Physics 131 Fall 2000 Exam 2 Solutions

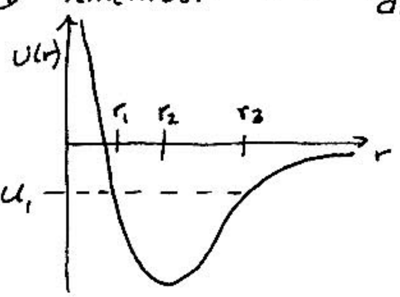
I. Multiple Choice

1. (B) We are told that a force is applied to a moving object in order to stop it. The Work-Energy Theorem tells us that $W = \Delta KE = \vec{F}\cdot\vec{d}$. $\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\frac{1}{2}mv_i^2$
$\vec{F}\cdot\vec{d} = -Fd$, So $-\frac{1}{2}mv_i^2 = -Fd$, $m = \frac{2Fd}{v_i^2} = 3\,kg$.

2. (B) Since the shopper is moving at constant velocity, the net force on him must be zero, so his apparent weight is unaffected.

3. (A) For each of the objects we know that the friction force must be equal to $\mu_k N = \mu_k mg$. So, F_k is proportional to m as well as to μ_k. In both cases, $F_k = ma = \mu_k mg$, so $a = \mu_k g \rightarrow$ masses cancel out. From kinematics, $v_f^2 = v_i^2 - 2ad \rightarrow$ 2nd block has half the acceleration, stops in 2d.

4. (A) Remember $F = -\frac{dU}{dr}$. As a particle is moved from r_1 towards $r = 0$, its potential energy increases, which means that if you let it go, it will move back towards positive r; this must mean the force points in the $+r$ direction. All of the other statements are true.

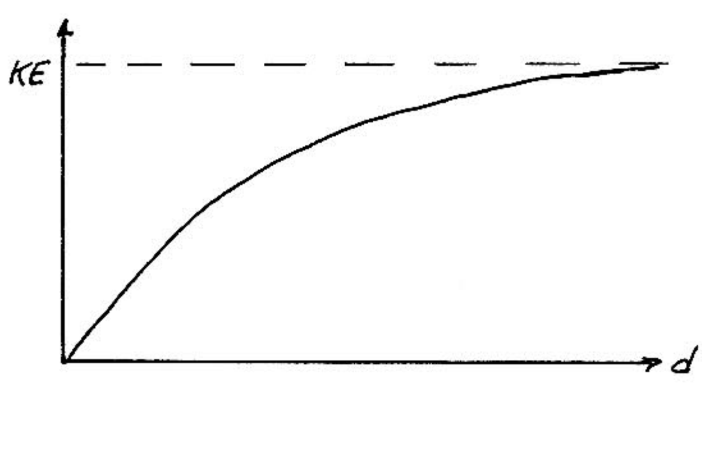


5. (D) $W = \vec{F}\cdot\vec{d}$. If $\vec{F}$ has a component opposite the displacement, then $\vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos\theta < 0$, since $\theta > 90°$, so you get negative work from this force.

Problems:

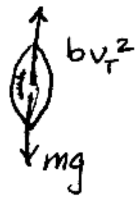
II.

(a)



The drag force opposes the motion, gradually building up until it exactly balances the downward pull of gravity. From this point on, $v = const.$, so $KE = const.$

II, (cont.) (b) We're told that $F = -bv^2$. At terminal velocity, the drag force just balances the downward pull of gravity, so that there is no net force on the ball $\Rightarrow$ $v = $ const.



$$\Sigma F_y = 0 = bv_T^2 - mg, \quad v_T^2 = mg/b, \quad v_T = \sqrt{mg/b}$$

Plugging in the numbers, we get $v_T = 12.1$ m/s

(c) The football takes <u>longer</u> to come down than to go up. The drag force is non-conservative and takes energy from the system (ball). This means that as the ball comes down, its kinetic energy at every point is <u>less</u> than it was at the same height coming up. Since $KE = \frac{1}{2}mv^2$, it's average <u>velocity</u> must also be less coming down, so it takes longer to cover the same distance.

III. (a) $P = W/\Delta t = \Delta KE/\Delta t = \frac{1}{2}mv_f^2/t$, where v_f is her velocity crossing the finish line, and t will be 10.75 seconds later on. This equation is valid at any time during her run, however. Rearranging, we get $\sqrt{\frac{2P}{m}}\, t^{1/2} = v$

(b) $v = dx/dt = \sqrt{\frac{2P}{m}}\, t^{1/2}$, $\quad dx = \sqrt{\frac{2P}{m}}\, t^{1/2} dt$, $\quad \int_0^{x_f} dx = \int_0^{t_f} \sqrt{\frac{2P}{m}}\, t^{1/2} dt$

(c.) $$\int_0^{x_f} dx = x_f = \sqrt{\frac{2P}{m}} \int_0^{t_f} t^{1/2} dt = \sqrt{\frac{2P}{m}} \left(\frac{2}{3} t^{3/2}\right)_0^{t_f} = \sqrt{\frac{8P}{9m}}\, t_f^{3/2} = x_f$$

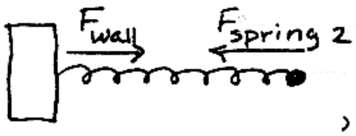
So, now we know a relationship between distance and time for constant power output. Given $x_f = 100$ m, $t_f = 10.75$ sec, $m = 60$ kg, we can solve for P, finding $P = 543$ Watts. Pretty impressive!

IV (a.) Assuming the plate stays vertical, the two springs are compressed the same amount. There are no non-conservative forces acting here, so the kinetic energy of the block will be entirely converted into potential energy stored in the springs: $KE_i + \underset{0}{\cancel{U_i}} = \underset{0}{\cancel{KE_f}} + U_f$

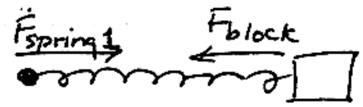
If we assume $U = 0$ for the uncompressed springs, $\frac{1}{2}mv^2 = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$, where x is the distance of compression.

$$mv^2 = x^2(k_1 + k_2) \Rightarrow x = v\sqrt{\frac{m}{k_1 + k_2}}$$

IV (cont.) (b.) In principle, this problem also uses conservation of energy, but we have to do a ~~little~~ work to figure out how much the springs are compressed. Consider the spring against the wall ("Spring 1") first. Its free body diagram looks like:



, the second spring:



Newton's 3rd law tells us that the springs push on each other with equal and opposite forces (their point of contact does not accelerate with respect to either spring), so $|\vec{F}_1| = |\vec{F}_2|$, or $k_1 x_1 = k_2 x_2$. The spring with the larger spring constant has to be compressed less to provide an equal restoring force. Now that we know this, we can go ahead and conserve energy:

$$KE_1 + U_1 = KE_2 + U_2 \Rightarrow \tfrac{1}{2}mv^2 = \tfrac{1}{2}k_1 x_1^2 + \tfrac{1}{2}k_2 x_2^2 = \tfrac{1}{2}k_1 x_1^2 + \tfrac{1}{2}k_2\left(\frac{k_1}{k_2}\right)^2 x_1^2,$$

where we substituted $x_2 = \frac{k_1}{k_2}x_1$.

So, $\frac{1}{2}mv^2 = \frac{1}{2}x_1^2\left[k_1 + \frac{k_1^2}{k_2}\right]$, $x_1^2 = \frac{mv^2}{k_1(1+k_1/k_2)}$, $x_1 = v\sqrt{\frac{m}{k_1(1+k_1/k_2)}}$

The total compression is equal to $x_1 + x_2 = x_1 + \frac{k_1}{k_2}x_1 = x_1\left(1+\frac{k_1}{k_2}\right) \equiv d$

$$d = \left(1+\frac{k_1}{k_2}\right)\cdot v\cdot\sqrt{\frac{m}{k_1(1+k_1/k_2)}} = v\sqrt{\frac{m}{k_1}\left(1+\frac{k_1}{k_2}\right)} = \boxed{v\cdot\sqrt{\frac{m}{k_1}+\frac{m}{k_2}}}$$

(c.) The force on the block is provided by the spring it's touching only. Assuming $k_1 < k_2$ and it's touching the spring with the smaller k, $F = k_1 x_1 = k_1 v\sqrt{\frac{m}{k_1(1+k_1/k_2)}}$

V (a.) You could do this with conservation of energy or projectile motion kinematics. We'll do cons. of E: $KE_1 + U_1 = KE_2 + U_2$, choose $U=0$ at horizon,

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_f^2 + U_2 \Rightarrow \tfrac{1}{2}\cancel{m}v_x^2 + \tfrac{1}{2}mv_y^2 = \tfrac{1}{2}\cancel{m}v_x^2 + mgh$$

$$h = \frac{v_y^2}{2g} = \frac{v_0^2\sin^2\theta}{2g}$$ } m cancels out!

(b.) At the highest point, $\vec{v} = v_x$, which has been constant $= v_0\cos\theta$

V. (cont.)(c) No matter how you look at it, the force due to gravity must give the acceleration of the warheads:

$$F = G\frac{M_E m_W}{r^2} = m_W a, \qquad a = \frac{GM_E}{r^2}$$

} still independent of m_W!

Looking at this from a projectile motion perspective, the acceleration from gravity decreases with increasing height. This implies that h, the maximum height, will be larger. The two objects still attain the same altitude with identical horizontal velocities.

(d) Since, for any two objects moving in a gravitational field, the acceleration is independent of mass, you can't tell which warhead to shoot down by looking at their trajectories. Score one debate point for physics...