Physics 131 Fall 2000 Final Exam Solutions

Multiple Choice:

1. Formally, $W_{force} = -\Delta U$, so gravity does positive work. Conceptually, in order for an object's gravitational potential energy to decrease, mgh must become smaller, so h must decrease. If a particle goes from h_1 to h_2, with $h_2 < h_1$, gravity does positive work, since the force is along the direction of the displacement, or $\vec{F} \cdot \vec{ds} > 0$. So, D is correct.

2. Draw a free-body diagram of the weight and its pulley:

For $\Sigma F_y = 0$, $2T - mg = 0$, or $T = mg/2 = F$

Numerically $F = 100/2 = 50\,N$ A.

3. Gravity provides the centripetal acceleration keeping the satellite in a circular orbit, so $F_{grav} = \frac{mv^2}{r} = \frac{mv^2}{R+h}$ C

4. The wavenumber k is given by $2\pi/\lambda$, and $\lambda = 20\,m$, so $k = \frac{2\pi}{20} = \frac{\pi}{10} = 0.314\ 1/m$ D.

5. $v = \lambda f = \omega/k = \frac{0.5}{\pi/10} = \frac{5}{\pi} = 1.59\ m/s$ C

6. The crest gets to $x = 5$ $T/4$ later, and $T = \frac{2\pi}{\omega} = 4\pi$, $T/4 = \pi$, so at time $t = 5\pi/4$ the crest reaches $x = 5$. Another way to see this is that the wave velocity is $5/\pi$ m/s, so it travels 5m in π seconds. B

7. If a force is acting on this particle, than, from $\vec{F} = m\vec{a}$, its acceleration $= d^2/dt^2 \neq 0$. So, for no force, $\vec{a} = 0$, or $\vec{v} = \frac{d}{dt} = \text{const.}$ The only choice which satifies this is A.

8. The definition of momentum is $F = \Delta P/\Delta t$, or $F\Delta t = \Delta p$. If both cars are pushed for the same time by the same force, their momenta will be the same. C.

Problems:

II. A.) $V_{car} = 100 \frac{km}{hr} \times \frac{1000m}{1km} \times \frac{1hr}{3600s} = 27.8 \; m/s$

$V_{throw} = 22.2 \; m/s$

$V_{turnip,g} = V_{car,g} + V_{throw,car} = 50 \; m/s$. (fast turnip.)

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(1)(50)^2 = 1250 \; J$

$p = mv = 50 \; kg \, m/s$

B.) $\Delta p = \text{Impulse} = F\Delta t$. If we use this formulation, F is the average force exerted by the victim on the turnip as it's being stopped. $F = \Delta p/\Delta t = 5000 \; N = 6.4$ times the victim's weight.

C.) For a completely inelastic collision, $mv_i = (m+M)v_f$, or

$$V_f = \frac{mv_i}{m+M} = \frac{1}{80+1}(50) = 0.62 \; m/s$$

III. A.) If we have $\tau = -K\theta$, then the energy stored as a function of θ is $U = \frac{1}{2}K\theta^2$. You can guess this directly from the form of the torque equation, or you can use $dW = \vec{\tau} \cdot d\vec{\theta}$ to get $\frac{1}{2}K\theta^2$, or you could use $-\frac{dU}{d\theta} = \tau$. In any case, we have $\theta = \pi/4$, so

$$W = \frac{1}{2}K\frac{\pi^2}{16} = \frac{K\pi^2}{32} = U_{spring}$$

B.) This potential energy becomes kinetic and gravitational energy at the release point:

$$U_{spring} = KE + U_{grav} \Rightarrow \frac{K\pi^2}{32} = KE + U_{grav} = KE + MgL\sin(45°),$$

so, $\boxed{KE = \frac{K\pi^2}{32} - MgL\frac{\sqrt{2}}{2}}$

C.) $KE = \frac{1}{2}MV_0^2 = \frac{K\pi^2}{32} - MgL\frac{\sqrt{2}}{2} = 2328 \; J$, $V_0^2 = 466 \; m^2/s^2$, $V_0 = 21.6 \; m/s$

$\Rightarrow V_{oy}^2 = 466 \times \sin^2(45°) = 233 \; m^2/s^2$. At the highest point in the trajectory, the y comp't of velocity $= 0 \Rightarrow \frac{1}{2}mv_{oy}^2 = mgh$, $h = \frac{V_{oy}^2}{2g} = 11.9 \; m$

The time needed to reach the top of the trajectory is given by

Problem III (cont.)

$0 = v_{f_y} = v_{0y} - gt, \quad t = v_{0y}/g = 1.56 \text{ sec.}$

To hit the ground again, the diaper bomb must fall an additional $2 \times \frac{\sqrt{2}}{2} = \sqrt{2}$ meters (the height of the bucket at release)

So, $d = \cancel{v_0 t}^{\,0} + \frac{1}{2} g t^2$ (+ is down) $\Rightarrow 11.9\,m + 1.41\,m = \frac{1}{2} g t^2$

$t^2 = 2.72 \text{ sec}^2, \quad \Rightarrow t = 1.65 \text{ sec.}$

So, the total flight time is $t_{tot} = 1.56 + 1.65 = 3.21$ sec.
In this time, the projectile travels $x = v_{0x} t_{tot} = \underline{49\ m}$.

→ not quite far enough! Maybe a lighter load next time...

IV A.) The angular momentum is given by $\vec{r} \times m\vec{v} = mvD$.

B.) $U = -\frac{GmM}{R}$

C.) KE can be obtained by getting v^2 from Newton's Laws:

$\sum F_r = F_{grav} = \frac{mv^2}{R}$ (centripetal force) $\Rightarrow \frac{G\cancel{m}M}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}}$

$v^2 = \frac{GM}{R}, \quad \underline{\frac{1}{2}mv^2 = \frac{GmM}{2R}}$

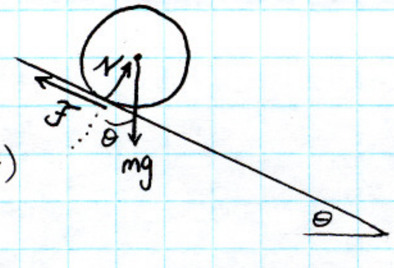
D.) From conservation of angular momentum, we can find the velocity at point C just before the rocket is fired:

$L_i = L_f$

$mv_i D = mv_f R, \quad v_f = \frac{D}{R} V.$

From part C, our final velocity needs to be $v = \sqrt{\frac{GM}{R}}$,
so $\Delta p = p_f - p_i = \underline{m\sqrt{\frac{GM}{R}} - mV\frac{D}{R}} < 0.$

V.



A)

B. Friction causes the rotation.
We can find F from Newton's Laws:

$\sum F_\perp = 0 = -mg\cos\theta + N \Rightarrow N = mg\cos\theta$

$F = \mu_s N = \mu_s mg\cos\theta \Rightarrow \tau_F = \mu_s mgR\cos\theta$

Problem V (cont.) C.) The ball will have more total kinetic energy, because in the case of rolling without slipping the friction force does no work, so no energy is lost. If the block slides with friction, some of the original potential energy is lost.

D. The acceleration of each object down the slope is given by $\Sigma F_x = ma_{cm} = mg\sin\theta - \mathcal{F}$. Since, in general, the coefficient of static friction is larger than the coefficient of kinetic friction, the rolling object will have a larger friction force and hence a smaller acceleration. Since $v_f = \sqrt{2ad}$ if an object is released from rest, this translates into a smaller linear momentum for the rolling object if they have the same mass. This assumes θ is sufficiently large. that $\mathcal{F} \approx \mu_s N$.

VI. A.) take the pivot point to be $h=0$. Then,

$$U = -m_1 g l_1 \sin\theta + m_2 g l_2 \sin\theta$$

B.) If $m_1 > m_2$, $U = -m_1 g l_1 \sin\theta + m_2 g l_2 \sin\theta$ is a minimum if $\theta = \pi/2$ $\rightarrow U = -m_1 l_1 g + m_2 l_2 g$.

Just to check, $\frac{dU}{d\theta} = -m_1 l_1 g\cos\theta + m_2 l_2 g \cos\theta = 0$

This can be true if $\cos\theta = 0$ $(\theta = \pi/2)$ or $m_1 l_1 = m_2 l_2$ (see part C). For $\theta = \pi/2$, we can see first of all that $-\frac{dU}{d\theta} = F = 0$, so that it is in fact an equilibrium. Since U is a minimum there, it must be true that it's stable.

C.) It's sufficient to look at $\Sigma \tau_{pivot} = 0 = m_1 l_1 g\cos\theta - m_2 l_2 g \cos\theta$. For this to be true for any θ,

$$m_1 g l_1 = m_2 g l_2 \implies m_1 l_1 = m_2 l_2$$

$$U = -m_1 l_1 g \sin\theta + m_2 l_2 g \sin\theta \equiv 0 \quad (= \text{const., really})$$

Given that $-\frac{dU}{dx} = 0$, this is a neutral equilibrium, i.e. there is no net force pulling on the system, nor a net torque, even if it's displaced from a given position.