

**INSTRUCTIONS:** Write your NAME and LECTURE SECTION (I: Ruchti, II: Hildreth) on the front of the blue exam booklet. The exam is closed book, and you may have only pens/pencils and a calculator (no stored equations or programs and no graphing). Show all of your work in the blue book. For problems II-V, an answer alone is worth very little credit, even if it is correct – so show how you get it.

Suggestions: Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

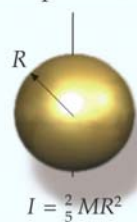
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#### Moments of Inertia of Uniform Bodies of Various Shapes

Thin rod about perpendicular line through center



Solid sphere about diameter



**I. Multiple Choice Questions** (4 points each) Please write the letter corresponding to your answer for each question in the grid stamped on the first *inside* page of your blue book. No partial credit is given for these questions.

1. An object of mass 1 kg is moving initially with a velocity 3 m/s, moving in the positive direction along the  $x$  axis. A force then acts on the object for 0.5 seconds. Afterwards, the object moves with a velocity of 4 m/s in the positive direction along the  $y$  axis. The average force that acted on the object had a magnitude of:

- (a) 5 N      (b) 10 N      (c) 1 N      (d) 7 N      (e) none of these

2. A truck of mass 2000 kg engages in a head-on, perfectly inelastic collision with a water buffalo of mass 500 kg (*i.e.*, their initial velocities are in opposite directions.). The initial velocity of the truck is 5 m/s, and the initial velocity of the buffalo is 2 m/s. What is the ratio of the final to the initial kinetic energy of the system?

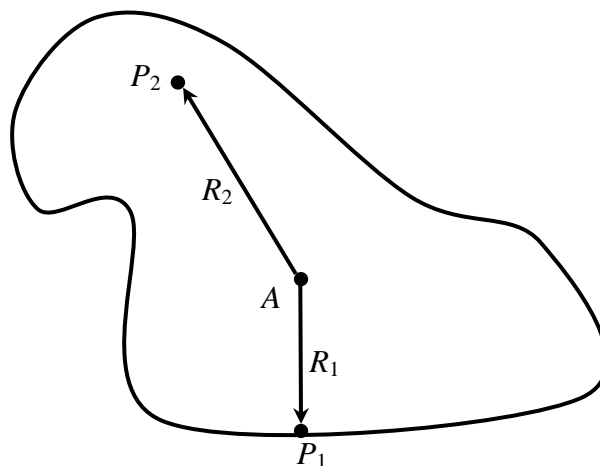
- (a) 1.25      (b) 0.62      (c) 0.68      (d) 0.31      (e) 1.96

3. A solid disk spins about its axis of symmetry with an angular velocity  $\omega_0 = 2\pi$  rad/s. It is decelerated to rest in 10 seconds. The total angle (in radians) through which the disk turns in this time interval is:

- (a)  $20\pi$       (b)  $4\pi$       (c)  $\pi$       (d)  $2\pi$       (e)  $10\pi$

4. In the figure below,  $R_2 > R_1$ , and the point  $A$  marks the center of mass of the object. The following moments of inertia are calculated about axes perpendicular to the plane of the paper: the moment of inertia about an axis through point  $P_1$  is  $I_1$ , the moment of inertia about an axis through  $P_2$  is  $I_2$ , and the moment of inertia about an axis through point  $A$ , the center of mass, is  $I_{\text{cm}}$ . Which of the relationships among the moments of inertia shown below is true?

- (a)  $I_1 = I_2 > I_{\text{cm}}$       (b)  $I_1 = I_2 < I_{\text{cm}}$       (c)  $I_1 > I_2 > I_{\text{cm}}$   
(d)  $I_2 > I_1 > I_{\text{cm}}$       (e)  $I_2 < I_1 < I_{\text{cm}}$



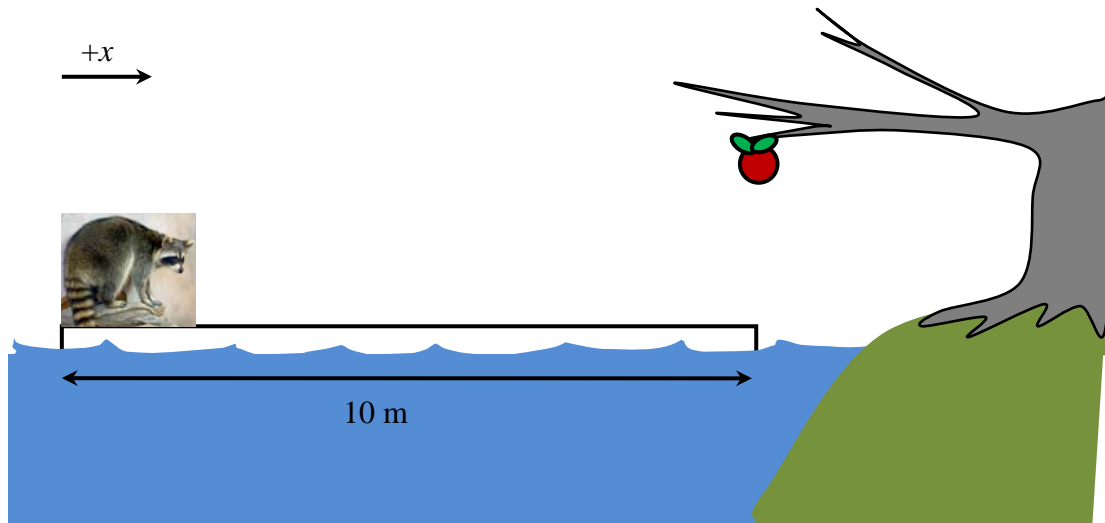
5. A cylindrical winch of radius 0.5 m rotates at a constant 100 revolutions per minute. The winch is driven by a motor that generates a total torque of 50000 Nm while lifting a heavy container off of a cargo ship. The total power generated by the winch is

- (a) 83 kW    (b) 261 kW    (c) 400 kW    (d) 523 kW    (e) 5000 kW

**Problems (20 points each)**

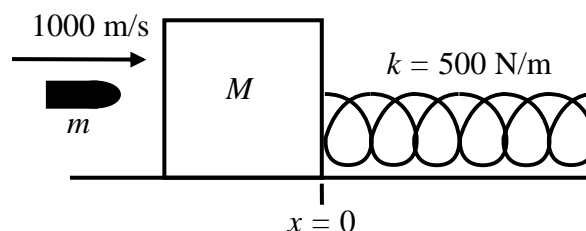
II. A raccoon standing at one end of a 10 meter-long floating log notices an apple hanging from a tree branch directly over the far end of the log. The raccoon starts walking along the log at a constant velocity of  $\mathbf{u} = 0.1 \text{ m/s } \hat{\mathbf{i}}$  toward the apple; this is the velocity of the raccoon *with respect to the log*. Assume the mass of the raccoon is  $m = 25 \text{ kg}$  and the mass of the log is  $M = 100 \text{ kg}$ . Neglect the size of the raccoon. (Depending on your choice of method, you may answer (b) before (a).)

- As the raccoon is walking, what is the velocity of the log with respect to the shore?
- When the raccoon reaches the opposite end of the log, how far is he horizontally from the apple he desires?



III. A bullet of mass  $m = 5 \text{ grams}$  travelling at 1000 m/s strikes and sticks instantaneously in a massive block of mass  $M = 20 \text{ kg}$  which is attached to a massless spring that is initially uncompressed. The spring constant of the spring is  $k = 500 \text{ N/m}$ .

- Find the recoil velocity  $V$  of the block/bullet system at the moment immediately after impact, before the spring begins to compress.
- Assuming the surface on which the block rests is frictionless, find the distance of maximum compression of the spring after the collision.
- Assuming the surface on which the block rests has a coefficient of kinetic friction  $\mu_k = 0.2$ , find the distance of maximum compression of the spring after the collision. (Hint: use the Work-Energy Theorem.)

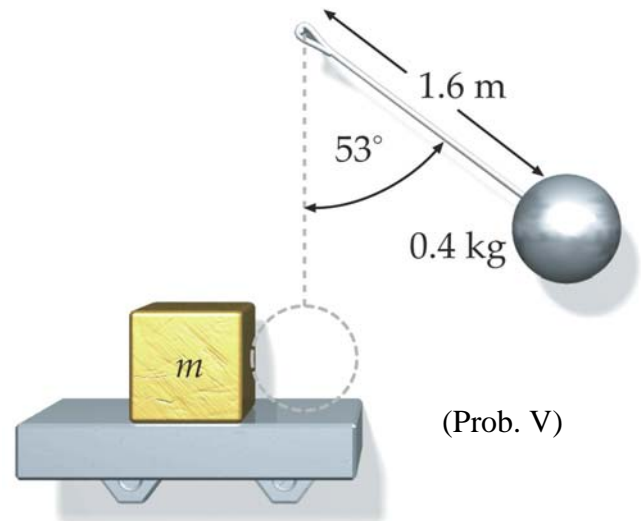
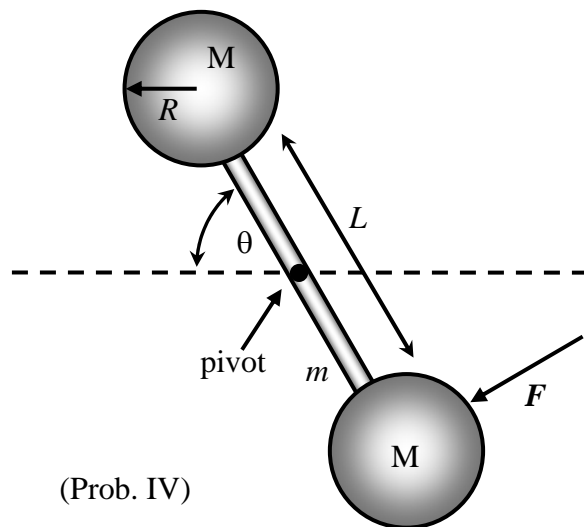


IV. A barbell spins in the vertical plane about a horizontal axis perpendicular to the bar separating the two uniform spherical weights, as shown in the figure below, left. Each of the spherical weights has mass  $M$  and radius  $R$ ; the thin bar separating them has mass  $m$  and length  $L$ .

- Find the moment of inertia of the barbell as it spins about the axis shown, which lies midway between the two spheres. (Hint: the parallel axis theorem may be useful here.)
- At an arbitrary angle  $\theta$  away from the horizontal, what is the total torque due to gravity on the barbell? Show your calculation.

Now, a uniform force  $F = 44 \text{ N}$  is applied to one of the weights, as shown. The force  $F$  remains perpendicular to the central bar as the system rotates. You are given the following dimensions:  $m=0.2 \text{ kg}$ ,  $L=20 \text{ cm}$ ,  $R=10 \text{ cm}$ .

- What is the torque due to the force  $F$ ?
- If the observed angular acceleration is  $10 \text{ rad/s}^2$ , what is the mass  $M$ ?



V. As shown in the figure, above right, a pendulum consists of a  $0.4 \text{ kg}$  bob attached to a string of length  $1.6 \text{ m}$ . A block of mass  $m$  rests on a horizontal frictionless surface. The pendulum is released from rest at an angle of  $53^\circ$  with the vertical, and the bob collides *elastically* with the block. Following the collision, the block slides away and the pendulum swings back up to a maximum angle of  $5.73^\circ$  from the vertical. Each of the velocities requested below should be given with an appropriate sign in your chosen coordinate system to indicate direction.

- Find the velocity of the pendulum bob just as it strikes the block.
- Find the velocity of the pendulum bob just after it strikes the block.
- Find the velocity of the block after the collision.
- Find the unknown mass  $m$ .