Physics 131 Fall 2000 Exam 3 Solutions

I. Multiple Choice:

1. Conservation of momentum tells us that $p_i = p_f$, or
$0 = Mv_1 - 5Mv_2$, $\Rightarrow v_2 = \frac{1}{5} v_1$

The total energy is $E = \frac{1}{2}Mv_1^2 + \frac{1}{2}(5M)v_2^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}M\frac{v_1^2}{5}$

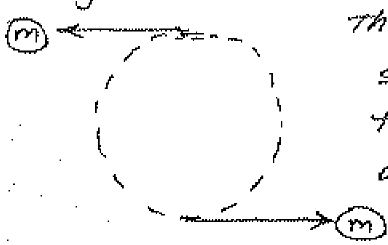
$E = \frac{6}{5}\left(\frac{1}{2}Mv_1^2\right) \Rightarrow M$ gets $\frac{\frac{1}{2}Mv_1^2}{\frac{6}{5}\left(\frac{1}{2}Mv_1^2\right)} = \frac{5}{6}$ of E

$\Rightarrow$ E

2. Conservation of momentum $\Rightarrow \vec{P}_{Earth} + \vec{P}_{chinese} = 0 \Rightarrow$ C

3. The force of gravity also attracts the earth to the chinese, so when they are about to land, the earth has equal and opposite momentum upwards $\Rightarrow \vec{P}_{earth} + \vec{P}_{chinese} = 0$ (is always 0)
$\Rightarrow$ C

4. Angular momentum is conserved as the masses fall:



The masses move along tangents to the circle, so they still have angular momentum, which they take with them. The angular momentum of the dizzy professor is also constant, so his rotational velocity doesn't change, since his moment of inertia is constant. $\Rightarrow$ B

5. A has the most mass farthest away from the axis.

II. Problems

II. 1.) Since there is no external force in the horizontal direction, the horizontal linear momentum must be conserved.

II (cont.) B.) $E_i = E_f \Longrightarrow mgR = mgr + \frac{1}{2}Mv_B^2 + \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$

Here, V_B is the velocity of the block, V_c is the velocity of the cylinder's CofM, and ω is the angular velocity of the cylinder. Note that the cylinder's CofM only falls by a distance $R-r$.

So, we have $\quad mg(R-r) = \frac{1}{2}Mv_B^2 + \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv_B^2 + \frac{1}{2}mv_c^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{v_c^2}{r^2} \leftarrow (v = r\omega)$$

$$mg(R-r) = \frac{1}{2}Mv_B^2 + \frac{3}{4}mv_c^2$$

Now, use cons. of momentum to relate V_B and V_c: $\quad 0 = P_i = -MV_B + mV_c = P_f$

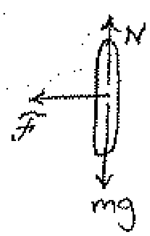
$MV_B = mV_c$, $V_B = \frac{m}{M}V_c$. $\longrightarrow$ put this into energy equa:

$$2\cancel{m}g(R-r) = \cancel{M}\frac{m^{\cancel{2}}}{M^{\cancel{2}}}v_c^2 + \frac{3}{2}\cancel{m}v_c^2 \Longrightarrow 2g(R-r) = v_c^2\left(\frac{m}{M} + 3/2\right)$$

$$V_c = \sqrt{\frac{2g(R-r)}{(m/M + 3/2)}} \quad , \quad V_B = \frac{m}{M}\sqrt{\frac{2g(R-r)}{(m/M + 3/2)}}$$

$\longrightarrow$ +x direction $\qquad\qquad$ −x direction

III. A.



B. The washer breaks free when the maximal centripetal force that can be provided by the gum is no longer sufficient to hold the washer on a circular path:

$$\sum F_r = \frac{mv^2}{d} = F_{max} \quad , \qquad F_{max} = \mu_s N = \mu_s mg$$

So, the maximum tolerable velocity² is $\quad v^2 = d\mu_s g$, $v = \sqrt{d\mu_s g}$

C. $v = d\omega = \sqrt{\mu_s dg} \quad , \quad \omega = \sqrt{\frac{\mu_s g}{d}}$

D. $W = \Delta KE = KE_f - KE_i = \frac{1}{2}I\omega^2 - 0$, $I = \underbrace{\frac{1}{3}ML^2}_{\text{rod}} + \underbrace{md^2}_{\text{washer}}$

$$KE_f = \frac{1}{2}\left(\frac{1}{3}ML^2 + md^2\right)\left(\frac{\mu_s g}{d}\right) = W$$

E. $\tau = I\alpha$, $\omega_f = \alpha T$, $\alpha = \frac{1}{T}\sqrt{\frac{\mu_s g}{d}} \Longrightarrow \tau = I\alpha = \frac{1}{T}\left(\frac{1}{3}ML^2 + md^2\right)\sqrt{\frac{\mu_s g}{d}}$

IV A.) Momentum is conserved, so $P_i = P_f \Rightarrow mv_i = (m+M)V_f$

$$V_f = \frac{m}{m+M} v_i = 5\ m/s$$

B.) Using conservation of energy: $(m+M)gh = \frac{1}{2}(m+M)V_f^2$

$$h = \frac{V_f^2}{2g} = 1.28\ m$$

C.) The motion of the center of mass is the <u>same</u> in both cases, so $V_f = 5\ m/s$.

D.) Similarly, $h = 1.28\ m$. If you need to be convinced, we can write $E_i = E_f \Rightarrow \frac{1}{2}(m+M)V_f^2 + \cancel{\frac{1}{2}I\omega^2} = (m+M)gh + \cancel{\frac{1}{2}I\omega^2}$

↳ rotational energy is constant, it cancels out.

E.) $KE_{block} = \frac{1}{2}(m+M)V_f^2$ (case 1) $KE_{block} = 75\ J$

$KE_{block} = \frac{1}{2}(m+M)V_f^2 + \frac{1}{2}I\omega^2$ (case 2) $KE_{block} = 75 + 81\ J$

↳ need to use cons. of angular momentum to find ω:

$L_i = \underbrace{mvd}_{(d=3cm)} = L_f = I\omega$, $\omega = \frac{mvd}{I} = 180\ Hz$

↳ $\frac{1}{2}I\omega^2 = 81\ J$

The kinetic energy of the bullet originally is $\frac{1}{2}mv^2 = 4500\ J$.
So, in case (1) $4500 - 75 = 4425\ J$ lost to friction
(2) $4500 - 156 = 4344\ J$ lost to friction

In case (2), less work is done by friction on the bullet, since extra energy is put into the block's rotation, so the bullet hole in case (2) is <u>shallower</u> than in case (1).

V. A.) In an elastic collision, momentum <u>and</u> energy are conserved.
So, $P_i = P_f$, $KE_i = KE_f$, $M = 3m_N$

$m_N v_i = MV - m_N v_f$ $\qquad$ $\frac{1}{2}m_N v_i^2 = \frac{1}{2}MV^2 + \frac{1}{2}m_N v_f^2$

↓ $\qquad$ ↓

$v_i = 3V - v_f$ $\qquad$ $v_i^2 - v_f^2 = 3V^2$

↓ $\qquad$ ↓

$v_i + v_f = 3V$ $\qquad$ $(v_i + v_f)(v_i - v_f) = 3V^2$

V (cont.) Now, divide the KE equation by the p eq'n:

$$\frac{(v_i + v_f)(v_i - v_f)}{(v_i + v_f)} = \frac{3V^2}{3V} \Rightarrow v_i - v_f = V$$

Putting this back into the momentum eq'n gives

$$v_i + v_f = 3(v_i - v_f) \qquad \left[\text{Remember, this is really } v_i + v_f = \frac{M}{m}(v_i - v_f)\right]$$

$$4v_f = 2v_i, \quad \underline{v_f = \tfrac{1}{2} v_i} \qquad \left[(M-m)v_i = (m+M)v_f, \quad \underline{v_f = \left(\frac{M-m}{m+M}\right) v_i} \right.$$

B. $KE_i = \frac{1}{2} m_N v_i^2, \quad KE_f = \frac{1}{2} m_N v_f^2 = \frac{1}{2} m_N \left(\frac{1}{4} v_i^2\right)$

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2} m_N v_i^2 \left(\frac{1}{4}\right)}{\frac{1}{2} m_N v_i^2} = \frac{1}{4} \qquad \left[\frac{v_f^2}{v_i^2} = \frac{KE_f}{KE_i} = \left(\frac{M-m}{m+M}\right)^2 \right]$$

C. For carbon, $M = 12 m_N \quad \frac{v_f^2}{v_i^2} = \left(\frac{12-1}{12+1}\right)^2 = 0.71$

∴ carbon is a less-efficient moderator

→ but it's much easier to deal with than deuterium!