equation to find v: $\frac{mv^2}{h} = kx - mg$, $v^2 = \frac{kxh}{m} - gh$

$v = \sqrt{1837.5} = 42.9 \ m/s$

d.) $|F| = kx = (26.46)(50) = 1323 \ N \approx 2g$

III. a.) $U = -\frac{GM_E m}{R_E + h_1} = -\frac{(6.7\times10^{-11})(6\times10^{24})(10^4)}{6.37\times10^6 + 2\times10^6} = -4.78\times10^{11} \ J$

b.) Use centripetal force to find v^2: $\frac{mv^2}{R_E+h_1} = \frac{GM_E m}{(R_E+h_1)^2}$, $mv^2 = \frac{GM_E m}{(R_E+h_1)} = |U|$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}|U| = 2.39\times10^{11} \ J$

c.) $E = KE + U = \frac{1}{2}|U| - |U| = -\frac{1}{2}|U| = -2.39\times10^{11} \ J$

d.) $E_{new} = -\frac{1}{2}|U_{new}|$, $|U_{new}| = \frac{GM_E m}{(R_E + h_2)} = 3.86\times10^{11} \ J \Rightarrow \underbrace{E_{new} = -1.93\times10^{11} \ J}$

higher orbit, E_{tot} is less negative.

$\Delta E = E_f - E_i$

$\Delta E = -1.93\times10^{11} J + 2.39\times10^{11} J = \underline{4.6\times10^{10} \ J}$

IV. a) $W_F = \Delta KE = -KE_i$, $W_F = -F_k \cdot d$, $F_k = \mu_k N = \mu_k (m_c + m_T) g$

$-\mu_k \cancel{(m_c+m_T)} g d = -\frac{1}{2}\cancel{(m_c+m_T)} V_F^2$, $V_F^2 = 2\mu_k g d$, $\underline{V_F = 16.6 \text{ m/s}}$

b.) Cons. of Momentum: P_x: $m_T V_T = (m_c+m_T) V_F \cos(30°)$

$$V_T = \frac{m_c + m_T}{m_T} V_F \cos(30°) = \underline{19.1 \text{ m/s}}$$

P_y: $m_c V_c = (m_c+m_T) V_F \sin(30°)$

$$V_c = \frac{m_c+m_T}{m_c} V_F \sin(30°) = \underline{33.1 \text{ m/s}}$$

c.) $\Delta KE = KE_f - KE_i = \frac{1}{2}(m_c+m_T)V_F^2 - \left[\frac{1}{2}m_c V_c^2 + \frac{1}{2}m_T V_T^2\right]$

$= 0.551 \times 10^6 - 1.096 \times 10^6 \text{ J} = -0.545 \times 10^6 \text{ J}$

$\Rightarrow$ $\underline{5.45 \times 10^5 \text{ J were lost}}$

V. a.) $I = \sum mr^2 = 8mR^2$

b.) $L = MvR$, Find v from Cons. of E: $Mgh = \frac{1}{2}Mv^2 + MgR$, $2g(h-R) = v^2$

$L = MR\sqrt{2g(h-R)}$

c.) $I_{new} = 8mR^2 + MR^2$, $L_{wheel} = I_{new}\omega = L_i = MR\sqrt{2g(h-R)}$

$$\omega = \frac{L_i}{I_{new}} = \frac{MR\sqrt{2g(h-R)}}{R^2(8m+M)} = \underline{\frac{M\sqrt{2g(h-R)}}{R(8m+M)}} = \omega$$

d.) $E_i = E_f$ $\quad \frac{1}{2}I_{new}\omega^2 + MgR = \frac{1}{2}I_{new}\omega_f^2$, $\omega_f^2 = \omega^2 + \frac{2MgR}{I_{new}}$

$$\omega_f^2 = \frac{M^2(2g(h-R))}{R^2(8m+M)^2} + \frac{2MgR}{R^2(8m+M)}, \quad \omega_f = \sqrt{\frac{M^2[2g(h-R)]}{R^2(8m+M)^2} + \frac{2Mg}{R(8m+M)}}$$

VI. a.) $I = \frac{1}{3}M_r L^2 + \frac{2}{5}M_s R^2 + M_s(L+R)^2 = 1.0 + 0.072 + 6.48 = 7.55 \text{ kg}\cdot\text{m}^2$

b.) $$d = \frac{m_r \cdot L/2 + m_s(L+R)}{m_r + m_s} = \frac{1.5 + 5.4}{7.5} = 0.92 \text{ m}$$

c.) $\tau = I\alpha$, $\tau_{grav} = -M_{tot} g d \sin\phi = I\frac{d\phi^2}{dt^2}$, $\frac{d\phi^2}{dt^2} = -\frac{M_{tot} g d}{I}\phi$, $[\sin\phi \approx \phi]$

d) $\omega = \sqrt{\frac{Mgd}{I}} = 3.0 \text{ rad/s}$, $\underline{T = \frac{2\pi}{\omega} = 2.1 \text{ sec.}}$