Physics 131 Exam I Solutions Fall 2000

I. Multiple Choice Questions

1. Two stones are dropped from a cliff, the second one 1 sec. after the first. We can write their speeds as:

$$v_2 = -gt \quad , \quad v_1 = -g(t+1)$$

dropped first

After the first second (once the second rock is falling) the difference in their speeds is

$$v_1 - v_2 = -gt - g(1) + gt = g(1)$$

← no time dependence

(C) Stays the same.

2. The separation between the rocks is given by

$$x_2 = -\tfrac{1}{2}gt^2 \,, \quad x_1 = -\tfrac{1}{2}(g)(t+1)^2 \Rightarrow x_1 - x_2 = -\tfrac{1}{2}g(2t+1).$$

This grows as t increases, so (A) is correct.
Another way to see this is that if $v_1 > v_2$ always, the first stone will always travel farther in a second than stone 2

3. They each take the same time to fall, so they hit the ground exactly 1 second apart. (B)

4. Car A: the distance it travels in each second increases, so $v = \Delta x/\Delta t$ increases with time, or it accelerates

Car B: $\Delta x/\Delta t$ = const, so it stays at constant velocity.

Car C: $\Delta x/\Delta t$ decreases, so it decelerates. ⇒ (C) is correct

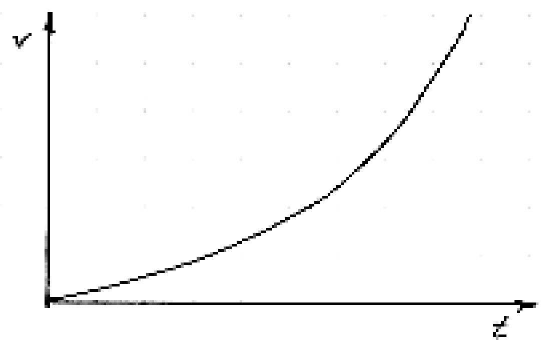
5. Without air resistance, the bombs fall straight down in the bomber reference frame, so to us on the ground they move at 620 km/h
Converting, $620\,\frac{km}{hr} \cdot \frac{1\,hr}{3600\,s} \cdot \frac{1000\,m}{1\,km} = 172$ m/s

So, each bomb travels a horizontal distance of $x = vt = (172\,m/s)(0.1\,s)$
$= 17.2$ m in 0.1 sec.

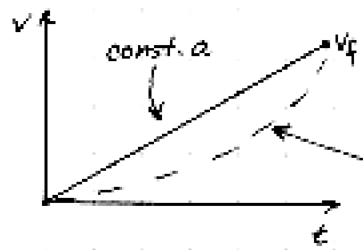
B is correct

Problems

II. A. We're told the space shuttle's acceleration constantly increases. This means that the slope of our v vs. t curve should also constantly increase, since $a = \Delta v/\Delta t$, and a is increasing. So, the graph should look like:



B. You can reason your way through this one in a couple of ways. First, imagine that $v(t) \propto t$, that is, constant acceleration. In this case, v vs. t looks like:



Here, the average value of v from $t=0$ to t_f is just given by the mean of v, or $(v_f - v_i)/2$. Now, add to the graph something like $v(t) \propto t^2$. It's easy to see right away that the mean value of v is less than th case of constant acceleration.

Another way to see this is that the shuttle spends more time at lower velocities before it gets to v_f than the case of consta acceleration, so the average v must be lower. Yet another way is to invoke some calculus. For $v(t) = at$,

$$\langle v \rangle = \frac{1}{t}\int_0^t at'\,dt' = \frac{1}{2t}at^2 = \frac{at}{2} = \frac{v_f}{2}$$

Note: the average vel is proprtional to the a under the curve - less the straight-line case.

for $v(t) = bt^2$, say, $\langle v \rangle = \frac{1}{t}\int_0^t bt'^2\,dt' = \frac{1}{3t}bt^3 = \frac{bt^2}{3} = v_f/3$

In fact, you can prove (probably!) that for $da/dt > 0$, $\langle v \rangle < v_f/2$ for any functional form of v.

No matter how you look at it, $\boxed{\langle v \rangle < v_f/2}$ if acceleration increas

Problems (cont.)

III (a) The length of time the electrons spend in the capacitor is just given by their (constant) horizontal velocity:

$$l = v_x t \Rightarrow 0.1\,m = (5\times10^6\,m/s)\,t \;,\quad t = 2.0\times10^{-8}\,sec.$$

(b.) The vertical deflection is just the distance the electrons are pushed by the accelerating force over the time they are in the capacitor:

$$y = \tfrac{1}{2}at^2 = \tfrac{1}{2}(5\times10^{13})(2\times10^{-8})^2 = .01\,m \quad \text{along } \vec{a}$$

IV A. In order for the bullet to hit the duck, both the x and y positions must coincide at some time t. (This is very similar to the monkey problem...) It's much easier to solve this with the x positions:

$$x_{duck} = vt \;,\qquad x_{bullet} = u\cos\theta\, t$$

In order to hit the duck, these must be equal at some time t':

$$vt' = u\cos\theta\, t' \Rightarrow \cos\theta = v/u \Rightarrow \boxed{\theta = \cos^{-1}(v/u)}$$

If this is true, the bullet always hits the duck as long as it has a maximum height greater than h.

B. We can prove the previous statement by now finding the time of impact and showing there is always a solution:
- the height of the bullet is given by $y = v_{0y}t - \tfrac{1}{2}gt^2$.
At the collision point,

$$h = u\sin\theta\, t - \tfrac{1}{2}gt^2 \Rightarrow \tfrac{1}{2}gt^2 - \underbrace{u\sin\theta}_{v_{0y}} + h = 0 \Rightarrow t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2gh}}{g}$$

This always has real solutions if $v_{0y}^2 \geq 2gh$, or $\frac{v_{0y}^2}{2g} > h$

But, $v_{0y}^2/2g$ is the maximum height achieved by a projectile (prove it!), so what was written in part A is true.

Now for numbers: $V = 20\,km/hr = 5.56\,m/s$, $\theta = \cos^{-1}\left(\frac{5.56}{300}\right) = 88.9°$

Problems (cont.)

IV. B (cont) So, $V_{oy} = u\sin\theta = 299.95\ m/s$

$$t = \frac{299.95 \pm 298.31}{g} \rightarrow \text{take early time} \Rightarrow \boxed{t = 0.17\ sec}$$

C. Since the duck has no vertical velocity to start with, the time it takes to fall can be obtained from

$$y = y_0 - \tfrac{1}{2}gt^2 \Rightarrow 0 = 50m - \tfrac{1}{2}(9.8\,m/s^2)t^2 \Rightarrow \boxed{t = 3.2\ sec}$$

V A. As always, the vertical and horizontal motions are independent. So, it takes exactly the same amount of time for the projectile to go up as it does to come down $\Rightarrow$ $\boxed{1:1 \text{ is correct}}$

If you wanted to prove this mathematically, you could do the following:

time to apex: $V_{fy} = 0 = V_{oy} - gt_{1/2}$, $t_{1/2} = \frac{V_{oy}}{g}$

time for total flight: $y_f = y_0 + V_{oy}t - \frac{1}{2}gt^2$

$$0 = 0 + V_{oy}t - \tfrac{1}{2}gt^2 \Rightarrow t = \frac{2V_{oy}}{g}, \text{ exactly } \underline{\text{twice}}\ t_{1/2}$$

B. In the total flight time $2V_{oy}/g = \frac{2v\sin\theta}{g} \equiv t_{tot}$, the projectile horizontal position is:

$$x = V_{ox}t_{tot} + \tfrac{1}{2}a_x t_{tot}^2 \equiv \text{Range}$$

$$= \frac{2V^2\cos\theta\sin\theta}{g} + \tfrac{1}{2}a_x\left(\frac{4v^2\sin^2\theta}{g^2}\right) = \frac{2V^2}{g}\sin\theta\left[\cos\theta + \frac{a_x\sin\theta}{g}\right]$$